

**A FUZZY DECISION TREE MODEL TO  
USING IN PRIVATE TRAVELLING TRANSPORT SYSTEMS**

**M. VIJAYA**

**Head, PG and Research Department of Mathematics,  
Marudupandiyar College, Vallam. Thanjavur-613403.**

**M. ARTHI\***

**Ph.D. Scholar, Department of mathematics  
Marudupandiyar College, Vallam. Thanjavur-613403.**

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**ABSTRACT**

*In this paper deals about the fuzzy decision tree used to travelling transport systems.*

**Keywords:** *Fuzzy decision trees, Transport systems.*

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**1. INTRODUCTION**

The planning and management of urban public transport systems [6]. Such as bus networks [7], is a relevant problem that can be decomposed in a sequence of tasks, including the network design, frequency setting, timetable development, as well as bus and drivers scheduling [1], among others. Regarding bus network planning, a strong restriction for the bus system is the number of available vehicles. In this sense, researchers usually focus on the scenario of distributing a previously defined set of vehicles into a given number of bus lines in order to define a bus network. Proposals for this specific task usually consider one single objective function [10], such as the optimization of the average time a traveller waits for a service. The planning and management of urban public transport systems also involves a series of practical problems that are usually tackled by human experts. One of these problems is the redistribution of the buses of a bus network in case of mechanical problems in vehicles and absent drivers. For this specialized task, instead of preplanning the whole bus network if no extra vehicles or drivers exist, a simple solution involves studying the impact of reallocating buses or drivers from non -affected lines to affected ones.

Decision trees are popular models in machine learning, especially for classification problems, due to the fact that they produce graphical models, as well as text rules, that are easily understandable for final users. Moreover, their induction process is usually fast, requiring low computational power. Fuzzy systems, on the other hand, provide mechanisms to handle imprecision and uncertainty in data, based on the fuzzy logic and fuzzy sets theories [14][15]. The combination of fuzzy systems and decision trees[12][4][2][8][9] has produced fuzzy decision tree models, which benefit from both techniques to provide simple, accurate, and highly interpretable models at low computational cost.

In this sense, we tackle the bus network reallocation problem using a fuzzy decision tree model. The idea is to provide the human planner a support system to evaluate possible options. In other words, the decision tree can be used to decide the best bus line of a previously planned bus network to be modified in case a bus in the bus network breaks down or a driver is unable to work.

**2. FUZZY CLASSIFICATION SYSTEMS**

Classification is a relevant task of machine learning that can be applied to pattern recognition, decision making, and data mining, among others. The classification task can be roughly described as: given a set of objects  $E = \{e_1, e_2, \dots, e_n\}$ , also named examples or cases, which are described by  $m$  features, also named variables or attributes, assign a class  $C_i$  from a set of classes  $C = \{C_1, C_2, \dots, C_j\}$  to an object  $e_p$ ,  $e_p = (a_{p1}, a_{p2}, \dots, a_{pm})$ .

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**Corresponding Author: M. Arthi\***

**Ph.D. Scholar, Department of mathematics  
Marudupandiyar College, Vallam. Thanjavur-613403.  
E-mail: [arthib1992@gmail.com](mailto:arthib1992@gmail.com)**

Fuzzy classification systems are rule-based fuzzy systems that require the granulation of the features domain by means of fuzzy sets and partitions. The linguistic attributes in the antecedent part of the rules represent features, or attributes, and the consequent part represents a class. A typical fuzzy classification rule can be expressed by

$$R_k : \text{IF } X_1 \text{ is } A_{1l1} \text{ AND } \dots \text{ AND } X_m \text{ is } A_{mlm} \text{ THEN Class} = C_i$$

Where  $R_k$  is the rule identifier,  $X_1, \dots, X_m$  are the features of the example considered in the problem (represented by linguistic attributes),  $A_{1l1}, \dots, A_{mlm}$  are the linguistic values used to represent the feature values, and  $C_i \in C$  is the class. The inference mechanism compares the example to the rules in the fuzzy rule base in order to assign a class to the example.

The classic and general fuzzy reasoning methods [5] are widely used in the literature. Given a set of fuzzy rules (fuzzy rule base) and an input example, the classic fuzzy reasoning method classifies this input example using the class of the rule with maximum compatibility to the input example, while the general fuzzy reasoning method calculates the sum of compatibility degrees for each class and uses the class with highest sum to classify the input example. The classic fuzzy reasoning method is also known as the best rule method, while the general fuzzy reasoning method is also known as the best class method.

### 3. FUZZY DT

Decision trees also require low computational power and usually produce competitive models that can be expressed graphically or as a set of rules. Another important aspect of decision trees is the fact that their induction process selects only the relevant attributes for the definition of the final model. Thus, the process of inducing the decision tree model performs an embedded attribute selection process, which simplifies the final model, improving its interpretability.

In this work, we adopt the Fuzzy DT [4] algorithm to generate the fuzzy decision trees in order to support the task of bus rescheduling. Fuzzy DT uses the same measures of the classic C4.5 [11] algorithm, one of the most relevant and well-known decision tree algorithms, to decide on the importance of the features. Thus, Fuzzy DT uses the information gain and entropy measures to sequentially select the features to induce the models, which can be numerical and/or categorical. The entropy of a set  $S$  containing  $k$  possible classes is defined as [13]:

$$E(S) = - \frac{\sum_j^k \text{freq}(C_j S)}{|S|} \cdot \log_2 \frac{\text{freq}(C_j S)}{|S|}$$

Where  $\text{freq}(C_j; S)$  represents the number of examples in  $S$  that belongs to class  $C_j$  and  $|S|$  is the number of examples in  $S$ . The entropy indicates the average amount of information necessary to classify an example in  $S$ .

The information gain (or entropy reduction) of an attribute  $At_i$ , i.e., how much information is gained by splitting  $S$  using the values of  $At_i$ , can be defined as:

$$IG(S|At_i) = E(S) - E(S|At_i)$$

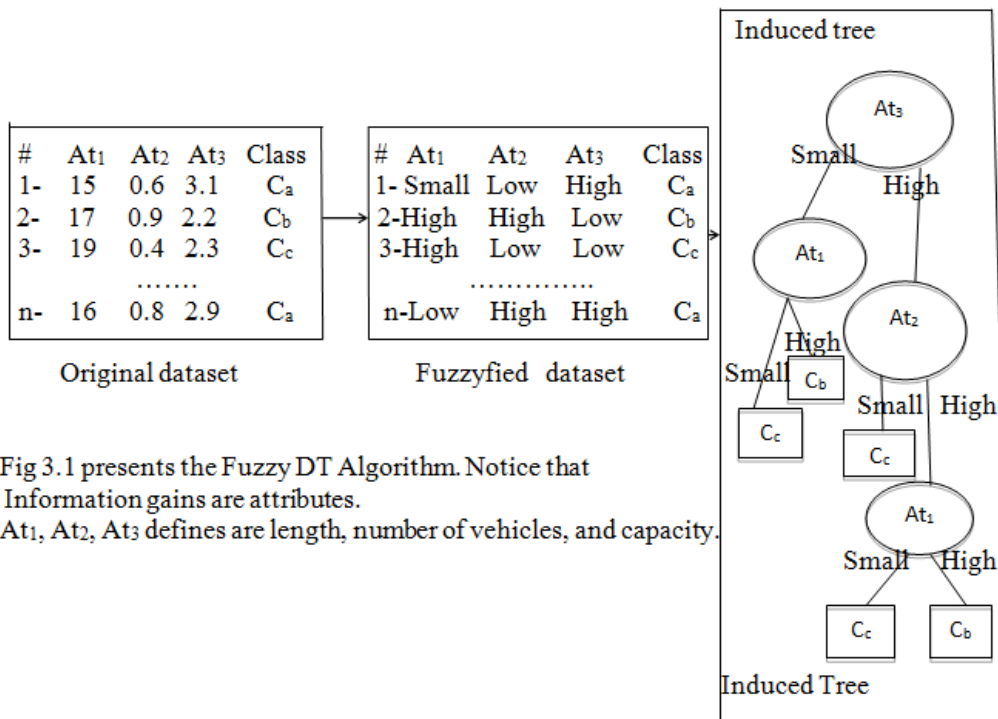


Fig 3.1 presents the Fuzzy DT Algorithm. Notice that Information gains are attributes. At<sub>1</sub>, At<sub>2</sub>, At<sub>3</sub> defines are length, number of vehicles, and capacity.

Figure-3.1: The Fuzzy DT algorithms

Fuzzy DT recursively creates branches corresponding to the values of the selected features until a class is assigned as a terminal node. Each branch of the tree can be seen as a rule, whose conditions are formed by their attributes and respective tests.

In order to avoid over fitting, Fuzzy DT adopts the same strategy of C4.5 of applying a post-pruning process. This way, the pruning process takes place after the tree is completely induced. The pruning process of Fuzzy DT basically assesses the error rates of the tree and its components directly on the set of training examples [4].

The post-pruning method implemented in Fuzzy DT replaces sub-trees with leaf nodes. The class assigned to a leaf is the most frequent one found in the examples of the training set covered by that leaf. This pruning method analyses the error rate of the tree using just the training examples with which the tree is built. The basic idea is to estimate the real error of a sub-tree, which, in fact, cannot be determined using only the examples of the training set. If the estimated real error is smaller than the apparent error, i.e., the error calculated using the set of training examples, the sub-tree is pruned.

The main steps of the Fuzzy DT algorithm to induce a fuzzy decision tree are listed next.

- Define the fuzzy data base, i.e., the fuzzy granulation for the domains of the continuous features;
- Replace the continuous attributes of the training set using the linguistic labels of the fuzzy sets with highest compatibility with the input values;
- Calculate the entropy and information gain of each feature to split the training set and define the test nodes of the tree until all features are used or all training examples are classified;
- Apply a post-pruning process, similarly to C4.5, using 25% confidence limits as default.

As the fuzzyfication of the training data is done before the induction of the tree, the third step of Fuzzy DT corresponds to the same step of the classic decision tree algorithm.

Figure 3.1 illustrates the process of data fuzzyfication and tree induction for a toy dataset with  $n$  examples, 3 attributes ( $At_1$ ,  $At_2$ , and  $At_3$ ), and 3 classes ( $C_a$ ,  $C_b$ , and  $C_c$ ). The first block of Figure 3.1 illustrates a dataset with  $n$  examples, three attributes ( $At_1$ ,  $At_2$ , and  $At_3$ ) and a class attribute.

The fuzzyfied version of this dataset is presented in the second block. This fuzzyfied set of examples is used to induce the final fuzzy decision tree, which is illustrated in the last block of Figure 3.1.

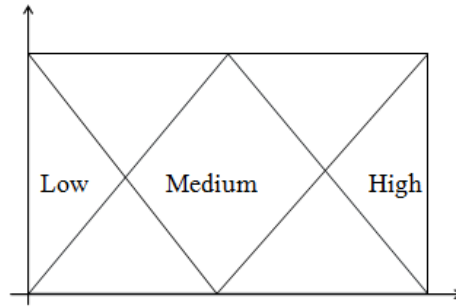
Next Section presents the experiments, as well as the attributes and data definition procedure and induced fuzzy decision trees for the task of bus rescheduling.

**Table-3.1:** General characteristics of the involved attributes

No.	Attribute	Min.	Max.	Avg.
1	Length (m)	7,610.00	30,900.00	16,583.14
2	Number of vehicles	2.00	12.00	7.22
3	Buses per Km	80.60	504.50	178.73
4	Interval between buses (seconds)	308.00	1,670.00	836.39
5	Rotation time (seconds)	2,822.00	7,378.00	4,584.51
6	Average speed (km/h)	9.70	17.00	12.89
7	Capacity	91.00	134.00	94.67
8	Number of travellers	225.00	2,716.00	1,137.73
9	Travellers per km	680.00	13,173.00	4,176.63
10	Average occupation rate	0.00	0.64	0.22
11	Maximum occupation rate	0.19	0.68	0.41
12	Commuting travellers	61.00	860.00	338.80
13	Average length of a route (km)	65.90	4,879.00	3,323.70
14	Average time (seconds)	497.00	1,121.00	864.51
15	Buses per km per travellers	0.10	0.39	0.21
16	Load	125.00	967.00	468.71
17	Average waiting time (seconds)	400.00	830.00	650.25

The generated decision tree is based on 52 real examples collected for 26 bus lines in the city of Grenoble, France. Table 3.1 presents the name of the 17 attributes included in the data, as well as their Minimum (Min), maximum (Max), and average (Avg.) values.

The input variables shown in the table 3.1 were used in order to induce a decision tree model that can be classifies the resulting average waiting time for a given bus service. Fig 3.1 presents the definition of the input variables in terms of fuzzy sets



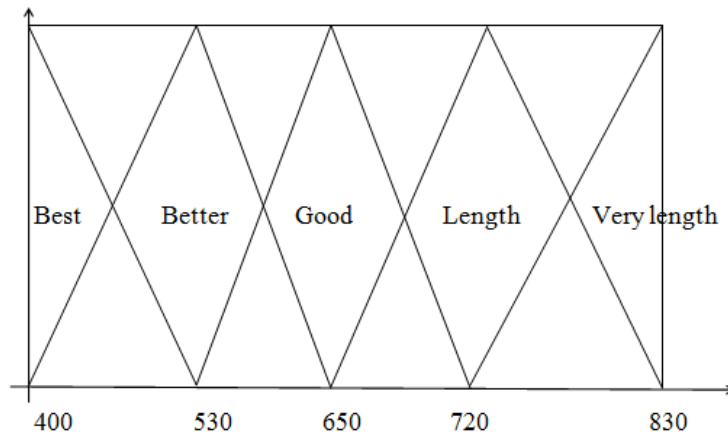
**Figure-3.1:** Definition of the input variables using 3 triangular fuzzy sets, evenly distributed in their domains.

#### 4. EXPERIMENTS

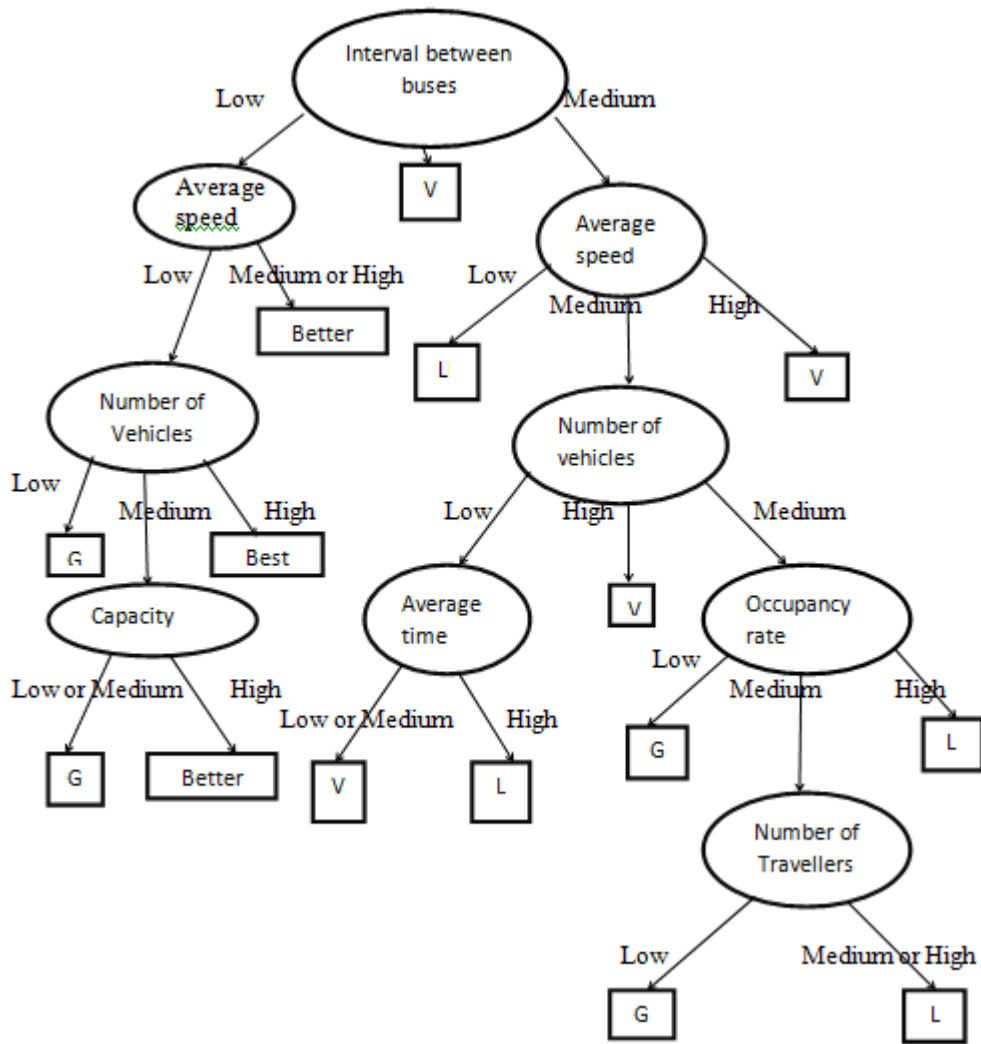
Most of the attributes are self-explanatory. Attribute Load refers to the number of travellers in the busiest section of the line. The Average Time attribute refers to the time taken to travel from the first to the last stop of a line. The Average Waiting Time attribute refers to the average time a traveller has to wait for a bus to arrive at any bus stop of the given bus line.

Each original attribute was used to define a linguistic attribute, according to the fuzzy logic theory. The linguistic attributes were defined by triangular shaped fuzzy sets, evenly distributed in the domains, according to the equalized universe method [3].

Specifically for the output attribute of the fuzzy decision tree, the Average Waiting Time, we adopted 5 triangular fuzzy sets, also evenly distributed in its domain, for all experiments. The linguistic values were chosen in order to reflect the adequacy of the waiting time of a passenger for a bus service. The set of linguistic values is composed of, Best, Better, Good, Length, and Very length.



**Figure-4.1:** Definition of the Average Waiting Time linguistic variable using 5 triangular fuzzy sets, evenly distributed in the domain, which ranges from 400 to 830 seconds, or 8 to 15 minutes.



**Figure-4.2:** Induced fuzzy decision tree to support the task of bus reallocation.

Such linguistic values properly define the Average Waiting Time and can be easily understood and interpreted by an expert in bus network planning.

- If Interval Between Buses is High then Average Waiting Time is Very Length
- If Interval Between Buses is Low & average speed is Medium or High then Average Waiting Time is Better
- If Interval Between Buses is Low & Average speed is Low and Number of vehicles is Low then Average Waiting Time is Good
- If Interval Between Buses is Low & Average speed is Low & Number of vehicles is High then Average Waiting Time is Best
- If Interval Between Buses is Low & Average speed is Low and Number of vehicles is Medium & Capacity is High Then Average Waiting Time is Better
- If Interval Between Buses is Low & Average speed is Low & Number of vehicles is Medium & Capacity is Low or Medium then Average Waiting Time is Good
- If Interval Between Buses is Medium & Average speed is Low then Average Waiting Time is Length
- If Interval Between Buses is Medium & Average speed is High then Average Waiting Time is Very Length
- If Interval Between Buses is Medium & Average speed is Medium & Number of vehicles is High then Average Time is Very Length
- If Interval Between Buses is Medium & Average speed is Medium & Number of vehicles is Low then Average Time is Low or Medium then Average Waiting Time is Very Length
- If Interval Between Buses is Medium & Average speed is Medium & Number of vehicles is Low then Average Time is High then Average Waiting Time is Length
- If Interval Between Buses is Medium & Average speed is Medium & Number of vehicles is Medium and Occupancy Rate is Low then Average Waiting Time is Good
- If Interval Between Buses is Medium & Average speed is Medium & Number of vehicles is Medium and Occupancy Rate is High then Average Waiting Time is Length

- If Interval Between Buses is Medium & Average speed is Medium & Number of vehicles is Medium and Occupancy Rate is Medium then Average Waiting Time is Good
- If Interval Between Buses is Medium & Average speed is Medium & Number of vehicles is Medium and Occupancy Rate is Medium and Number of Travellers is Medium or High then Average Waiting Time is Length

Although the previous set of rules can be used to evaluate the reallocation of buses, its graphical representation as a decision tree (Figure 4.2) is easier and more intuitive to make inferences about a real situation and, thus, take decisions.

## CONCLUSIONS

We present fuzzy decision tree models to support the reallocation of buses, specifically in case of buses breakdowns or absence of drivers. The models were induced using real data collected for a bus system with 26 bus lines. The 16 attributes involved in the induction process are related to a series of characteristics of the bus lines, such as their length and interval between buses, as well as characteristics of the travellers and the usage of the bus services, such as the average number of travellers, average number of commuting travellers, among others. Fuzzy DT was used to induce fuzzy decision trees. As the induced models use linguistic attributes, instead of continuous ones, the models become more intuitive and interpretable to human experts.

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