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STUDY ON STRONG INTUITIONISTIC FUZZY GRAPHS OF SECOND TYPE AND THEIR PROPERTIES

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ABSTRACT

In this paper, we define some new operations like direct product, semi-strong product, strong product, cartesian product and composition of intuitionistic fuzzy graphs of second type and also define strong intuitionistic fuzzy graphs of second type. Further we establish some of their properties.

Keywords: *Intuitionistic fuzzy graphs, Intuitionistic fuzzy graphs of second type, direct, semi-strong, cartesian, composition, strong.*

2010 AMS Subject Classification: *05C72, 03E72.*

1. INTRODUCTION

Fuzzy sets were introduced by Lotfi. A. Zadeh [10] in 1965 as a generalisation of classical (crisp) sets. Further the fuzzy sets are generalised by Krassimir.T. Atanassov [1] in which he has taken non-membership values also into consideration. He introduced Intuitionistic fuzzy sets [IFS] and their extensions like Intuitionistic fuzzy sets of second type [IFSST], Intuitionistic L-fuzzy sets [ILFS] and Temporal Intuitionistic fuzzy sets [TIFS]. A. Shannon and K. T. Atanassov [6] discussed the theory of Intuitionistic fuzzy graphs. R. Parvathi and M. G. Karunambigai [3,4] introduced Intuitionistic Fuzzy Graphs [IFG] elaborately and analyzed its components also defined strong intuitionistic fuzzy graphs. After that they introduced and studied the operations cartesian product, composition on IFG. S. Ismail Mohideen, A. NagoorGani, B. Fathima Kani and C.Yasmin [2] discussed the properties of operations on regular IFG. Further Sankar Sahoo and Madhumangal Pal [5] defined and studied various operations like direct product, semi-strong product and strong product on IFG. The present authors [7,8,9] introduced the extension of IFG namely Intuitionistic Fuzzy Graphs of Second Type [IFGST] and defined some basic operations like union and join on IFGST. In section 2, we give some basic definitions and in section 3, we define some new operations direct product, semi-strong product, strong product, cartesian product and composition of IFGST. In section 4, we define the concept of strong intuitionistic fuzzy graphs of second type and establish some of their properties. The paper is concluded in section 5.

2. PRELIMINARIES

In this section, we give some basic definitions.

Definition 2.1: [3] An Intuitionistic Fuzzy Graph [IFG] is of the form $G = [V, E]$ where

(i) $V = \{v_1, v_2, \ldots, v_n\}$ such that $\mu_1: V \to [0,1]$ and $\nu_1: V \to [0,1]$ denote the degree of membership and nonmembership of the element $v_i \in V$, respectively, and $0 \le \mu_1(v_i) + \nu_1(v_i) \le 1$ for every $v_i \in V$, $(i = 1,2, \ldots n)$,

(ii) $E \subseteq V \times V$ where $\mu_2: V \times V \to [0,1]$ and $\nu_2: V \times V \to [0,1]$ are such that
$\mu_2(v_i, v_j) \le \min[\mu_1(v_i), \mu_1(v_j)]$,
$\nu_2(v_i, v_j) \le \max[\nu_1(v_i), \nu_1(v_j)]$
and $0 \le \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \le 1$
for every $(v_i, v_j) \in E$, $(i, j = 1,2, \ldots n)$.

Definition 2.2: [3] An IFG, $G = [V, E]$ is said to be a strong IFG if $\mu_{2ij} = \min(\mu_{1i}, \mu_{1j})$ and $\nu_{2ij} = \max(\nu_{1i}, \nu_{1j})$ for every $(v_i, v_j) \in E$.

Definition 2.3: [7] An Intuitionistic Fuzzy Graphs of Second Type [IFGST] is of the form $G = [V, E]$ where

(i) $V = \{v_1, v_2, \ldots, v_n\}$ such that $\mu_1 : V \to [0,1]$ and $\nu_1 : V \to [0,1]$ denote the degree of membership and nonmembership of the element $v_i \in V$, respectively, and $0 \le \mu_1(v_i)^2 + \nu_1(v_i)^2 \le 1$ for every $v_i \in V$, $(i = 1,2, \ldots n)$,

(ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \to [0,1]$ and $\nu_2 : V \times V \to [0,1]$ are such that
$\mu_2(v_i, v_j) \le \min[\mu_1(v_i)^2, \mu_1(v_j)^2]$,
$\nu_2(v_i, v_j) \le \max[\nu_1(v_i)^2, \nu_1(v_j)^2]$
and $0 \le \mu_2(v_i, v_j)^2 + \nu_2(v_i, v_j)^2 \le 1$
for every $(v_i, v_j) \in E$, $(i, j = 1,2, \ldots n)$.

3. OPERATIONS ON INTUITIONISTIC FUZZY GRAPHS OF SECOND TYPE

In this section, we define some new operations like direct product, semi-strong product, strong product, cartesian product and composition of IFGST with suitable examples.

Definition 3.1: Let $G_1 = [V_1, E_1]$ and $G_2 = [V_2, E_2]$ be two $IFGST$ then the direct product $G = G_1 \sqcap G_2$ of G_1 and G_2 is also an IFGST defined by,

(i) $(\mu_1 \sqcap \mu_1')(u_1, u_2) = \min(\mu_1(u_1), \mu_1'(u_2))$ $\quad for\ every\ (u_1, u_2 \in V)$
$\quad (\nu_1 \sqcap \nu_1')(u_1, u_2) = \max(\nu_1(u_1), \nu_1'(u_2))$ $\quad for\ every\ (u_1, u_2 \in V)$

(ii) $(\mu_2 \sqcap \mu_2')(u_1, u_2)(v_1, v_2) = \min(\mu_2(u_1 v_1), \mu_2'(u_2 v_2))$ $\quad for\ every\ u_1 v_1 \in E_1,\ \ u_2 v_2 \in E_2$
$\quad (\nu_2 \sqcap \nu_2')(u_1, u_2)(v_1, v_2) = \max(\nu_2(u_1 v_1), \nu_2'(u_2 v_2))$ $\quad for\ every\ u_1 v_1 \in E_1,\ \ u_2 v_2 \in E_2$

where $V = V_1 \times V_2$ and $E = \{(u_1, u_2)(v_1, v_2) : u_1 v_1 \in E_1,\ u_2 v_2 \in E_2\}$. Also (μ_1, ν_1), (μ_1', ν_1') are the vertex degree of membership and non-membership of the elements of V_1 in G_1 and V_2 in G_2 respectively and (μ_2, ν_2) and (μ_2', ν_2') are the edge degree of membership and non-membership of the elements of E_1 in G_1 and E_2 in G_2 respectively.

Example 3.1:

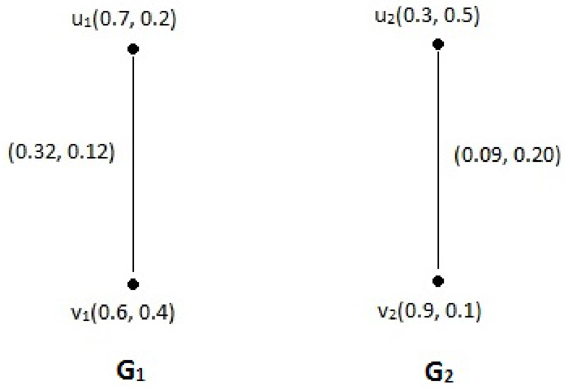

Figure-1: G_1 and G_2

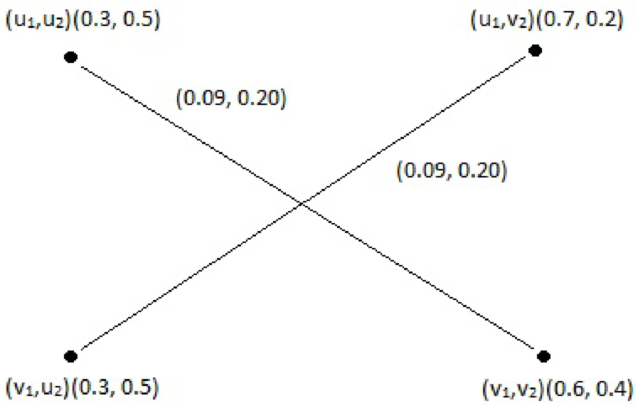

Figure-2: direct Product of G_1 and G_2 ($G_1 ó G_2$)

Definition 3.2: Let $G_1 = [V_1, E_1]$ and $G_2 = [V_2, E_2]$ be two $IFGST$ then the semi-strong product $G = G_1 \bullet G_2$ of G_1 and G_2 is also an IFGST defined by,

(i) $(\mu_1 \bullet \mu_1')(u_1, u_2) = \min(\mu_1(u_1), \mu_1'(u_2))$ $\quad for\ every\ (u_1, u_2 \in V)$
$\quad (\nu_1 \bullet \nu_1')(u_1, u_2) = \max(\nu_1(u_1), \nu_1'(u_2))$ $\quad for\ every\ (u_1, u_2 \in V)$
(ii) $(\mu_2 \bullet \mu_2')(u, u_2)(u, v_2) = \min(\mu_1^2(u), \mu_2'(u_2 v_2))$ $\quad for\ every\ u \in V_1, \quad u_2 v_2 \in E_2$
$\quad (\nu_2 \bullet \nu_2')(u, u_2)(u, v_2) = \max(\nu_1^2(u), \nu_2'(u_2 v_2))$ $\quad for\ every\ u \in V_1,\ u_2 v_2 \in E_2$
(iii) $(\mu_2 \bullet \mu_2')(u_1, u_2)(v_1, v_2) = \min(\mu_2(u_1 v_1), \mu_2'(u_2 v_2))$ $\quad for\ every\ u_1 v_1 \in E_1,\ u_2 v_2 \in E_2$
$\quad (\nu_2 \bullet \nu_2')(u_1, u_2)(v_1, v_2) = \max(\nu_2(u_1 v_1), \nu_2'(u_2 v_2))$ $\quad for\ every\ u_1 v_1 \in E_1,\ u_2 v_2 \in E_2$

where $V = V_1 \times V_2$ and $E = \{(u, u_2)(u, v_2): u \in V_1,\ u_2 v_2 \in E_2\} \cup \{(u_1, u_2)(v_1, v_2): u_1 v_1 \in E_1,\ u_2 v_2 \in E_2\}$. Also (μ_1, ν_1), (μ_1', ν_1') are the vertex degree of membership and non-membership of the elements of V_1 in G_1 and V_2 in G_2 respectively and (μ_2, ν_2) and (μ_2', ν_2') are the edge degree of membership and non-membership of the elements of E_1 in G_1 and E_2 in G_2 respectively.

Example 3.2:

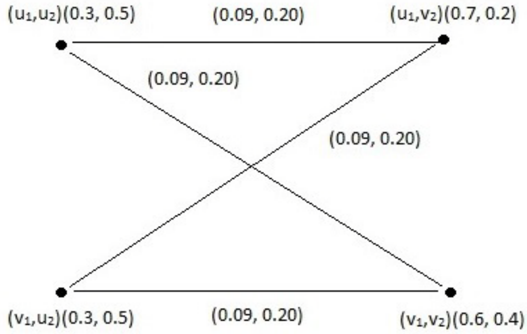

Figure-3: semi-strong product of G_1 and G_2 $(G_1 \bullet G_2)$

Definition 3.3: Let $G_1 = [V_1, E_1]$ and $G_2 = [V_2, E_2]$ be two $IFGST$ then the strong product $G = G_1 \otimes G_2$ of G_1 and G_2 is also an IFGST defined by,

(i) $(\mu_1 \otimes \mu_1')(u_1, u_2) = \min(\mu_1(u_1), \mu_1'(u_2))$ $\quad for\ every\ (u_1, u_2 \in V)$
$\quad (\nu_1 \otimes \nu_1')(u_1, u_2) = \max(\nu_1(u_1), \nu_1'(u_2))$ $\quad for\ every\ (u_1, u_2 \in V)$
(ii) $(\mu_2 \otimes \mu_2')(u, u_2)(u, v_2) = \min(\mu_1^2(u), \mu_2'(u_2 v_2))$ $\quad for\ every\ u \in V_1, \quad u_2 v_2 \in E_2$
$\quad (\nu_2 \otimes \nu_2')(u, u_2)(u, v_2) = \max(\nu_1^2(u), \nu_2'(u_2 v_2))$ $\quad for\ every\ u \in V_1,\ u_2 v_2 \in E_2$
(iii) $(\mu_2 \otimes \mu_2')(u_1, w)(v_1, w) = \min(\mu_2(u_1 v_1), (\mu_1')^2(w))$ $\quad for\ every\ w \in V_2, \quad u_1 v_1 \in E_1$
$\quad (\nu_2 \otimes \nu_2')(u_1, w)(v_1, w) = \max(\nu_2(u_1 v_1), (\nu_1')^2(w))$ $\quad for\ every\ w \in V_2,\ u_1 v_1 \in E_1$
(iv) $(\mu_2 \otimes \mu_2')(u_1, u_2)(v_1, v_2) = \min(\mu_2(u_1 v_1), \mu_2'(u_2 v_2))$ $\quad for\ every\ u_1 v_1 \in E_1,\ u_2 v_2 \in E_2$
$\quad (\nu_2 \otimes \nu_2')(u_1, u_2)(v_1, v_2) = \max(\nu_2(u_1 v_1), \nu_2'(u_2 v_2))$ $\quad for\ every\ u_1 v_1 \in E_1,\ u_2 v_2 \in E_2$

where $V = V_1 \times V_2$ and $E = \{(u, u_2)(u, v_2): u \in V_1,\ u_2 v_2 \in E_2\} \cup \{(u_1, w)(v_1, w): w \in V_2,\ u_1 v_1 \in E_1\} \cup \{(u_1, u_2)(v_1, v_2): u_1 v_1 \in E_1,\ u_2 v_2 \in E_2\}$. Also (μ_1, ν_1), (μ_1', ν_1') are the vertex degree of membership and non-membership of the elements of V_1 in G_1 and V_2 in G_2 respectively and (μ_2, ν_2) and (μ_2', ν_2') are the edge degree of membership and non-membership of the elements of E_1 in G_1 and E_2 in G_2 respectively.

Example 3.3:

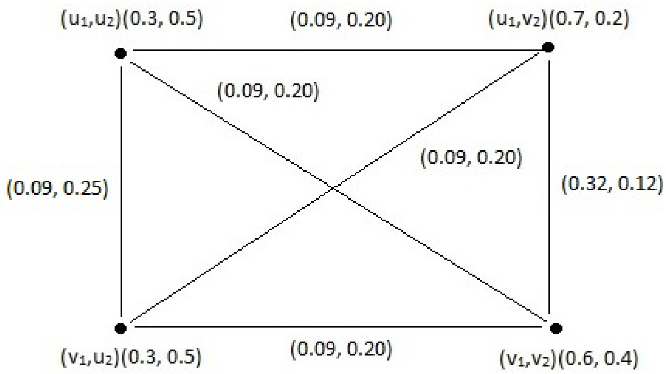

Figure-4: strong product of G_1 and G_2 $(G_1 \otimes G_2)$

Definition 3.4: Let $G_1 = [V_1, E_1]$ and $G_2 = [V_2, E_2]$ be two $IFGST$ then the cartesian product $G = G_1 \times G_2$ of G_1 and G_2 is also an IFGST defined by,

(i) $(\mu_1 \times \mu_1')(u_1, u_2) = \min(\mu_1(u_1), \mu_1'(u_2))$ $for\ every\ (u_1, u_2 \in V)$
$(\nu_1 \times \nu_1')(u_1, u_2) = \max(\nu_1(u_1), \nu_1'(u_2))$ $for\ every\ (u_1, u_2 \in V)$

(ii) $(\mu_2 \times \mu_2')(u, u_2)(u, v_2) = \min(\mu_1^2(u), \mu_2'(u_2 v_2))$ $for\ every\ u \in V_1,\quad u_2 v_2 \in E_2$
$(\nu_2 \times \nu_2')(u, u_2)(u, v_2) = \max(\nu_1^2(u), \nu_2'(u_2 v_2))$ $for\ every\ u \in V_1,\ u_2 v_2 \in E_2$

(iii) $(\mu_2 \times \mu_2')(u_1, w)(v_1, w) = \min(\mu_2(u_1 v_1), (\mu_1')^2(w))$ $for\ every\ u_1 v_1 \in E_1,\quad w \in V_2$
$(\nu_2 \times \nu_2')(u_1, w)(v_1, w) = \max(\nu_2(u_1 v_1), (\nu_1')^2(w))$ $for\ every\ u_1 v_1 \in E_1,\ w \in V_2$

where $V = V_1 \times V_2$ and $E = \{(u, u_2)(u, v_2): u \in V_1,\ u_2 v_2 \in E_2\} \cup \{(u_1, w)(v_1, w): u_1 v_1 \in E_1,\ w \in V_2\}$. Also (μ_1, ν_1), (μ_1', ν_1') are the vertex degree of membership and non-membership of the elements of V_1 in G_1 and V_2 in G_2 respectively and (μ_2, ν_2) and (μ_2', ν_2') are the edge degree of membership and non-membership of the elements of E_1 in G_1 and E_2 in G_2 respectively.

Example 3.4:

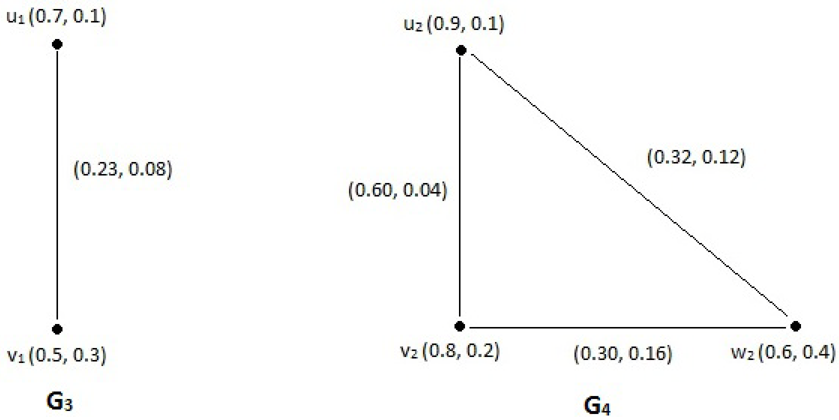

Figure-5: G_3 and G_4

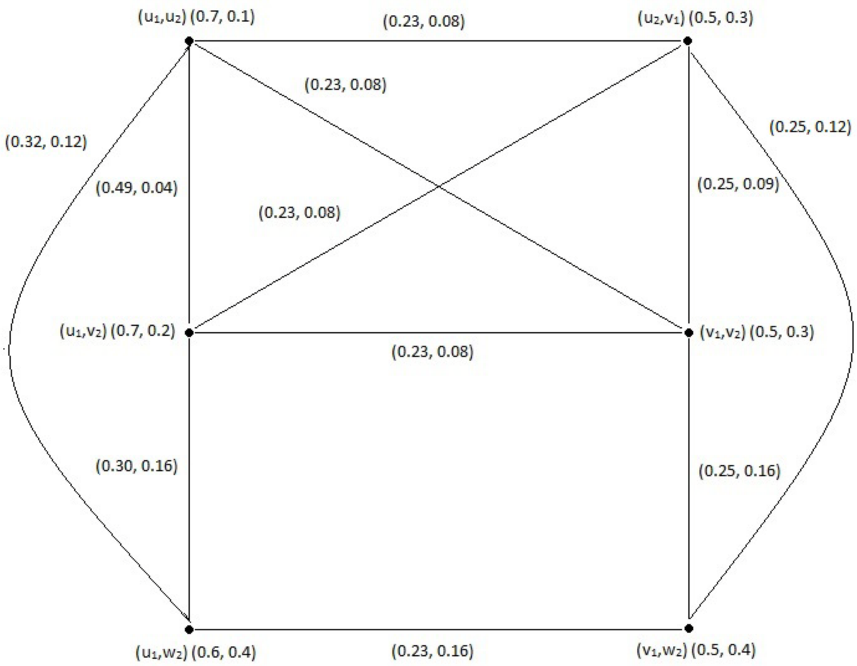

Figure-6: cartesian product of G_3 and G_4 $(G_3 \times G_4)$

Definition 3.5: Let $G_1 = [V_1, E_1]$ and $G_2 = [V_2, E_2]$ be two $IFGST$ then the composition $G = G_1 \circ G_2$ of G_1 and G_2 is also an IFGST defined by,

(i) $(\mu_1 \circ \mu_1')(u_1, u_2) = \min(\mu_1(u_1), \mu_1'(u_2))$ $for\ every\ (u_1, u_2 \in V)$
$(\nu_1 \circ \nu_1')(u_1, u_2) = \max(\nu_1(u_1), \nu_1'(u_2))$ $for\ every\ (u_1, u_2 \in V)$

(ii) $(\mu_2 \circ \mu_2')(u, u_2)(u, v_2) = \min(\mu_1^2(u), \mu_2'(u_2 v_2))$ $for\ every\ u \in V_1,\quad u_2 v_2 \in E_2$
$(\nu_2 \circ \nu_2')(u, u_2)(u, v_2) = \max(\nu_1^2(u), \nu_2'(u_2 v_2))$ $for\ every\ u \in V_1,\ u_2 v_2 \in E_2$

(iii) $(\mu_2 \circ \mu_2')(u_1, w)(v_1, w) = \min(\mu_2(u_1 v_1), (\mu_1')^2(w))$ $for\ every\ u_1 v_1 \in E_1,\quad w \in V_2$
$(\nu_2 \circ \nu_2')(u_1, w)(v_1, w) = \max(\nu_2(u_1 v_1), (\nu_1')^2(w))$ $for\ every\ u_1 v_1 \in E_1,\ w \in V_2$

(iv) $(\mu_2 \circ \mu_2')(u_1,u_2)(v_1,v_2) = \min(\mu_2(u_1v_1), (\mu_1')^2(u_2), (\mu_1')^2(v_2))$ $for\ every\ u_1v_1 \in E_1,$ $u_2,v_2 \in V_2,$ $u_2 \neq v_2$

$(\nu_2 \circ \nu_2')(u_1,u_2)(v_1,v_2) = \max(\nu_2(u_1v_1), (\nu_1')^2(u_2), (\nu_1')^2(v_2))$ $for\ every\ u_1v_1 \in E_1, u_2,v_2 \in V_2, u_2 \neq v_2$ where $V = V_1 \times V_2$ and $E = \{(u,u_2)(u,v_2): u \in V_1,\ u_2v_2 \in E_2\} \cup \{(u_1,w)(v_1,w): u_1v_1 \in E_1,\ w \in V_2\} \cup \{(u_1,u_2)(v_1,v_2): u_1v_1 \in E_1,\ u_2,v_2 \in V_2,\ u_2 \neq v_2\}$. Also (μ_1,ν_1), (μ_1',ν_1') are the vertex degree of membership and non-membership of the elements of V_1 in G_1 and V_2 in G_2 respectively and (μ_2,ν_2) and (μ_2',ν_2') are the edge degree of membership and non-membership of the elements of E_1 in G_1 and E_2 in G_2 respectively.

Example 3.5:

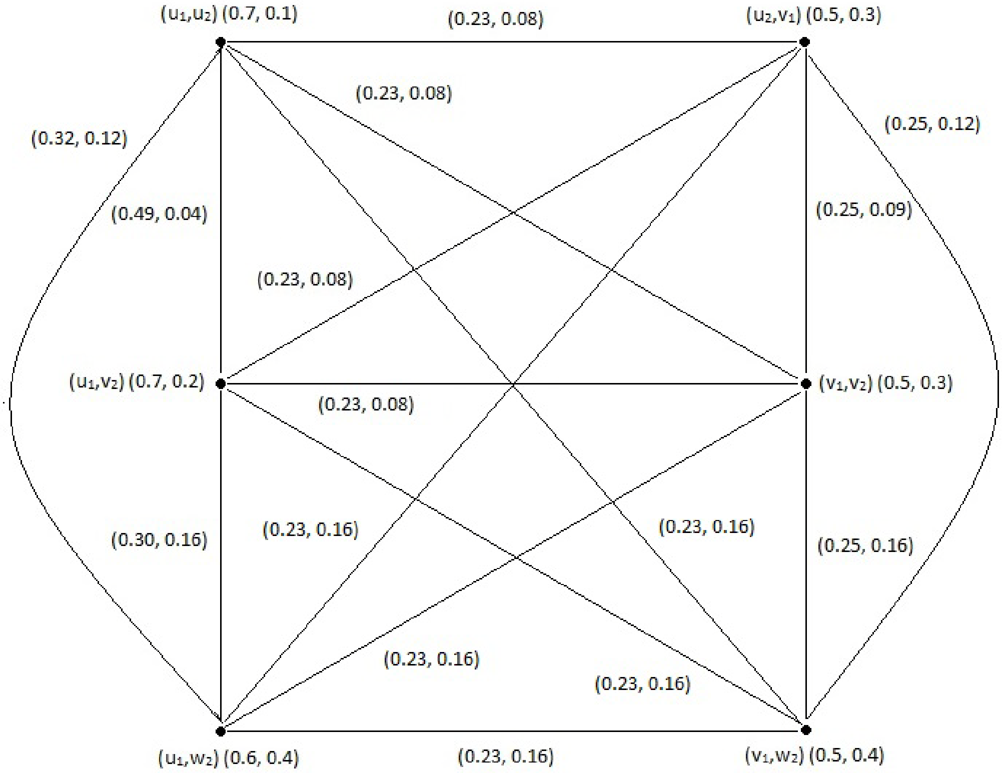

Figure-7: composition of G_3 and G_4 $(G_3 \circ G_4)$

4. PROPERTIES OF STRONG INTUITIONISTIC FUZZY GRAPHS OF SECOND TYPE

In this section, we define strong intuitionistic fuzzy graphs of second type and establish some of their properties based on newly defined operations.

Definition 4.1: An IFGST, $G = [V,E]$ is said to be a strong IFGST if $\mu_{2ij} = \min(\mu_{1i}^2, \mu_{1j}^2)$ and $\nu_{2ij} = \max(\nu_{1i}^2, \nu_{1j}^2)$ for every $(v_i,v_j) \in E$

Example 4.1:

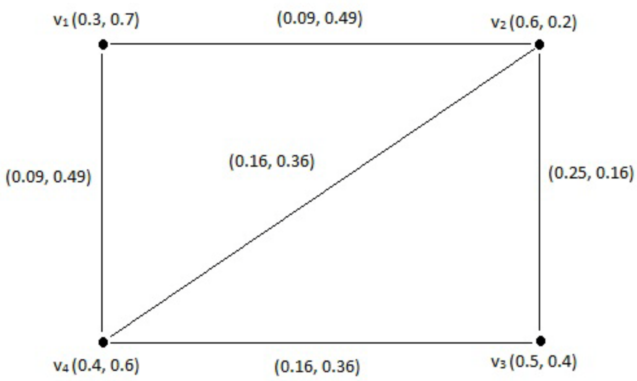

Figure-8: strong IFGST

Theorem 4.1: Let $G_1 = [V_1, E_1]$ and $G_2 = [V_2, E_2]$ be any two IFGSTs. If G_1 and G_2 are strong IFGST, then $G_1 \sqcap G_2$ is also strong IFGST.

Proof: Let G_1 and G_2 are strong IFGST then for all $(u_1, v_1) \in E_1$ and $(u_2, v_2) \in E_2$ we have,

$$\mu_2(u_1, v_1) = \min(\mu_1^2(u_1), \mu_1^2(v_1))$$
$$\nu_2(u_1, v_1) = \max(\nu_1^2(u_1), \nu_1^2(v_1))$$
$$\mu_2'(u_2, v_2) = \min((\mu_1')^2(u_2), (\mu_1')^2(v_2))$$
$$\nu_2'(u_2, v_2) = \max((\nu_1')^2(u_2), (\nu_1')^2(v_2))$$

Now,

$$\begin{aligned}(\mu_2 \sqcap \mu_2')(u_1, u_2)(v_1, v_2) &= \min(\mu_2(u_1v_1), \mu_2'(u_2v_2)) \\ &= \min(\min(\mu_1^2(u_1), \mu_1^2(v_1)), \min((\mu_1')^2(u_2), (\mu_1')^2(v_2))) \\ &= \min(\min(\mu_1^2(u_1), (\mu_1')^2(u_2)), \min(\mu_1^2(v_1), (\mu_1')^2(v_2))) \\ &= \min((\mu_1 \sqcap \mu_1')^2(u_1, u_2), (\mu_1 \sqcap \mu_1')^2(v_1, v_2)) \end{aligned} \tag{4.1}$$

Also,

$$\begin{aligned}(\nu_2 \sqcap \nu_2')(u_1, u_2)(v_1, v_2) &= \max(\nu_2(u_1v_1), \nu_2'(u_2v_2)) \\ &= \max(\max(\nu_1^2(u_1), \nu_1^2(v_1)), \max((\nu_1')^2(u_2), (\nu_1')^2(v_2))) \\ &= \max(\max(\nu_1^2(u_1), (\nu_1')^2(u_2)), \max(\nu_1^2(v_1), (\nu_1')^2(v_2))) \\ &= \max((\nu_1 \sqcap \nu_1')^2(u_1, u_2), (\nu_1 \sqcap \nu_1')^2(v_1, v_2)) \end{aligned} \tag{4.2}$$

From equation (4.1) and (4.2) $G_1 \sqcap G_2$ is also strong IFGST.

This completes the proof.

Theorem 4.2: Let $G_1 = [V_1, E_1]$ and $G_2 = [V_2, E_2]$ be any two IFGSTs. If G_1 and G_2 are strong IFGST, then $G_1 \bullet G_2$ is also strong IFGST.

Proof: Let G_1 and G_2 are strong IFGST then for all $(u_1, v_1) \in E_1$ and $(u_2, v_2) \in E_2$ we have,

$$\mu_2(u_1, v_1) = \min(\mu_1^2(u_1), \mu_1^2(v_1))$$
$$\nu_2(u_1, v_1) = \max(\nu_1^2(u_1), \nu_1^2(v_1))$$
$$\mu_2'(u_2, v_2) = \min((\mu_1')^2(u_2), (\mu_1')^2(v_2))$$
$$\nu_2'(u_2, v_2) = \max((\nu_1')^2(u_2), (\nu_1')^2(v_2))$$

If $u \in V_1$ and $(u_2, v_2) \in E_2$ then,

$$\begin{aligned}(\mu_2 \bullet \mu_2')(u, u_2)(u, v_2) &= \min(\mu_1^2(u), \mu_2'(u_2v_2)) \\ &= \min(\mu_1^2(u), \min((\mu_1')^2(u_2), (\mu_1')^2(v_2))) \\ &= \min(\min(\mu_1^2(u), (\mu_1')^2(u_2)), \min(\mu_1^2(u), (\mu_1')^2(v_2))) \\ &= \min((\mu_1 \bullet \mu_1')^2(u, u_2), (\mu_1 \bullet \mu_1')^2(u, v_2)) \end{aligned} \tag{4.3}$$

Similarly we have

$$(\nu_2 \bullet \nu_2')(u, u_2)(u, v_2) = \max((\nu_1 \bullet \nu_1')^2(u, u_2), (\nu_1 \bullet \mu_1')^2(u, v_2)) \tag{4.4}$$

Again if $(u_1, v_1) \in E_1$ and $(u_2, v_2) \in E_2$ then,

$$\begin{aligned}(\mu_2 \bullet \mu_2')(u_1, u_2)(v_1, v_2) &= \min(\mu_2(u_1v_1), \mu_2'(u_2v_2)) \\ &= \min(\min(\mu_1^2(u_1), \mu_1^2(v_1)), \min((\mu_1')^2(u_2), (\mu_1')^2(v_2))) \\ &= \min(\min(\mu_1^2(u_1), (\mu_1')^2(u_2)), \min(\mu_1^2(v_1), (\mu_1')^2(v_2))) \\ &= \min((\mu_1 \bullet \mu_1')^2(u_1, u_2), (\mu_1 \bullet \mu_1')^2(v_1, v_2)) \end{aligned} \tag{4.5}$$

Similarly we have

$$(\nu_2 \bullet \nu_2')(u_1, u_2)(v_1, v_2) = \max((\nu_1 \bullet \nu_1')^2(u_1, u_2), (\nu_1 \bullet \nu_1')^2(v_1, v_2)) \tag{4.6}$$

From the equations (4.3),(4.4),(4.5) and (4.6), $G_1 \bullet G_2$ is also strong IFGST.

This completes the proof.

Theorem 4.3: Let $G_1 = [V_1, E_1]$ and $G_2 = [V_2, E_2]$ be any two IFGSTs. If G_1 and G_2 are strong IFGST, then $G_1 \otimes G_2$ is also strong IFGST.

Proof: Let G_1 and G_2 are strong IFGST then for all $(u_1, v_1) \in E_1$ and $(u_2, v_2) \in E_2$ we have,

$$\mu_2(u_1, v_1) = \min(\mu_1^2(u_1), \mu_1^2(v_1))$$
$$\nu_2(u_1, v_1) = \max(\nu_1^2(u_1), \nu_1^2(v_1))$$
$$\mu_2'(u_2, v_2) = \min((\mu_1')^2(u_2), (\mu_1')^2(v_2))$$
$$\nu_2'(u_2, v_2) = \max((\nu_1')^2(u_2), (\nu_1')^2(v_2))$$

If $u \in V_1$ and $(u_2, v_2) \in E_2$ then,

$$\begin{aligned}(\mu_2 \otimes \mu_2')(u,u_2)(u,v_2) &= \min(\mu_1^2(u), \mu_2'(u_2v_2)) \\ &= \min(\mu_1^2(u), \min((\mu_1')^2(u_2), (\mu_1')^2(v_2))) \\ &= \min(\min(\mu_1^2(u), (\mu_1')^2(u_2)), \min(\mu_1^2(u), (\mu_1')^2(v_2))) \\ &= \min((\mu_1 \otimes \mu_1')^2(u,u_2), (\mu_1 \otimes \mu_1')^2(u,v_2)) \qquad (4.7)\end{aligned}$$

Similarly we have

$$(\nu_2 \otimes \nu_2')(u,u_2)(u,v_2) = \max((\nu_1 \otimes \nu_1')^2(u,u_2), (\nu_1 \otimes \mu_1')^2(u,v_2)) \qquad (4.8)$$

If $(u_1, v_1) \in E_1$ and $w \in V_2$ then,

$$\begin{aligned}(\mu_2 \otimes \mu_2')(u_1,w)(v_1,w) &= \min(\mu_2(u_1v_1), (\mu_1')^2(w)) \\ &= \min(\min(\mu_1^2(u_1), \mu_1^2(v_1)), (\mu_1')^2(w)) \\ &= \min(\min(\mu_1^2(u_1), (\mu_1')^2(w)), \min(\mu_1^2(v_1), (\mu_1')^2(w))) \\ &= \min((\mu_1 \otimes \mu_1')^2(u_1,w), (\mu_1 \otimes \mu_1')^2(v_1,w)) \qquad (4.9)\end{aligned}$$

Similarly we have

$$(\nu_2 \otimes \nu_2')(u_1,w)(v_1,w) = \max((\nu_1 \otimes \nu_1')^2(u_1,w), (\nu_1 \otimes \nu_1')^2(v_1,w)) \qquad (4.10)$$

Again if $(u_1, v_1) \in E_1$ and $(u_2, v_2) \in E_2$ then,

$$\begin{aligned}(\mu_2 \otimes \mu_2')(u_1,u_2)(v_1,v_2) &= \min(\mu_2(u_1v_1), \mu_2'(u_2v_2)) \\ &= \min(\min(\mu_1^2(u_1), \mu_1^2(v_1)), \min((\mu_1')^2(u_2), (\mu_1')^2(v_2))) \\ &= \min(\min(\mu_1^2(u_1), (\mu_1')^2(u_2)), \min(\mu_1^2(v_1), (\mu_1')^2(v_2))) \\ &= \min((\mu_1 \otimes \mu_1')^2(u_1,u_2), (\mu_1 \otimes \mu_1')^2(v_1,v_2)) \qquad (4.11)\end{aligned}$$

Similarly we have

$$(\nu_2 \otimes \nu_2')(u_1,u_2)(v_1,v_2) = \max((\nu_1 \otimes \nu_1')^2(u_1,u_2), (\nu_1 \otimes \nu_1')^2(v_1,v_2)) \qquad (4.12)$$

From the equations (4.7), (4.8), (4.9), (4.10), (4.11) and (4.12) it follows that $G_1 \otimes G_2$ is also strong IFGST.

This completes the proof.

Theorem 4.4: Let $G_1 = [V_1, E_1]$ and $G_2 = [V_2, E_2]$ be any two IFGSTs. If G_1 and G_2 are strong IFGST, then $G_1 \times G_2$ is also strong IFGST.

Proof: Let G_1 and G_2 are strong IFGST then for all $(u_1, v_1) \in E_1$ and $(u_2, v_2) \in E_2$ we have,

$$\begin{aligned}\mu_2(u_1,v_1) &= \min(\mu_1^2(u_1), \mu_1^2(v_1)) \\ \nu_2(u_1,v_1) &= \max(\nu_1^2(u_1), \nu_1^2(v_1)) \\ \mu_2'(u_2,v_2) &= \min((\mu_1')^2(u_2), (\mu_1')^2(v_2)) \\ \nu_2'(u_2,v_2) &= \max((\nu_1')^2(u_2), (\nu_1')^2(v_2))\end{aligned}$$

If $u \in V_1$ and $(u_2, v_2) \in E_2$ then,

$$\begin{aligned}(\mu_2 \times \mu_2')(u,u_2)(u,v_2) &= \min(\mu_1^2(u), \mu_2'(u_2v_2)) \\ &= \min(\mu_1^2(u), \min((\mu_1')^2(u_2), (\mu_1')^2(v_2))) \\ &= \min(\min(\mu_1^2(u), (\mu_1')^2(u_2)), \min(\mu_1^2(u), (\mu_1')^2(v_2))) \\ &= \min((\mu_1 \times \mu_1')^2(u,u_2), (\mu_1 \times \mu_1')^2(u,v_2)) \qquad (4.13)\end{aligned}$$

Similarly we have

$$(\nu_2 \times \nu_2')(u,u_2)(u,v_2) = \max((\nu_1 \times \nu_1')^2(u,u_2), (\nu_1 \times \mu_1')^2(u,v_2)) \qquad (4.14)$$

Again if $(u_1, v_1) \in E_1$ and $w \in V_2$ then,

$$\begin{aligned}(\mu_2 \times \mu_2')(u_1,w)(v_1,w) &= \min(\mu_2(u_1v_1), (\mu_1')^2(w)) \\ &= \min(\min(\mu_1^2(u_1), \mu_1^2(v_1)), (\mu_1')^2(w)) \\ &= \min(\min(\mu_1^2(u_1), (\mu_1')^2(w)), \min(\mu_1^2(v_1), (\mu_1')^2(w))) \\ &= \min((\mu_1 \times \mu_1')^2(u_1,w), (\mu_1 \times \mu_1')^2(v_1,w)) \qquad (4.15)\end{aligned}$$

Similarly we have

$$(\nu_2 \times \nu_2')(u_1,w)(v_1,w) = \max((\nu_1 \times \nu_1')^2(u_1,w), (\nu_1 \times \nu_1')^2(v_1,w)) \qquad (4.16)$$

From the equations (4.13),(4.14),(4.15) and (4.16), $G_1 \times G_2$ is also strong IFGST.

This completes the proof.

Theorem 4.5: Let $G_1 = [V_1, E_1]$ and $G_2 = [V_2, E_2]$ be any two IFGSTs. If G_1 and G_2 are strong IFGST, then $G_1 \circ G_2$ is also strong IFGST.

Proof: Let G_1 and G_2 are strong IFGST then for all $(u_1, v_1) \in E_1$ and $(u_2, v_2) \in E_2$ we have,

$$\mu_2(u_1, v_1) = \min(\mu_1^2(u_1), \mu_1^2(v_1))$$
$$\nu_2(u_1, v_1) = \max(\nu_1^2(u_1), \nu_1^2(v_1))$$
$$\mu_2'(u_2, v_2) = \min((\mu_1')^2(u_2), (\mu_1')^2(v_2))$$
$$\nu_2'(u_2, v_2) = \max((\nu_1')^2(u_2), (\nu_1')^2(v_2))$$

If $u \in V_1$ and $(u_2, v_2) \in E_2$ then,

$$\begin{aligned}(\mu_2 \circ \mu_2')(u, u_2)(u, v_2) &= \min(\mu_1^2(u), \mu_2'(u_2 v_2)) \\ &= \min(\mu_1^2(u), \min((\mu_1')^2(u_2), (\mu_1')^2(v_2))) \\ &= \min(\min(\mu_1^2(u), (\mu_1')^2(u_2)), \min(\mu_1^2(u), (\mu_1')^2(v_2))) \\ &= \min((\mu_1 \circ \mu_1')^2(u, u_2), (\mu_1 \circ \mu_1')^2(u, v_2)) \qquad (4.17)\end{aligned}$$

Similarly we have

$$(\nu_2 \circ \nu_2')(u, u_2)(u, v_2) = \max((\nu_1 \circ \nu_1')^2(u, u_2), (\nu_1 \circ \mu_1')^2(u, v_2)) \qquad (4.18)$$

If $(u_1, v_1) \in E_1$ and $w \in V_2$ then,

$$\begin{aligned}(\mu_2 \circ \mu_2')(u_1, w)(v_1, w) &= \min(\mu_2(u_1 v_1), (\mu_1')^2(w)) \\ &= \min(\min(\mu_1^2(u_1), \mu_1^2(v_1)), (\mu_1')^2(w)) \\ &= \min(\min(\mu_1^2(u_1), (\mu_1')^2(w)), \min(\mu_1^2(v_1), (\mu_1')^2(w))) \\ &= \min((\mu_1 \circ \mu_1')^2(u_1, w), (\mu_1 \circ \mu_1')^2(v_1, w)) \qquad (4.19)\end{aligned}$$

Similarly we have

$$(\nu_2 \circ \nu_2')(u_1, w)(v_1, w) = \max((\nu_1 \circ \nu_1')^2(u_1, w), (\nu_1 \circ \nu_1')^2(v_1, w)) \qquad (4.20)$$

Again if $(u_1, v_1) \in E_1$, $(u_2, v_2) \in E_2$ and $u_2 \neq v_2$ then,

$$\begin{aligned}(\mu_2 \circ \mu_2')(u_1, u_2)(v_1, v_2) &= \min(\mu_2(u_1 v_1), (\mu_1')^2(u_2), (\mu_1')^2(v_2)) \\ &= \min(\min(\mu_1^2(u_1), \mu_1^2(v_1)), (\mu_1')^2(u_2), (\mu_1')^2(v_2)) \\ &= \min(\min(\mu_1^2(u_1), (\mu_1')^2(u_2)), \min(\mu_1^2(v_1), (\mu_1')^2(v_2))) \\ &= \min((\mu_1 \circ \mu_1')^2(u_1, u_2), (\mu_1 \circ \mu_1')^2(v_1, v_2)) \qquad (4.21)\end{aligned}$$

Similarly we have

$$(\nu_2 \circ \nu_2')(u_1, u_2)(v_1, v_2) = \max((\nu_1 \circ \nu_1')^2(u_1, u_2), (\nu_1 \circ \nu_1')^2(v_1, v_2)) \qquad (4.22)$$

From the equations (4.17),(4.18),(4.19),(4.20),(4.21) and (4.22), $G_1 \circ G_2$ is also strong IFGST.

This completes the proof.

5. CONCLUSION

In this paper, we have defined some new operations direct product, semi-strong product, strong product, cartesian product and composition on IFGST. Also we have defined the concept of strong intuitionistic fuzzy graphs of second type and established some of their properties. In future we will study some more properties and applications of IFGST.

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