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# ARITHMETIC OPERATIONS ON SYMMETRIC OCTAGONAL INTUITIONISTIC FUZZY NUMBER

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***ABSTRACT***

*This paper introduces Symmetric Octagonal Intuitionistic Fuzzy Number (SOIFN) and its arithmetic operations. The arithmetic operations addition, subtraction, scalar multiplication and additive inverse are performed using (α, β) cuts. An example illustrates the proposed arithmetic operations of Symmetric Octagonal Intuitionistic Fuzzy Numbers(SOIFN).*

***Keywords:*** *Fuzzy set, Intuitionistic Fuzzy Set, Octagonal Intuitionistic Fuzzy Number (OIFN), Symmetric Octagonal Intuitionistic Fuzzy Number (SOIFN).*

## 1. INTRODUCTION

In real life, there are many situations, where it is impossible to get precise data for the cost parameters, due to complexity and uncertainty of information. Therefore, it is desirable to apply fuzzy sets to eliminate the impreciseness and vagueness..

The concept of fuzzy set was introduced by Zadeh [15] in 1965 and it dealt with imprecision, vagueness in real life situations. In 1970, Bellman & Zadeh [4] proposed the concept of decision making problems involving uncertainty and imprecision. K.T. Attanasov [1], [2], [3] introduced Intuitionistic fuzzy sets and their applications to deal with vagueness. W.L. Gau and D.J. Buehrer [7] discussed the concept of vague sets. Bustine and Burillo [5] insisted that vague sets are intuitionistic fuzzy sets. Dhanalakshmi .V, Felbin .C Kennedy, [6] proposed the ranking algorithm for symmetric octagonal fuzzy numbers. G. Menaka[8] and Rajarajeswari, G.Menaka [12] proposed a new approach for ranking of octagonal intuitionistic fuzzy numbers. G. Sahaya sudha, K.R. Vijayalakshmi [13] presented a value and ambiguity based ranking of a symmetric hexagonal intuitionistic fuzzy numbers in decision making. R. Parvathi, C. Malathi [10] listed the Arithmetic Operations on Symmetric Trapezoidal Intuitionistic Fuzzy Numbers using (α, β) cuts. A. Sahaya Sudha and R.Gokilamani [14] proposed an Arithmetic Operations on Hexadecagonal Fuzzy Numbers. K. Ponnivalavan and T. Pathinathan, [11] defined Intuitionistic Pentagonal Fuzzy Numbers and the Arithmetic Operations on Intuitionistic Pentagonal Fuzzy numbers. A. Mohammed Shapique [9] introduced the Heptagonal Fuzzy numbers and the Arithmetic Operations on Heptagonal Fuzzy Numbers using α cut method.

In this paper, SOIFN have been introduced. Arithmetic operations of SOIFN have been proposed using (α, β) cuts. A numerical example for each arithmetic operation is provided.

The rest of this paper organized as follows. In Section 2, definitions of fuzzy set, intuitionistic fuzzy set and OIFN are given. Section 3, presents introduction of SOIFN. In Section 4, arithmetic operations of SOIFN based on (α, β) cuts are proposed. A numerical example showing each arithmetic operation is illustrates in section5, followed by conclusion in section-6.

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## 2. DEFINITIONS

**Definition 2.1: Fuzzy Set**
Let A be a classical set, $\mu_A(x)$ be a function from A to [0, 1]. A fuzzy set A with the membership function $\mu_A(x)$ is defined as $A = \{x, \mu_A(x); x \in A \text{ and } \mu_A(x) \in [0,1]\}$.

**Definition 2.2: Intuitionistic Fuzzy Set**
Let *X* be a given set. An Intuitionistic Fuzzy Set *A* in *X* is given by, where A={( x, $\mu_A$(x),$\vartheta_A(x)$ ) | x ϵ X} $X \to [0, 1]$, where $\mu_A$(x) is the degree of membership of the element *x* in *A* and, $\vartheta_A(x)$ is the degree of non membership of *x* in *A* and $0 \le \mu_A(x) + \vartheta_A(x) \le 1$.

**Definition 2.3: Octagonal Intuitionistic Fuzzy Number (OIFN)**
An OIFN is specified by A= $[(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8); (a_1', a_2', a_3', a_4, a_5, a_6', a_7', a_8')]$
where $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_1', a_2', a_3', a_6', a_7', a_8'$ are real numbers and its membership and non-membership functions are given below.

$$\mu_A(x) = \begin{cases} 0 & if\ x < a_1 \\ k\left(\dfrac{x-a_1}{a_2-a_1}\right) & if\ a_1 \le x \le a_2 \\ k & if\ a_2 \le x \le a_3 \\ k+(1-k)\left(\dfrac{x-a_3}{a_4-a_3}\right) & if\ a_3 \le x \le a_4 \\ 1 & if\ a_4 \le x \le a_5 \\ k+(1-k)\left(\dfrac{a_6-x}{a_6-a_5}\right) & if\ a_5 \le x \le a_6 \\ k & if\ a_6 \le x \le a_7 \\ k\left(\dfrac{a_8-x}{a_8-a_7}\right) & if\ a_7 \le x \le a_8 \end{cases}$$

$$\vartheta_A(x) = \begin{cases} 1 & if\ a_1' \le x \\ k+(1-k)\left(\dfrac{a_2'-x}{a_2'-a_1'}\right) & if\ a_1' \le x \le a_2' \\ k & if\ a_2' \le x \le a_3' \\ k\left(\dfrac{a_4-x}{a_4-a_3'}\right) & if\ a_3' \le x \le a_4 \\ 0 & if\ a_4 \le x \le a_5 \\ k\left(\dfrac{x-a_5}{a_6'-a_5}\right) & if\ a_5 \le x \le a_6' \\ k & if\ a_6' \le x \le a_7' \\ k+(1-k)\left(\dfrac{x-a_7'}{a_8'-a_7'}\right) & if\ a_7' \le x \le a_8' \end{cases}$$

Where k =1/2

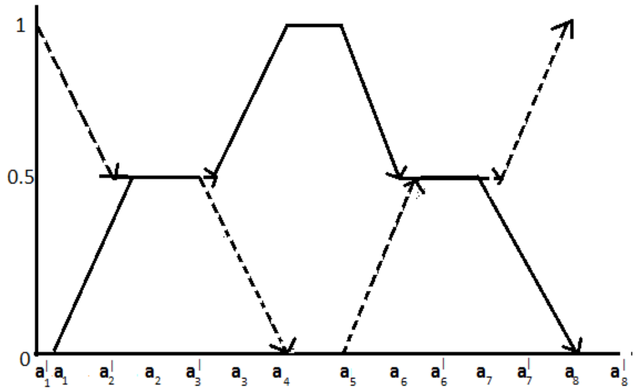


**Figure-1:** Diagrammatic Representation of OIFN

### 3. SYMMETRIC OCTAGONAL INTUITIONISTIC FUZZY NUMBER (SOIFN)

A **SOIFN** is given by $A_o = [(a_L - r - s - t, a_L - r - s, a_L - r,\ a_L,\ a_U,\ a_U + r,\ a_U + r + s,\ a_U + r + s + t)\ ;\ (a_L' - r' - s' - t',\ a_L' - r' - s',\ a_L' - r',\ a_L,\ a_U,\ a_U{}' + r', a_U{}' + r' + s',\ a_U{}' + r' + s' + t')]$ where $a_L - r - s - t, a_L - r - s, a_L - r, a_L, a_U,\ a_U + r, a_U + r + s, a_U + r + s + t, a_L' - r' - s' - t', a_L{}' - r' - s', a_L{}' - r', a_L, a_U, a_U{}' + r', a_U{}' + r' + s', a_U{}' + r' + s' + t'$ are real numbers such that $(a_L' - r' - s' - t' \leq a_L - r - s - t \leq a_L{}' - r' - s' \leq a_L - r - s \leq\ a_L{}' - r' \leq a_L - r \leq\ a_L \leq a_U \leq a_U + r \leq a_U{}' + r' \leq a_U + r + s \leq a_U{}' + r' + s' \leq a_U + r + s + t \leq a_U{}' + r' + s' + t')$ and its membership and non-membership functions are given below

$$\mu_{A_0}(x) = \begin{cases} \dfrac{x - (a_L - r - s - t)}{2t}, & a_L - r - s - t \leq x \leq a_L - r - s \\ \dfrac{1}{2}, & a_L - r - s \leq x \leq a_L - r \\ \dfrac{1}{2} + \dfrac{(x - (a_L - r))}{2r} & a_L - r \leq x \leq a_L \\ 1 & a_L \leq x \leq a_U \\ \dfrac{1}{2} + \dfrac{((a_U + r) - x)}{2r} & a_U \leq x \leq a_U + r \\ \dfrac{1}{2}, & a_U + r \leq x \leq a_U + r + s \\ \dfrac{\left((a_u + r + s + t) - x\right)}{2t}, & a_U + r + s \leq x \leq a_U + r + s + t \end{cases}$$

$$\vartheta_{A_0}(x) = \begin{cases} \dfrac{1}{2} + \dfrac{(a_L{}' - r' - s') - x}{2t'}, & a_L{}' - r' - s' - t' \leq x \leq a_L{}' - r' - s' \\ \dfrac{1}{2}, & a_L{}' - r' - s' \leq x \leq a_L{}' - r' \\ \dfrac{1}{2}\left(\dfrac{a_L - x}{a_L - (a_L' - r')}\right) & a_L{}' - r' \leq x \leq a_L \\ 0 & a_L \leq x \leq a_U \\ \dfrac{1}{2}\left(\dfrac{x - a_U}{(a_U{}' + r') - a_U}\right) & a_U \leq x \leq a_U{}' + r' \\ \dfrac{1}{2}, & a_U' + r' \leq x \leq a_U' + r' + s' \\ \dfrac{1}{2} + \left(\dfrac{\left(x - (a_U{}' + r' + s')\right)}{2t'}\right), & a_U{}' + r' + s' \leq x \leq a_U{}' + r' + s' + t' \end{cases}$$

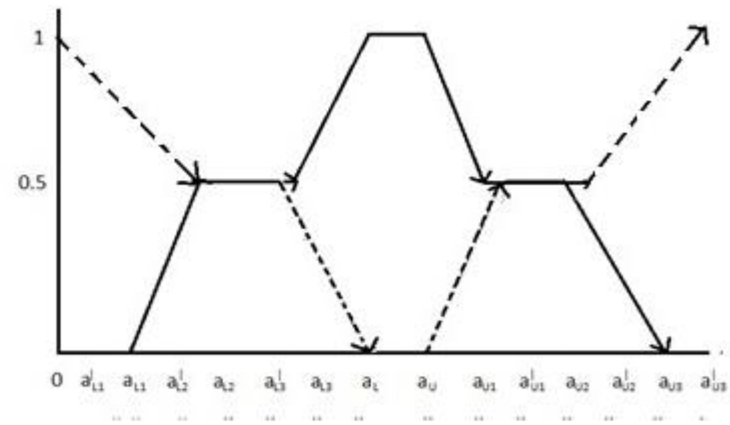


**Figure-2:** Diagrammatic representation of SOIFN

Where $a_{L1} = a_L - r - s - t\ , a_{L2} =\ a_L - r - s, a_{L3} = a_L - r, a_{U1} = a_U + r, a_{U2} = a_U + r + s, a_{U3} = a_U + r + s + t\ , a_{L1}' = a_L' - r' - s' - t', a_{L2}' = a_L' - r' - s', a_{L3}' = a_L' - r', a_{U1}' = a_U' + r', a_{U2}' = a_U' + r' + s', a_{U3}' = a_U' + r' + s' + t'$

**4. ARITHMETIC OPERATIONS ON SOIFN BY (α, β) CUTS**

**1. Addition:**
If $A_o = [(a_L - r_1 - s_1 - t_1,\ a_L - r_1 - s_1,\ a_L - r_1,\ a_L,\ a_U,\ a_U + r_1,\ a_U + r_1 + s_1,\ a_U + r_1 + s_1 + t_1);$
$(a_L' - r_1' - s_1' - t_1',\ a_L' - r_1' - s_1', a_L' - r_1',\ a_L,\ a_U,\ a_U' + r_1', a_U' + r_1' + s_1',\ a_U' + r_1' + s_1' + t_1')]$
$B_o$ =$[(b_L - r_2 - s_2 - t_2,\ b_L - r_2 - s_2,\ b_L - r_2,\ b_L,\ b_U,\ b_U + r_2, b_U + r_2 + s_2,\ b_U + r_2 + s_2 + t_2);$
$(b_L' - r_2' - s_2' - t_2',\ b_L' - r_2' - s_2',\ b_L' - r_2',\ b_L,\ b_U,\ b_U' + r_2', b_U' + r_2' + s_2',\ b_U' + r_2' + s_2' + t_2')]$ are two SOIFNs then $C_o = A_o + B_o$ ia also a SOIFNs and is given by $C_0 =(((a_L + b_L) - (r + s + t),\ (a_L + b_L) - (r + s),\ (a_L + b_L) - r, a_L + b_L, a_U + b_U, (a_U + b_U) + r, (a_U + b_U) + (r + s), (a_U + b_U) + (r + s + t)); ( (a_L' + b_L')(r' + s' + t'), (a_L' + b_L') - (r' + s'), (a_L' + b_L') - r', a_L + b_L, a_U + b_U, (a_U' + b_U') + r', (a_U' + b_U') + (r' + s'),\ (a_U' + b_U') + (r' + s' + t')))$
Where $r_1 + r_2 = r\ , s_1 + s_2 = s, t_1 + t_2 = t, t_1' + t_2' = t', s_1' + s_2' = s', r_1' + r_2' = r'$

**Proof:** The membership and non-membership function of IFS $C_o = A_o + B_o$ can be obtained by (α, β) cuts with the transformation z = x+y. α-cut for membership function of $A_o$ is $\left((a_L - r_1 - s_1 - t_1) + 2\alpha t_1\ , 2\alpha r_1 - r_1 + (a_L - r_1)\ , r_1 - 2\alpha r_1 + (a_U + r_1), (a_U + r_1 + s_1 + t_1) - 2\alpha t_1\right)$ for every α € [0,1]. That is x € $((a_L - r_1 - s_1 - t_1) + 2\alpha t_1\ , 2\alpha r_1 - r_1 + (a_L - r_1)\ , r_1 - 2\alpha r_1 + (a_U + r_1), (a_U + r_1 + s_1 + t_1) - 2\alpha t_1)$ . α-cut for membership function of $B_o\ is$ $((b_L - r_2 - s_2 - t_2) + 2\alpha t_2\ , 2\alpha r_2 - r_2 + (b_L - r_2),\ r_2 - 2\alpha r_2 + (b_U + r_2), (b_U + r_2 + s_2 + t_2) - 2\alpha t_2)$ for every α € [0,1]. That is y € $((b_L - r_2 - s_2 - t_2) + 2\alpha t_2\ , 2\alpha r_2 - r_2 + (b_L - r_2),\ r_2 - 2\alpha r_2 + (b_U + r_2), (b_U + r_2 + s_2 + t_2) - 2\alpha t_2)$. So z = (x+y) €$((a_L + b_L) - (r + s + t) + 2\alpha t\ , r(2\alpha - 1) + [(a_L + b_L) - r], r(1 - 2\alpha) + [(a_U + b_U) + r], [(a_U + b_U) + (r + s + t)] - 2\alpha t)$.
Where $r_1 + r_2 = r\ , s_1 + s_2 = s, t_1 + t_2 = t$.

Therefore the membership function of $C_o = A_o + B_o$ is given by

$$\mu_{C_0}(z) = \begin{cases} \dfrac{z - [(a_L + b_L) - (r + s + t)]}{2t}, & (a_L + b_L) - (r + s + t) \le z \le (a_L + b_L) - (r + s) \\ \dfrac{1}{2} & (a_L + b_L) - (r + s) \le z \le (a_L + b_L) - r \\ \dfrac{1}{2} + \dfrac{(z - ((a_L + b_L) - r))}{2r} & (a_L + b_L) - r \le z \le a_L + b_L \\ 1 & a_L + b_L \le z \le a_U + b_U \\ \dfrac{1}{2} + \dfrac{(((a_U + b_U) + r) - z)}{2r} & a_U + b_U \le z \le (a_U + b_U) + r \\ \dfrac{1}{2}, & (a_U + b_U) + r \le z \le (a_U + b_U) + r + s \\ \dfrac{\left(((a_U + b_U) + r + s + t) - z\right)}{2t}, & (a_U + b_U) + r + s \le z \le (a_U + b_U) + r + s + t \end{cases}$$

For non-membership function β-cut for $A_o'$ is $((t_1' - 2\beta t_1') + (a_L' - r_1' - s_1'),\ a_L - 2\beta\left(a_L - (a_L' - r_1')\right), a_U + 2\beta\left((a_U' + r_1') - a_U\right), 2\beta t_1' - t_1' + (a_U' + r_1' + s_1'))$ for every $\beta$€[0,1]. That is x € $\left((t_1' - 2\beta t_1') + (a_L' - r_1' - s_1'), a_L - 2\beta\left(a_L - (a_L' - r_1')\right),\ a_U + 2\beta\left((a_U' + r_1') - a_U\right), 2\beta t_1' - t_1' + (a_U' + r_1' + s_1')\right)$ β-cut for non membership function of $B_o'$ is $((t_2' - 2\beta t_2') + (b_L' - r_2' - s_2'),\ b_L - 2\beta\left(b_L - (b_L' - r_2')\right), b_U + 2\beta\left((b_U' + r_2') - b_U\right), 2\beta t_2' - t_2' + \left(b_U' + r_2' + s_2'\right))$ for every $\beta$€[0,1]. That is y € $((t_2' - 2\beta t_2') + (b_L' - r_2' - s_2'),\ b_L 2\beta\left(b_L - (b_L' - r_2')\right), b_U + 2\beta\left((b_U' + r_2') - b_U\right), 2\beta t_2' - t_2' + \left(b_U' + r_2' + s_2'\right))$. $so\ z = (x + y)$ € $((1 - 2\beta)t'(a_L' + b_L') - (r' + s'), (a_L + b_L) - 2\beta((a_L + b_L) - (a_L' + b_L') - r'), (a_U + b_U) + 2\beta((a_U' + b_U') + r' - (a_U + b_U), (2\beta - 1)t' + \left((a_U' + b_U') + (r' + s')\right))$
Where $t_1' + t_2' = t', s_1' + s_2' = s', r_1' + r_2' = r$.

Therefore the non-membership function of $C_o' = A_o' + B_o'$ is given by

 

$$\vartheta_{C_0'}(z) = \begin{cases} \frac{1}{2}+\frac{((a_L'+b_L')-(r'+s'))-z}{2t'}, & (a_L'+b_L')-(r'+s'+t') \le z \le (a_L'+b_L')-(r'+s') \\ \frac{1}{2}, & (a_L'+b_L')-(r'+s') \le z \le (a_L'+b_L')-r' \\ \frac{1}{2}\left(\frac{(a_L+b_L)-z}{(a_L+b_L)-((a_L'+b_L')-r')}\right) & (a_L'+b_L')-r' \le z \le a_L+b_L \\ 0 & a_L+b_L \le z \le a_U+b_U \\ \frac{1}{2}\left(\frac{z-(a_U+b_U)}{((a_U'+b_U')+r')-(a_U+b_U)}\right) & a_U+b_U \le z \le (a_U'+b_U')+r' \\ \frac{1}{2}, & (a_U'+b_U')+r' \le z \le (a_U'+b_U')+(r'+s') \\ \frac{1}{2}+\left(\frac{(x-[(a_U'+b_U')+(r'+s')])}{2t'}\right), & (a_U'+b_U')+(r'+s') \le z \le (a_U'+b_U')+(r'+s'+t') \end{cases}$$

**2. Subtraction**

If $A_o = [(a_L - r_1 - s_1 - t_1,\ a_L - r_1 - s_1,\ a_L - r_1,\ a_L,\ a_U,\ a_U + r_1,\ a_U + r_1 + s_1,\ a_U + r_1 + s_1 + t_1);(a_L' - r_1' - s_1' - t_1',\ a_L' - r_1' - s_1', a_L - r_1',\ a_L,\ a_U,\ a_U' + r_1', a_U' + r_1' + s_1',\ a_U' + r_1' + s_1' + t_1')\ ]$ $B_o = [(b_L - r_2 - s_2 - t_2,\ b_L - r_2 - s_2,\ b_L - r_2, b_L, b_U,\ b_U + r_2, b_U + r_2 + s_2,\ b_U + r_2 + s_2 + t_2);(b_L' - r_2' - s_2' - t_2', b_L' - r_2' - s_2', b_L r_2',\ b_L,\ b_U,\ b_U' + r_2', b_U' + r_2' + s_2',\ b_U' + r_2' + s_2' + t_2')]$ are two SOIFNs then $C_o = A_o - B_o$ ia also a SOIFNs and is given by

$C_o = (((a_L - b_U) - (r + s + t), (a_L - b_u) - (r + s), (a_L - b_U) - r, a_L - b_L, a_U - b_U, (a_U - b_L) + r, (a_U - b_L + (r + s), (a_U - b_L) + (r + s + t)); ((a_L' - b_U') - (r' + s' + t'), (a_L' - b_U') - (r' + s'), (a_L' - b_U') - r', a_L - b_L, a_U - b_U, (a_U' - b_L') + r', (a_U' - b_L') + (r' + s'), (a_U' - b_L') + (r' + s' + t')))$

Where $r_1 + r_2 = r$ , $s_1 + s_2 = s, t_1 + t_2 = t,\ t_1' + t_2' = t', s_1' + s_2' = s', r_1' + r_2' = r'$ .

**Proof:** The membership and non-membership function of IFS $C_o = A_o - B_o$ can be obtained by (α, β) cuts with the transformation z = x - y. α-cut for membership function of $A_o$ is $[(a_L - r_1 - s_1 - t_1) + 2\alpha t_1 , 2\alpha r_1 - r_1 + (a_L - r_1) , r_1 - 2\alpha r_1 + (a_U + r_1), (a_U + r_1 + s_1 + t_1) - 2\alpha t_1]$ for every α € [0,1]. That is x € $[(a_L - r_1 - s_1 - t_1) + 2\alpha t_1 , 2\alpha r_1 - r_2 + (a_L - r_1) , r_1 - 2\alpha r_1 + (a_U + r_1), (a_U + r_1 + s_1 + t_1) - 2\alpha t_1]$. α-cut for membership function of $B_o$ is $[(b_L - r_2 - s_2 - t_2) + 2\alpha t_2 , 2\alpha r_2 - r_2 + (b_L - r_2) , r_2 - 2\alpha r_2 + (b_U + r_2), (b_U + r_2 + s_2 + t_2) - 2\alpha t_2]$ for every α € [0,1]. That is y € $[(b_L - r_2 - s_2 - t_2) + 2\alpha t_2 , 2\alpha r_2 - r_2 + (b_L - r_2), r_2 - 2\alpha r_2 + (b_U + r_2), (b_U + r_2 + s_2 + t_2) - 2\alpha t_2]$ . So z = (x - y) € $[(a_L - b_U) - (r + s + t) + 2\alpha t , r(2\alpha - 1) + (a_L - b_U) - r , r(1 - 2\alpha) + (a_U - b_L) + r , (a_U - b_L) + (r + s + t) - 2\alpha t]$ Where $r_1 + r_2 = r$ , $s_1 + s_2 = s, t_1 + t_2 = t$.

Therefore the membership function of $C_o = A_o - B_o$ is given by

$$\mu_{C_0}(z) = \begin{cases} \frac{z-[(a_L-b_U)-(r+s+t)]}{2t}, & (a_L-b_U)-(r+s+t) \le z \le (a_L-b_U)-(r+s) \\ \frac{1}{2} & (a_L-b_U)-(r+s) \le z \le (a_L-b_U)-r \\ \frac{1}{2}+\frac{(z-((a_L-b_U)-r))}{2r} & (a_L-b_U)-r \le z \le a_L-b_L \\ 1 & a_L-b_L \le z \le a_U-b_U \\ \frac{1}{2}+\frac{((a_U-b_L)+r)-z)}{2r} & a_U-b_U \le z \le (a_U-b_L)+r \\ \frac{1}{2}, & (a_U-b_L)+r \le z \le (a_U-b_L)+(r+s) \\ \frac{((a_U-b_L)+(r+s+t))-z)}{2t}, & (a_U-b_L)+r+s \le z \le (a_U-b_L)+r+s+t \end{cases}$$

For non-membership function β-cut for $A_o'$ is $((t_1' - 2\beta t_1') + (a_L' - r_1' - s_1'),\ a_L - 2\beta(a_L - (a_L' - r_1')), a_U + 2\beta((a_U' + r_1') - a_U), 2\beta t_1' - t_1' + (a_U' + r_1' + s_1'))$ for every $\beta$€[0,1]. That is x €$((t_1' - 2\beta t_1') + (a_L' - r_1' - s_1'), a_L - 2\beta(a_L - (a_L' - r_1')), a_U + 2\beta((a_U' + r_1') - a_U), 2\beta t_1' - t_1' + (a_U' + r_1' + s_1'))$. β-cut for non membership function of $B_o'$ is $((t_2' - 2\beta t_2') + (b_L' - r_2' - s_2'), b_L - 2\beta(b_L - (b_L' - r_2')), b_U + 2\beta((b_U' + r_2') - b_U), 2\beta t_2' - t_2' + (b_U' + r_2' + s_2'))$ for every $\beta$€[0,1]. That is y €$((t_2' - 2\beta t_2') + (b_L' - r_2' - s_2'), b_L - 2\beta(b_L - (b_L' - r_2')), b_U + 2\beta((b_U' + r_2') - b_U), 2\beta t_2' - t_2' + (b_U' + r_2' + s_2'))$. *So* $z = (x - y)$€ $[(1 - 2\beta)t' + (a_L' - b_U') - (r' + s'), (a_L - b_L) - 2\beta((a_L - b_L) - (a_L' - b_U') - r'), (a_U - b_U) + 2\beta((a_U' - b_L') + r' - (a_U - b_U),\ (2\beta - 1)t' + ((a_U' - b_L') + (r' + s'))]$

Where $t_1' + t_2' = t', s_1' + s_2' = s', r_1' + r_2' = r'$

Therefore the non-membership function of $C_o' = A_o{}' - B_o{}'$ is given by

$$\vartheta_{C_0{}'}(z) = \begin{cases} \frac{1}{2} + \frac{((a_L' - b_U{}') - (r' + s')) - z}{2t'}, & (a_L' - b_U{}') - (r' + s' + t') \le z \le (a_L{}' - b_U{}') - (r' + s') \\ \frac{1}{2}, & (a_L' - b_U{}') - (r' + s') \le z \le (a_L{}' - b_U{}') - r' \\ \frac{1}{2}\left(\frac{(a_L - b_L) - z}{(a_L - b_L) - ((a_L' - b_U{}') - r')}\right) & (a_L' - b_U{}') - r' \le z \le a_L - b_L \\ 0 & a_L - b_L \le z \le a_U - b_U \\ \frac{1}{2}\left(\frac{z - (a_U - b_U)}{((a_U' - b_L{}') + r') - (a_U - b_U)}\right) & a_U - b_U \le z \le (a_U' - b_L{}') + r' \\ \frac{1}{2}, & (a_U' - b_L{}') + r' \le z \le (a_U{}' - b_L{}') + (r' + s') \\ \frac{1}{2} + \left(\frac{(z - (a_U' - b_L{}') + (r' + s'))}{2t'}\right), & (a_U' - b_L{}') + (r' + s') \le z \le (a_U{}' - b_L{}') + (r' + s' + t') \end{cases}$$

**3. Scalar Multiplication**

Let k € R if $A_o = [(a_L - r - s - t,\ a_L - r - s,\ a_L - r,\ a_L,\ a_U,\ a_U + r,\ a_U + r + s,\ a_U + r + s + t);$ $(a_L' - r' - s' - t',\ a_L' - r' - s', a_L - r',\ a_L,\ a_U,\ a_U' + r', a_U{}' + r' + s',\ a_U{}' + r' + s' + t')]$ is a SOIFN then $C_o = kA_0$ is also a SOIFN and is given by

$$C_o = \begin{cases} [k(a_L - r - s - t), k(a_L - r - s), k(a_L - r), ka_L, k\,a_U, k(a_U + r), k(a_U + r + s),\ k(a_U + r + s + t); \\ k(a_L' - r' - s' - t'), k(a_L' - r' - s'), k(a_L{}' - r'), ka_L,\ ka_U,\ k(a_U' + r'), k(a_U{}' + r' + s'), k(a_U{}' + r' + s' + t')]\ ;\ k > 0 \\ [-k(a_U + r + s + t), -k(a_U + r + s), -k(a_U + r), -ka_U, -ka_L, -k(a_U - r), -k(a_U - r - s), -k(a_U - r - s - t); \\ -k(a_U' + r' + s' + t'), -k(a_U' + r' + s'), -k(a_L{}' + r'), -ka_U, -ka_L, -k(a_U' - r'), -k(a_U{}' - r' - s'), -k(a_U{}' - r' - s' - t')]\ ;\ k < 0 \end{cases}$$

**Proof:**

**Case-(i):** k >0

The membership and non-membership function of IFS $C_o = kA_0$ can be obtained by (α, β) cuts with the transformation z = kx. α - cut for membership function of $A_o$ is $[(a_L - r - s - t) + 2\alpha t\ , 2\alpha r - r + (a_L - r)\ , r - 2\alpha r + (a_U + r), (a_U + r + s + t) - 2\alpha t]$ for every α € [0,1]. That is x € $[(a_L - r - s - t) + 2\alpha t\ , 2\alpha r - r + (a_L - r)\ , r - 2\alpha r + (a_U + r), (a_U + r + s + t) - 2\alpha t]$. So z = kx € $[k(a_L - r - s - t) + 2\alpha kt\ , 2k\alpha r - kr + k(a_L - r)\ , rk - 2k\alpha r + k(a_U + r), k(a_U + r + s + t) - 2k\alpha t]$Where $r_1 + r_2 = r\ , s_1 + s_2 = s, t_1 + t_2 = t.$

Therefore the membership function of $C_o = kA_0$ is given by

$$\mu_{C_0}(z) = \begin{cases} \frac{z - k(a_L - r - s - t)}{2kt}, & k(a_l - r - s - t) \le z \le k(a_L - r - s) \\ \frac{1}{2}, & k(a_l - r - s) \le z \le k(a_L - r) \\ \frac{1}{2} + \frac{(z - k(a_L - r))}{2kr} & k(a_L - r) \le z \le ka_L \\ 1 & ka_L \le z \le ka_U \\ \frac{1}{2} + \frac{(k(a_U + r) - z)}{2kr} & ka_U \le z \le k(a_U + r) \\ \frac{1}{2}, & k(a_U + r) \le z \le k(a_U + r + s) \\ \frac{(k(a_U + r + s + t) - z)}{2kt}, & k(a_U + r + s) \le z \le k(a_U + r + s + t) \end{cases}$$

For non-membership function β-cut for $A_o$ is $((t' - 2\beta t') + (a_L' - r' - s'),\ a_L - 2\beta(a_L - (a_L' - r')), a_U + 2\beta((a_U{}' + r') - a_U), 2\beta t' - t' + (a_U{}' + r' + s'))$ for every $\beta$€[0,1]. That is x € $((t' - 2\beta t') + (a_L' - r' - s'), a_L - 2\beta(a_L - (a_L' - r')), a_U + 2\beta((a_U{}' + r') - a_U), 2\beta t' - t' + (a_U' + r' + s'))$. So z = kx € $(k(t' - 2\beta t') + k(a_L' - r' - s'),\ ka_L - 2k\beta(a_L - (a_L' - r')), ka_U + 2k\beta((a_U{}' + r') - a_U), k(2\beta - 1)t' + k(a_U' + r' + s'))$

Therefore the non membership function of $C_o = kA_0$ is given by

$$\vartheta_{C_0}(z) = \begin{cases} \frac{1}{2} + \frac{k(a_L{}' - r' - s') - z}{2kt'}, & k(a_L{}' - r' - s' - t') \le z \le k(a_L{}' - r' - s') \\ \frac{1}{2}, & k(\,a_L' - r' - s') \le z \le k(a_L' - r') \\ \frac{1}{2}\left(\frac{ka_L - z}{k(a_L - (a_L' - r'))}\right) & k(\,a_L' - r') \le z \le ka_L \\ 0 & k\,a_L \le z \le ka_U \\ \frac{1}{2}\left(\frac{z - ka_U}{k((a_U{}' + r') - a_U)}\right) & k\,a_U \le z \le k(a_U' + r') \\ \frac{1}{2}, & k(\,a_U' + r') \le z \le k(a_U' + r' + s') \\ \frac{1}{2} + \left(\frac{(z - (a_U{}' + r' + s'))}{2kt'}\right), & k(a_U{}' + r' + s') \le z \le k(a_U{}' + r' + s' + t') \end{cases}$$

**Case-(ii):** k<0

The membership and non-membership function of IFS $C_o = -kA_0$ can be obtained by (α, β) cuts with the transformation z = -kx. α-cut for membership function of $A_o$ is $[(a_L - r - s - t) + 2\alpha t\,, 2\alpha r - r + (a_L - r)\,, r - 2\alpha r + (a_U + r), (a_U + r + s + t) - 2\alpha t]$for every α € [0,1]. That is x € $((a_L - r - s - t) + 2\alpha t\,, 2\alpha r - r + (a_L - r)\,, r - 2\alpha r + (a_U + r), (a_U + r + s + t) - 2\alpha t)$. So z = - kx € $(-k(a_L - r - s - t) - 2k\alpha t\,, -k(2\alpha r - r) - k(a_L - r)\,, -k(r - 2\alpha r) - k(a_U + r), -k(a_U + r + s + t) + 2k\alpha t)$.

Therefore the membership function of $C_o = -kA_0$ is given by

$$\mu_{C_0}(z) = \begin{cases} \frac{z - (-k(a_U + r + s + t))}{2kt}, & -k(a_U + r + s + t) \le z \le -k(a_U + r + s) \\ \frac{1}{2}, & -\,k(\,a_U + r + s) \le z \le -k(a_U + r) \\ \frac{1}{2} + \frac{z - (-k(a_U + r))}{2kr} & -\,k(a_U + r) \le z \le -ka_U \\ 1 & -ka_U \le z \le -ka_L \\ \frac{1}{2} + \frac{(-k(a_L - r) - z)}{2kr} & -ka_L \le z \le -k(a_L - r) \\ \frac{1}{2}, & -k(a_L - r) \le z \le -k(a_L - r - s) \\ \frac{-k(a_L - r - s - t) - z}{2kt}, & -\,k(\,a_L - r - s) \le z \le -k(a_L - r - s - t) \end{cases}$$

For non-membership function β-cut for $A_o$ is$(t' - 2\beta t') + (a_L' - r' - s'),\ a_L - 2\beta\big(a_L - (a_L' - r')\big), a_U + 2\beta\big((a_U{}' + r') - a_U\big), 2\beta t' - t' + (a_U' + r' + s')$ for every $\beta$€[0,1]. That is x €$((t' - 2\beta t') + (a_L' - r' - s'),\ a_L - 2\beta\big(a_L - (a_L' - r')\big), a_U + 2\beta\big((a_U' + r') - a_U\big), 2\beta t' - t' + \big(a_U{}' + r' + s'\big))$ . So z = -kx € $(-k(t' - 2\beta t') - k(a_L' - r' - s'), -ka_L + 2k\beta\big(a_L - (a_L{}' - r')\big), -ka_U - 2k\beta\big((a_U{}' + r') - a_U\big), -k(2\beta t' - t') - k(a_U{}' + r' + s'))$

Therefore the non membership function of $C_o = -kA_0$ is given by

$$\vartheta_{C_0}(z)=\begin{cases}\frac{1}{2}+\frac{(-k)(a_U'+r'+s')-z}{2kt'}, & -k(a_U'+r'+s'+t')\le z\le -k(a_U'+r'+s')\\ \frac{1}{2}, & -k(a_U'+r'+s')\le z\le -k(a_U'+r')\\ \frac{1}{2}\left(\frac{(-k)a_U-z}{k((a_U'+r')-a_U)}\right) & -k(a_U'+r')\le z\le -ka_U\\ 0 & -ka_U\le x\le -ka_L\\ \frac{1}{2}\left(\frac{z-(-k)a_L}{k(a_L-(a_L'-r'))}\right) & -ka_L\le z\le -k(a_L'-r')\\ \frac{1}{2}, & -k(a_L'-r')\le z\le -k(a_L'-r'-s')\\ \frac{1}{2}+\left(\frac{(z-(-k)(a_L'-r'-s'))}{2kt'}\right), & -k(a_L'-r'-s')\le z\le -k(a_L'-r'-s'-t')\end{cases}$$

**4. Additive Image**

If $A_o=[(a_L-r-s-t,\ a_L-r-s,\ a_L-r,\ a_L,\ a_U,\ a_U+r,\ a_U+r+s,\ a_U+r+s+t);(a_L'-r'-s'-t',\ a_L'-r'-s', a_L-r',\ a_L,\ a_U,\ a_U'+r', a_U'+r'+s',\ a_U'+r'+s'+t')]$ additive image of $A_o{}'$ is also a SOIFN and is given by $A_o{}'=(-(a_U+r+s+t),-(a_U+r+s),-(a_U+r),-a_U,-a_L,\ -(a_L-r),-(a_L-r-s),-(a_L-r-s-t)\ -(a_U{}'+r'+s'+t'),-(a_U{}'+r'+s'),(-(a_U{}'+r'),\ -a_U,-a_L,\ -(a_L{}'-r'),-(a_L{}'-r'-s'),-(a_L{}'-r'-s'-t')$

**Proof:** The membership and non-membership function of IFS $A_o'=-A_0$ can be obtained by (α, β) cuts with the transformation z = -x. α-cut for membership function of $A_o$ is $[(a_L-r-s-t)+2\alpha t\ ,2\alpha r-r+(a_L-r)\ ,r-2\alpha r+(a_U+r),(a_U+r+s+t)-2\alpha t]$ for every α € [0,1]. That is x € $((a_L-r-s-t)+2\alpha t\ ,2\alpha r-r+\ (a_L-r)\ ,r-2\alpha r+(a_U+r),(a_U+r+s+t)-2\alpha t)$. So z = -x € $[-(a_U+r+s+t)+\ 2\alpha t\ ,r(2\alpha-1)-(a_U+r)\ ,r(1-2\alpha)-(a_L-r)\ ,(a_L-r-s-t)-2\alpha t]$.

Therefore the membership function of $A_o'=-A_0$ is given by

$$\mu_{A_0{}'}(z)=\begin{cases}\frac{z-(-(a_U+r+s+t))}{2t}, & -(a_U+r+s+t)\le z\le -(a_U+r+s)\\ \frac{1}{2}, & -(a_U+r+s)\le z\le -(a_U+r)\\ \frac{1}{2}+\frac{z-(-(a_U+r))}{2r} & -(a_U+r)\le z\le -a_U\\ 1 & -a_U\le z\le -a_L\\ \frac{1}{2}+\frac{(-(a_L-r)-z)}{2r} & -a_L\le z\le -(a_L-r)\\ \frac{1}{2}, & -(a_L-r)\le z\le -(a_L-r-s)\\ \frac{(-(a_L-r-s-t)-z)}{2t}, & -(a_L-r-s)\le z\le -(a_L-r-s-t)\end{cases}$$

For non-membership function β-cut for $A_o$ is $((t'-2\beta t')+(a_L'-r'-s'),\ a_L-2\beta\big(a_L-(a_L'-r')\big),a_U+2\beta\big((a_U'+r')-a_U\big),2\beta t'-t'+\big(a_U{}'+r'+s'\big))$ for every $\beta$€[0,1]. That is x € $((t'-2\beta t')+(a_L'-r'-s'),a_L-2\beta\big(a_L-(a_L'-r')\big),a_U+2\beta\big((a_U'+r')-a_U\big),2\beta t'-t'+(a_U'+r'+s'))$. So z = -x € $((t'-2\beta t')-(a_U'+r'+s'),\ \ -a_U-2\beta\big((a_U'+r')-a_U\big),\ -a_L+2\beta\big(a_L-(a_L'-r')\big),(2\beta t'-t')-(a_L'-r'-s'))$

Therefore the non membership function of $A_o'=-A_0$ is given by

$$\vartheta_{A_0{}'}(z) = \begin{cases} \frac{1}{2} + \frac{-[(a'_U + r' + s') - z]}{2t'}, & -(a'_U + r' + s' + t') \le z \le -(a_U{}' + r' + s') \\ \frac{1}{2}, & -(a'_U + r' + s') \le z \le -(a'_U + r') \\ \frac{1}{2}\left(\frac{-a_U - z}{(a'_U + r') - a_U}\right) & -(a'_U + r') \le z \le -a_U \\ 0 & -a_U \le x \le -a_L \\ \frac{1}{2}\left(\frac{z - (-a_L)}{a_L - (a'_L - r')}\right) & -a_L \le z \le -(a'_L - r') \\ \frac{1}{2}, & -(a'_L - r') \le z \le -(a'_L - r' - s') \\ \frac{1}{2} + \left(\frac{(z - (-)(a'_L - r' - s'))}{2t'}\right) & -(a'_L - r' - s') \le z \le -(a'_L - r' - s' - t') \end{cases}$$

## 5. NUMERICAL EXAMPLE

Consider two SOIFNS A=[(7,9,11,13,15,17,19,21);(4,7,10,13,15,18,21,24)] and
B=[(8,10,12,14,16,18,20,22);(5,8,11,14,16,19,22,25)] then

**(i) Addition**

A+B = [(7+8, 9+10, 11+12, 13+14, 15+16, 17+18, 19+20, 21+22);(4+5 7+8, 10+11, 13+14, 15+16, 18+19, 21+22, 24+25)] = [(15,19,23,27,31,35,39,43);(9,15,21,27,31,37,43,49)]

**(ii) Subtraction**

A - B = [(7 - 22, 9 - 20, 11 – 18 , 13 - 14, 15 - 16, 17 - 12, 19 - 10, 21 - 8);(4 – 25, 7 - 22, 10 - 19, 13 - 14, 15 - 16, 18 -11, 21 - 8, 24 - 5)] = [(-15,-11,-7,-1,-1,5,9,13);(-21,-15,-9,-1,-1,7,13,19)]

**(iii) Scalar Multiplication**

**Case-(i):** If k > 0 . Let k =2
kA=[(14,18,22,26,30,34,38,42);(8,14,20,26,30,36,42,48)]

**Case-(ii):** If k < 0. Let k = -2
-kA = [(-42,-38,-34,-30,-26,-22,-18,-14);(-48,-42,-36,-30,-26,-20,-14,-8)]

**(iv) Additive Image**

- A= [(-21,-19,-17,-15,-13,-11.-9,-7);(-24,-21,-18,-15,-13,-10,-7,-4)]

## 6. CONCLUSION

In this paper we have introduced a new type of fuzzy number called SOIFN. The four arithmetic operations of SOIFN are obtained using (α, β) cuts. The SOIFN are used to solve fuzzy optimization problems and industrial applications concerning decision making in an uncertain environment.

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