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PRE-rg-CLOSED FUNCTIONS IN TOPOLOGY

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ABSTRACT

Levine in 1970, introduced the concept of generalized closed (g-closed) sets in topological space and a class of topological spaces called $T_{1/2}$-spaces. Palaniappan et al in 1993, introduced the notions of regular generalized (in brief, rg-) closed sets, rg-continuous functions and rg-irresoluteness and in 1997, Arokia Rani et al. introduced and studied the concepts of, rg-openness and rg-closedness in topology. In 2013, Navalagi et al have defined and studied the concepts of pre-rg - openness and other allied rg-open functions. The purpose of this paper is to investigate the concept of pre-regular generalized (in brief, pre-rg-) closed functions and some allied rg-closed functions in topological spaces. Also, we study some more properties of rg-closed functions.

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Key words and Phrases: *regular open sets, regular closed sets, g-closed sets and rg-closed sets, rg-openness, rg-closedness, rg-continuity, rg-irresoluteness, pre-rg-openness.*

1. INTRODUCTION

Levine [6] generalized the closed set to generalized closed set (in brief, g-closed set) in topology for the first time. Since then it is noticed that some of the weaker forms of closed sets have been generalized in the literature. Thereafter, several authors have obtained many interesting results on these generalized closed sets [cf. 2]. Palaniappan and Rao [12] introduced and studied the notions of regular generalized (in brief, rg-) closed sets, rg-continuous functions, rg-irresoluteness. In 1997, Arokia Rani *et al.* [1] have studied the further properties of rg-continuity, rg-irresoluteness, rg-open functions, rg-closed functions, rg-closure operator, strongly rg-continuous functions, perfectly rg-continuous functions, T_{rg} –spaces, $T^*_{½}$ -spaces and rg-normal spaces in topology. The purpose of this paper is to investigate some more properties of regular generalized closed (i.e. in brief, rg-closed) functions and other allied rg-closed functions in topology and introduce and study pre-rg-closed functions.

2. PRELIMINARIES

Throughout this paper (X, τ), (Y, σ), (Z, γ) (or simply X, Y and Z) always means topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of space X. We denote the closure of A and the interior of A by Cl(A) and Int(A) respectively. A subset A of a space X is called regular open (resp. regular closed) if A = Int Cl(A) (resp. A = Cl Int(A)).

Definition 2.1 [7]**:** A subset A of a space X is said to be a pre-open if $A\subset Int(Cl(A))$.

The family of all preopen sets in a space X is denoted by PO(X). The complement of a preopen set of a space X is called preclosed [4].

Definition 2.2 [8]**:** The union of all preopen sets contained in A is called the preinterior of A and is denoted by pInt(A)

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Definition 2.3[4]: The intersection of all preclosed sets containing A is called the preclosure of A and is denoted by pCl(A).

Definition 2.4 [5]: A subset A of a space X is called a generalized closed set (g-closed) set if $Cl(A)\subset U$ whenever $A \subset U$ and U is open.

Clearly, every closed set is a g-closed set. The complement of a g-closed set in X is called g-open set. Clearly, every open set is a g-open set.

Definition 2.5 [11]**:** A subset A of a space X is called a generalized preclosed set (in brief ,gp-closed) set if $pCl(A)\subset U$ whenever $A \subset U$ and U is open. The complement of a gp-closed set in X is called gp-open set.

It is obvious that every preopen set is gp-open set.

Definition 2.6 [12]**:** A subset A of a space X is called a regular generalized closed (rg-closed) set if $Cl(A)\subset U$ whenever $A\subset U$ and U is regular open. The complement of a rg-closed set of a space is called rg-open. The family of all rg-open sets of a space X is denoted by RGO(X).

Definition 2.7: A function $f: X \to Y$ is said to be :

i) preopen [8] if the image of each open set U of X, f(U) is preopen in Y.
ii) preclosed[4] if the image of each closed set F of X, f(F) is preclosed in Y.
iii) preirresolute [13] if the inverse image of each preopen set of Y is preopen in X.
iv) generalized preclosed(gp-closed[10]) if the image of each closed set of X is gp-closed in Y.

Definition 2.8: A function $f: X\to Y$ is called

i) rg-open [1] if image of each open set of X is rg-open in Y.
ii) rg-closed [1] if image of each closed set of X is rg-closed in Y.
iii) g-closed [6] if the image of each closed set of X is g-closed in Y.
iv) almost closed [14] if the image of each regular closed set of X is closed in Y.

Definition 2.9: A function $f: X \to Y$ is said to be :

i) rg-continuous [12] if the inverse image of each closed set of Y is rg-closed in X.
ii) rg- irresolute [12] if $f^{-1}(V)$ is rg-open in X for every rg- open set V of Y.
iii) perfectly rg-continuous [1] if the inverse image of each rg-open set in Y is both open and closed in X.

Definition 2.10[9]: A function $f: X \to Y$ is said to be M-preclosed if the image of each preclosed set of X is preclosed in Y.

3. Pre-rg – closed functions

In view of g-closedness and rg-closedness definitions, we can observe that every g-closed function is rg-closed function .

We, characterize the rg-closedness in the following.

Theorem 3.1: Let $f: X\to Y$ be a function. Then the following are equivalent:

i) f is rg-closed.
ii) The image of each closed set in X is rg-closed in Y.
iii) The image of each open set in X is rg-open in Y.

Proof:
(i) ⇔ (ii): Follows from Definition.

(i)⇔ (iii): Let F be any open set in X. Then $X- F$ is closed in X. Since f is rg-closed, $f(X- F)$ is rg-closed in Y. But $f(X- F) = Y- f(F)$ is rg-closed in X. Therefore f (F) is rg-open in Y.

We, define the following.

Definition 3.2: A function $f: X\to Y$ is called strongly rg-closed if the image of each rg-closed set of X is closed in Y.

Theorem 3.3: If $f : X\to Y$ is rg-closed and $g:Y\to Z$ is strongly rg-closed, then the composition $gof : X\to Z$ is closed .

Proof: Obvious.

Theorem 3.4: If f: $X\to Y$ is closed and g: $Y\to Z$ is rg-closed, then the composition gof : $X\to Z$ is rg-closed.

Easy proof is omitted.

We, recall the following.

Definition 3.5[1]: A topological space (X,τ) is said to be $T^*_{1/2}$ iff every rg-closed set is closed.

Theorem 3.6: Let X and Z be any topological spaces and Y be a $T^*_{1/2}$ space. Then the composition gof: $X\to Z$ of two rg-closed functions f : $X\to Y$ and g : $Y\to Z$ is also rg-closed.

Proof: Easy.

We now define the following.

Definition 3.7: A function f: $X\to Y$ is said to be pre-rg-closed if the image of each regular closed set of X is rg-closed in Y.

Clearly, every rg-closed function is pre-rg-closed.

Theorem 3.8: If a function f: $X \to Y$ is strongly rg-closed then it is an almost-closed .

Proof: Let F be any regular closed set in X. Then F is rg-closed in X since every regular open set is rg-open. Again, as f is strongly rg-closed and F is rg-closed in X, f(F) is an closed in Y. So, f is almost closed.

Theorem 3.9: A function f: $X \to Y$ is pre rg-closed iff the image of every regular open set in X is rg-open in X.

Proof: Easy.

We, define the following.

Definition 3.10: A function f: $X\to Y$ is said to be always rg-closed, if the image of each rg-closed set of X is rg-closed in Y.

Definition 3.11: A function f: $X\to Y$ is said to be completely rg-closed, if the image of each rg-closed set of X is regular closed in Y.

Theorem 3.12: If f: $X\to Y$ is always -rg-closed and g: $Y\to Z$ is completely rg-closed, then gof: $X\to Y$ is completely rg- closed.

Proof: Obvious.

Theorem 3.13: If f : $X\to Y$ is completely rg-closed and g: $Y\to Z$ is pre- rg- closed, then gof: $X\to Y$ is always rg-closed.

Proof: Let F be rg-closed set in X. Since f is completely rg-open, f(F) is regular closed in Y. Hence, g(f(F)) = gof(F) is rg-closed in Z as g is pre-rg-closed function. Therefore, gof is always rg- closed.

We, define the following.

Definition 3.14: A function f : $X\to Y$ is said to be prg-closed if image of each preclosed set of X is rg-closed in Y.

The following is a characterization of prg-open functions.

Theorem 3.15: Let f: $X\to Y$ be a function. Then the following are equivalent.

i) f is prg-closed .
ii) The image of each preclosed set in X is rg-closed in Y.
iii) The image of each preopen set in X is rg-open in Y.

Proof: Follows from the definition.

Theorem 3.16: If f: X→Y is M-preclosed and g: Y→Z is prg-closed then the composition gof : X →Z is prg-closed

Proof: Let F be preclosed set in X. Since f is M-preclosed, f(F) is preclosed in Y. Therefore g(f(F)) = gof (F) is rg-closed in Z since h is prg-closed. Hence, the composition function gof is prg-closed.

Theorem 3.17: If f: X→Y is preclosed and g: Y→Z is prg-closed, then the composition gof : X→Y is rg-closed.

Proof: Obvious.

We, define the following**.**

Definition 3.18: A function f: X→Y is called strongly p-closed if the image of each preclosed set of X is closed in Y.

Clearly, every strongly p-closed function is M-preclosed.

Theorem 3.19: If f: X→Y is prg-closed function and g: Y→Z is strongly rg-closed then the composition gof : X→Z is strongly p-closed .

Proof: Let F be preclosed set in X. Since f is prg-closed, f(F) is rg-closed in Y. Therefore g(f(F)) = gof(F) is closed in Z. Hence, gof is strongly p-closed function.

Theorem 3.20: If f: X→Y is strongly p-closed and g: Y→Z is rg-closed then the composition is gof : X→Z is prg-closed .

Proof: Obvious.

Theorem 3.21: If f: X→Y is rg-closed and g: Y→Z is always rg-closed then the composition gof:X → Z is rg-closed.

Proof: Let F be a closed set in X. Since f is rg-closed, f(F) is rg-closed. Therefore, g(f(F)) = gof(F) is rg-closed set in Z since g is always rg-closed function. Hence, gof is rg-closed .

Now, we define the following.

Definition 3.22: A function f: X→Y is said to be contra strongly rg-closed if the image of each rg-closed set of X is open in Y.
It is easy to see that a function f: X→Y is contra strongly rg-closed iff image of each rg-open set of X is closed in Y.

Theorem 3.23: If f: X→Y is contra-strongly rg-closed and g: Y→Z is contra-open then the composition gof :X→Z is contra-strongly rg-closed .

Proof: Obvious.

Definition 3.24: A function f: X→Y is said to be contra rg-closed if the image of each closed set of X is rg-open in Y.
Routine proof of the following Theorem is omitted.

Theorem 3.25: Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be two functions. Then, the following statements are valid:

(i) If f is preclosed and g is prg-closed, then gof is rg-closed.
(ii) If f is M-preclosed and g is strongly p-closed, then gof is strongly p-closed .
(iii) If f is prg-closed and g is always rg-closed, then gof is prg-closed.
(iv) If f is pre-rg-closed and g is strongly rg-closed, then gof is almost - closed.
(v) If gof is always rg-closed and f is rg-irresolute surjection, then g is always rg-closed
(vi) If gof is pre-rg-closed and f is completely rg-continuous, then g is always rg-closed .
(vii) If gof is strongly rg-closed and f is rg-continuous surjection, then g is closed .
(viii) If gof is prg-closed and f is precontinuous surjective, then g is rg-closed.
(ix) If gof is strongly rg-closed and f is rg-continuous surjection, then g is closed .
(x) If gof is M-preclosed and g is prg-continuous injection, then f is prg-closed .
(xi) If gof is always rg-closed and g is rg-irresolute injection, then f is always rg-closed.
(xii) If gof is pre-rg-closed and g is strongly rg-continuous injection, then f is almost oprn.
(xiii) If gof is contra rg-closed and g is rg-irresolute injection, then f is rg-closed.
(xiv) If gof is prg-closed and g is rg-irresolute injection, then f is prg-closed.
(xv) If gof is rg-closed and g is strongly rg-continuous injective, then f is open.

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