STUDY ON STRONG INTUITIONISTIC FUZZY GRAPHS
OF SECOND TYPE AND THEIR PROPERTIES

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ABSTRACT

In this paper, we define some new operations like direct product, semi-strong product, strong product, cartesian product
and composition of intuitionistic fuzzy graphs of second type and also define strong intuitionistic fuzzy graphs of second
type. Further we establish some of their properties.

Keywords: Intuitionistic fuzzy graphs, Intuitionistic fuzzy graphs of second type, direct, semi-strong, cartesian,
composition, strong.

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1. INTRODUCTION

Fuzzy sets were introduced by Lotfi. A. Zadeh [10] in 1965 as a generalisation of classical (crisp) sets. Further the fuzzy
sets are generalised by Krassimir.T. Atanassov [1] in which he has taken non-membership values also into consideration.
He introduced Intuitionistic fuzzy sets [IFS] and their extensions like Intuitionistic fuzzy sets of second type [IFSST],
Intuitionistic L-fuzzy sets [ILFS] and Temporal Intuitionistic fuzzy sets [TIFS]. A. Shannon and K. T. Atanassov [6]
discussed the theory of Intuitionistic fuzzy graphs. R. Parvathi and M. G. Karunambigai [3,4] introduced Intuitionistic
Fuzzy Graphs [IFG] elaborately and analyzed its components also defined strong intuitionistic fuzzy graphs. After that
they introduced and studied the operations cartesian product, composition on IFG. S. Ismail Mohideen, A. NagoorGani,
B. Fathima Kani and C. Yasmin [2] discussed the properties of operations on regular IFG. Further Sankar Sahoo and
Madhumangal Pal [5] defined and studied various operations like direct product, semi-strong product and strong product
on IFG. The present authors [7,8,9] introduced the extension of IFG namely Intuitionistic Fuzzy Graphs of Second Type
[IFGST] and defined some basic operations like union and join on IFGST. In section 2, we give some basic definitions
and in section 3, we define some new operations direct product, semi-strong product, strong product, cartesian product
and composition of IFGST. In section 4, we define the concept of strong intuitionistic fuzzy graphs of second type
and establish some of their properties. The paper is concluded in section 5.

2. PRELIMINARIES

In this section, we give some basic definitions.

Definition 2.1: [3] An Intuitionistic Fuzzy Graph [IFG] is of the form $G = [V, E]$ where
(i) $V = \{v_1, v_2, \ldots, v_n\}$ such that $\mu_1: V \rightarrow [0,1]$ and $v_1: V \rightarrow [0,1]$ denote the degree of membership and
non-membership of the element $v_i \in V$, respectively, and $0 \leq \mu_1(v_i) + v_1(v_i) \leq 1$ for every $v_i \in V$,
($i = 1,2, \ldots n$),
(ii) $E \subseteq V \times V$ where $\mu_2: V \times V \rightarrow [0,1]$ and $v_2: V \times V \rightarrow [0,1]$ are such that
$\mu_2(v_i, v_j) \leq \min[\mu_1(v_i), \mu_1(v_j)]$,
$v_2(v_i, v_j) \leq \max[\mu_1(v_i), \mu_1(v_j)]$
and $0 \leq \mu_2(v_i, v_j) + v_2(v_i, v_j) \leq 1$
for every $(v_i, v_j) \in E$, ($i, j = 1,2, \ldots n$).
Definition 2.2: [3] An IFG, $G = [V,E]$ is said to be a strong IFG if $\mu_{2ij} = \min(\mu_{ij}, \mu_{ij})$ and $\nu_{2ij} = \max(\nu_{ij}, \nu_{ij})$ for every $(v_i, v_j) \in E$.

Definition 2.3: [7] An Intuitionistic Fuzzy Graphs of Second Type [IFGST] is of the form $G = [V,E]$ where

(i) $V = \{v_1, v_2, \ldots, v_n\}$ such that $\mu_1: V \rightarrow [0,1]$ and $\nu_1: V \rightarrow [0,1]$ denote the degree of membership and non-membership of the element $v_i \in V$, respectively, and $0 \leq \mu_1(v_i)^2 + \nu_1(v_i)^2 \leq 1$ for every $v_i \in V$, $(i = 1,2,\ldots,n)$.

(ii) $E \subseteq V \times V$ where $\mu_2: V \times V \rightarrow [0,1]$ and $\nu_2: V \times V \rightarrow [0,1]$ are such that

$\mu_2(v_i, v_j) \leq \min(\mu_1(v_i)^2, \mu_1(v_j)^2)$

$\nu_2(v_i, v_j) \leq \max(\nu_1(v_i)^2, \nu_1(v_j)^2)$

and $0 \leq \mu_2(v_i, v_j)^2 + \nu_2(v_i, v_j)^2 \leq 1$ for every $(v_i, v_j) \in E$, $(i, j = 1,2,\ldots,n)$.

3. OPERATIONS ON INTUITIONISTIC FUZZY GRAPHS OF SECOND TYPE

In this section, we define some new operations like direct product, semi-strong product, strong product, cartesian product and composition of IFGST with suitable examples.

Definition 3.1: Let $G_1 = [V_1, E_1]$ and $G_2 = [V_2, E_2]$ be two IFGST then the direct product $G = G_1 \otimes G_2$ of $G_1$ and $G_2$ is also an IFGST defined by,

(i) $(\mu_1 \sqcap \mu'_1)(u_1, u_2) = \min(\mu_1(u_1), \mu'_1(u_2))$ for every $(u_1, u_2) \in V$

$(v_1 \sqcap v'_1)(u_1, u_2) = \max(v_1(u_1), v'_1(u_2))$ for every $(u_1, u_2) \in V$

(ii) $(\mu_2 \sqcap \mu'_2)(u_1, u_2)(v_1, v_2) = \min(\mu_2(u_1, v_1), \mu'_2(u_2, v_2))$ for every $u_1v_1 \in E_1, \ u_2v_2 \in E_2$

$(v_2 \sqcap v'_2)(u_1, u_2)(v_1, v_2) = \max(v_2(u_1, v_1), v'_2(u_2, v_2))$ for every $u_1v_1 \in E_1, \ u_2v_2 \in E_2$

where $V = V_1 \times V_2$ and $E = \{(u_1, u_2)(v_1, v_2): u_1v_1 \in E_1, \ u_2v_2 \in E_2\}$. Also $(\mu_1, v_1)$, $(\mu'_1, v'_1)$ are the vertex degree of membership and non-membership of the elements of $V_1$ in $G_1$ and $V_2$ in $G_2$ respectively and $(\mu_2, v_2)$ and $(\mu'_2, v'_2)$ are the edge degree of membership and non-membership of the elements of $E_1$ in $G_1$ and $E_2$ in $G_2$ respectively.

Example 3.1:

Figure-1: $G_1$ and $G_2$

Figure-2: direct Product of $G_1$ and $G_2$ ($G_1 \otimes G_2$)
Definition 3.2: Let $G_1 = [V_1, E_1]$ and $G_2 = [V_2, E_2]$ be two IFGST then the semi-strong product $G = G_1 \odot G_2$ of $G_1$ and $G_2$ is also an IFGST defined by,

(i) $(\mu_1 \cdot \mu'_1)(u_1, u_2) = \min(\mu_1(u_1), \mu'_1(u_2))$ for every $(u_1, u_2) \in V$ 

$(v_1 \cdot v'_1)(u_1, u_2) = \max(v_1(u_1), v'_1(u_2))$ for every $(u_1, u_2) \in V$

(ii) $(\mu_2 \cdot \mu'_2)(u_1, u_2)(u, v) = \min(\mu_2(u_1), u_2)(u_2(v))$ for every $u \in V_1, \ u_2 v_2 \in E_2$

$(v_2 \cdot v'_2)(u_1, u_2)(u, v) = \max(v_2(u_1), u_2)(v_2(v))$ for every $u \in V_1, \ u_2 v_2 \in E_2$

(iii) $(\mu_2 \cdot \mu'_2)(u_1, u_2)(v_1, v_2) = \min(\mu_2(u_1), u_2)(v_2(v_2))$ for every $u_1 v_1, v_1, \ u_2 v_2 \in E_2$

$(v_2 \cdot v'_2)(u_1, u_2)(v_1, v_2) = \max(v_2(u_1), u_2)(v_2(v_2))$ for every $u_1 v_1, v_1, \ u_2 v_2 \in E_2$

where $V = V_1 \times V_2$ and $E = \{(u, u_2)(u_2, v_2); u \in V_1, u_2 v_2 \in E_2 \} \cup \{(u_2, v_2)(u_1, v_1); u_1 v_1, v_1, \ u_2 v_2 \in E_2 \}$. Also $(\mu_1, v_1)$, $(\mu'_1, v'_1)$ are the vertex degree of membership and non-membership of the elements of $V_1$ in $G_1$ and $V_2$ in $G_2$ respectively and $(\mu_2, v_2)$ and $(\mu'_2, v'_2)$ are the edge degree of membership and non-membership of the elements of $E_1$ in $G_1$ and $E_2$ in $G_2$ respectively.

Example 3.2:

![Figure-3: semi-strong product of $G_1$ and $G_2$ ($G_1 \odot G_2$)](image)

Definition 3.3: Let $G_1 = [V_1, E_1]$ and $G_2 = [V_2, E_2]$ be two IFGST then the strong product $G = G_1 \otimes G_2$ of $G_1$ and $G_2$ is also an IFGST defined by,

(i) $(\mu_1 \otimes \mu'_1)(u_1, u_2) = \min(\mu_1(u_1), \mu'_1(u_2))$ for every $(u_1, u_2) \in V$

$(v_1 \otimes v'_1)(u_1, u_2) = \max(v_1(u_1), v'_1(u_2))$ for every $(u_1, u_2) \in V$

(ii) $(\mu_2 \otimes \mu'_2)(u_1, u_2)(u, v) = \min(\mu_2(u_1), u_2)(u_2(v))$ for every $u \in V_1, \ u_2 v_2 \in E_2$

$(v_2 \otimes v'_2)(u_1, u_2)(u, v) = \max(v_2(u_1), u_2)(v_2(v))$ for every $u \in V_1, \ u_2 v_2 \in E_2$

(iii) $(\mu_2 \otimes \mu'_2)(u_1, u_2)(v_1, v_2) = \min(\mu_2(u_1), u_2)(v_2(v_2))$ for every $u_1 v_1, v_1, \ u_2 v_2 \in E_2$

$(v_2 \otimes v'_2)(u_1, u_2)(v_1, v_2) = \max(v_2(u_1), u_2)(v_2(v_2))$ for every $u_1 v_1, v_1, \ u_2 v_2 \in E_2$

where $V = V_1 \times V_2$ and $E = \{(u, u_2)(u_2, v_2); u \in V_1, u_2 v_2 \in E_2 \} \cup \{(u_2, v_2)(u_1, v_1); u_1 v_1, v_1, \ u_2 v_2 \in E_2 \} \cup \{(u_1, u_2)(v_1, v_2); u_1 v_1, v_1, \ u_2 v_2 \in E_2 \} \cup \{(u_1, u_2)(v_1, v_2); u_1 v_1, v_1, \ u_2 v_2 \in E_2 \}$. Also $(\mu_1, v_1)$, $(\mu'_1, v'_1)$ are the vertex degree of membership and non-membership of the elements of $V_1$ in $G_1$ and $V_2$ in $G_2$ respectively and $(\mu_2, v_2)$ and $(\mu'_2, v'_2)$ are the edge degree of membership and non-membership of the elements of $E_1$ in $G_1$ and $E_2$ in $G_2$ respectively.

Example 3.3:

![Figure-4: strong product of $G_1$ and $G_2$ ($G_1 \otimes G_2$)](image)
Definition 3.4: Let $G_1 = [V_1, E_1]$ and $G_2 = [V_2, E_2]$ be two IFGST then the cartesian product $G = G_1 \times G_2$ of $G_1$ and $G_2$ is also an IFGST defined by,

(i) $(\mu_1 \times \mu_1')(u_1, u_2) = \min(\mu_1(u_1), \mu_1(u_2))$ for every $(u_1, u_2) \in V$

(ii) $(\nu_1 \times \nu_1')(u_1, u_2) = \max(\nu_1(u_1), \nu_1(u_2))$ for every $(u_1, u_2) \in V$

(iii) $(\mu_2 \times \mu_2')(u_1, u_2)(u, v) = \min(\mu_2(u), \mu_2(u_2 v_2))$ for every $u \in V_1, u_2 v_2 \in E_2$

(iv) $(\nu_2 \times \nu_2')(u_1, u_2)(u, v) = \max(\nu_2(u), \nu_2(u_2 v_2))$ for every $u \in V_1, u_2 v_2 \in E_2$

Example 3.4:

Figure-5: $G_3$ and $G_4$

Figure-6: Cartesian product of $G_3$ and $G_4 (G_3 \times G_4)$

Definition 3.5: Let $G_1 = [V_1, E_1]$ and $G_2 = [V_2, E_2]$ be two IFGST then the composition $G = G_1 \circ G_2$ of $G_1$ and $G_2$ is also an IFGST defined by,

(i) $(\mu_1 \circ \mu_1')(u_1, u_2) = \min(\mu_1(u_1), \mu_1(u_2))$ for every $(u_1, u_2) \in V$

(ii) $(\nu_1 \circ \nu_1')(u_1, u_2) = \max(\nu_1(u_1), \nu_1(u_2))$ for every $(u_1, u_2) \in V$

(iii) $(\mu_2 \circ \mu_2')(u_1, u_2)(u, v) = \min(\mu_2(u), \mu_2(u_2 v_2))$ for every $u \in V_1, u_2 v_2 \in E_2$

(iv) $(\nu_2 \circ \nu_2')(u_1, u_2)(u, v) = \max(\nu_2(u), \nu_2(u_2 v_2))$ for every $u \in V_1, u_2 v_2 \in E_2$
(4) \((\mu_2 \circ \mu')(u_1, u_2)(v_1, v_2) = \min(\mu_2(u_1v_1), (\mu'_2)^2(u_2), (\mu'_1)^2(v_2))\) for every \(u_1v_1 \in E_1, u_2, v_2 \in V_2,
\) where \(u_2 \neq v_2\).

\((\nu_2 \circ \nu')(u_1, u_2)(v_1, v_2) = \max(\nu_2(u_1v_1), (\nu'_2)^2(u_2), (\nu'_1)^2(v_2))\) for every \(u_1v_1 \in E_1, u_2, v_2 \in V_2, u_2 \neq v_2\)
where \(V = V_1 \times V_2\) and \(E = \{ (u_1, u_2)(v_1, v_2) : u \in V_1, u_2, v_2 \in V_2 \} \cup \{ (u_1, w)(v_1, v) : u_1, v_1 \in E_1, w \in V_2 \} \cup \{ (u_1, u_2)(v_1, v_2) : u_1v_1 \in E_1, u_2, v_2 \in V_2, u_2 \neq v_2 \} \). Also \((\mu_1, \nu_1), (\mu'_1, \nu'_1)\) are the vertex degree of membership and non-membership of the elements of \(V_1\) in \(G_1\) and \(V_2\) in \(G_2\) respectively and \((\mu_2, \nu_2)\) and \((\mu'_2, \nu'_2)\) are the edge degree of membership and non-membership of the elements of \(E_1\) in \(G_1\) and \(E_2\) in \(G_2\) respectively.

Example 3.5:

\[\text{Figure-7: composition of } G_3 \text{ and } G_4 \ (G_3 \circ G_4)\]

4. PROPERTIES OF STRONG INTUITIONISTIC FUZZY GRAPHS OF SECOND TYPE

In this section, we define strong intuitionistic fuzzy graphs of second type and establish some of their properties based on newly defined operations.

Definition 4.1: An IFGST, \(G = [V, E]\) is said to be a strong IFGST if \(\mu_{2ij} = \min(\mu_{1i}, \mu'_{1j})\) and \(\nu_{2ij} = \max(\nu_{1i}, \nu'_{1j})\) for every \((v_i, v_j) \in E\).

Example 4.1:

\[\text{Figure-8: strong IFGST}\]
**Theorem 4.1:** Let $G_1 = [V_1, E_1]$ and $G_2 = [V_2, E_2]$ be any two IFGSTs. If $G_1$ and $G_2$ are strong IFGST, then $G_1 \cap G_2$ is also strong IFGST.

**Proof:** Let $G_1$ and $G_2$ are strong IFGST then for all $(u_1, v_1) \in E_1$ and $(u_2, v_2) \in E_2$ we have,

\[
\mu_2(u_1, v_1) = \min(\mu_1^2(u_1), \mu_2^2(v_1)) \\
v_2(u_1, v_1) = \max(v_1^2(u_1), v_2^2(v_1)) \\
\mu_2'(u_2, v_2) = \min((\mu_1')^2(u_2), (\mu_2')^2(v_2)) \\
v_2'(u_2, v_2) = \max((v_1')^2(u_2), (v_2')^2(v_2))
\]

Now,

\[
(\mu_2 \cap \mu_2')(u_1, u_2)(v_1, v_2) = \min(\mu_2(u_1, v_1), \mu_2'(u_2, v_2)) = \min(\min(\mu_1^2(u_1), \mu_2^2(v_1), (\mu_1')^2(u_2), (\mu_2')^2(v_2)))
\]

\[
= \min(\min(\mu_1^2(u_1), \mu_2^2(v_1)), \min((\mu_1')^2(u_2), (\mu_2')^2(v_2))) = \min((\mu_1 \cap \mu_1')^2(u_1, u_2), (\mu_2 \cap \mu_2')^2(v_1, v_2))
\]

(4.1)

Also,

\[
(v_2 \cap v_2')(u_1, u_2)(v_1, v_2) = \max(v_2(u_1, v_1), v_2'(u_2, v_2)) = \max(\max(v_1^2(u_1), v_2^2(v_1)), \max((v_1')^2(u_2), (v_2')^2(v_2)))
\]

\[
= \max(\max(v_1^2(u_1), (v_1')^2(v_1)), \max(v_2^2(v_2), (v_2')^2(v_2))) = \max((v_1 \cap v_1')^2(u_1, u_2), (v_2 \cap v_2')^2(v_1, v_2))
\]

(4.2)

From equation (4.1) and (4.2) $G_1 \cap G_2$ is also strong IFGST.

This completes the proof.

**Theorem 4.2:** Let $G_1 = [V_1, E_1]$ and $G_2 = [V_2, E_2]$ be any two IFGSTs. If $G_1$ and $G_2$ are strong IFGST, then $G_1 \cdot G_2$ is also strong IFGST.

**Proof:** Let $G_1$ and $G_2$ are strong IFGST then for all $(u_1, v_1) \in E_1$ and $(u_2, v_2) \in E_2$ we have,

\[
\mu_2(u_1, v_1) = \min(\mu_1^2(u_1), \mu_2^2(v_1)) \\
v_2(u_1, v_1) = \max(v_1^2(u_1), v_2^2(v_1)) \\
\mu_2(u_2, v_2) = \min((\mu_1')^2(u_2), (\mu_2')^2(v_2)) \\
v_2(u_2, v_2) = \max((v_1')^2(u_2), (v_2')^2(v_2))
\]

If $u \in V_1$ and $(u_2, v_2) \in E_2$ then,

\[
(\mu_2 \cdot \mu_2')(u_2, u_2)(u, v_2) = \min(\mu_1^2(u), \mu_2^2(u_2)) = \min(\mu_2(u), \min((\mu_1')^2(u_2), (\mu_2')^2(v_2))) = \min(\min(\mu_1^2(u), (\mu_1')^2(u_2)), (\mu_2^2(u_2), (\mu_2')^2(v_2))) = \min((\mu_1 \cdot \mu_1')^2(u, u_2), (\mu_1 \cdot \mu_2')^2(u_2, v_2))
\]

(4.3)

Similarly we have

\[
(v_2 \cdot v_2')(u_2, u_2)(u, v_2) = \max((v_1 \cdot v_1')^2(u, u_2), (v_2 \cdot v_2')^2(u_2, v_2))
\]

(4.4)

Again if $(u_1, v_1) \in E_1$ and $(u_2, v_2) \in E_2$ then,

\[
(\mu_2 \cdot \mu_2')(u_2, u_2)(v_1, v_2) = \min(\mu_2(u_1, v_1), \mu_2^2(u_2)) = \min(\min(\mu_1^2(u_1), \mu_2^2(v_1)), \min((\mu_1')^2(u_2), (\mu_2')^2(v_2)))
\]

\[
= \min(\min(\mu_1^2(u_1), (\mu_1')^2(u_2)), (\mu_2^2(v_2), (\mu_2')^2(v_2))) = \min(\mu_2(v_1), (\mu_2')^2(v_2)) = \min((\mu_1 \cdot \mu_1')(u_2, u_2), (\mu_2 \cdot \mu_2')^2(u_2, v_2))
\]

(4.5)

Similarly we have

\[
(v_2 \cdot v_2')(u_2, u_2)(v_1, v_2) = \max((v_1 \cdot v_1')^2(u_2, u_2), (v_2 \cdot v_2')^2(v_1, v_2))
\]

(4.6)

From the equations (4.3),(4.4),(4.5) and (4.6), $G_1 \cdot G_2$ is also strong IFGST.

This completes the proof.

**Theorem 4.3:** Let $G_1 = [V_1, E_1]$ and $G_2 = [V_2, E_2]$ be any two IFGSTs. If $G_1$ and $G_2$ are strong IFGST, then $G_1 \otimes G_2$ is also strong IFGST.

**Proof:** Let $G_1$ and $G_2$ are strong IFGST then for all $(u_1, v_1) \in E_1$ and $(u_2, v_2) \in E_2$ we have,

\[
\mu_2(u_1, v_1) = \min(\mu_1^2(u_1), \mu_2^2(v_1)) \\
v_2(u_1, v_1) = \max(v_1^2(u_1), v_2^2(v_1)) \\
\mu_2(u_2, v_2) = \min((\mu_1')^2(u_2), (\mu_2')^2(v_2)) \\
v_2(u_2, v_2) = \max((v_1')^2(u_2), (v_2')^2(v_2))
\]
Similarly we have
\[(v_2 \otimes v'_2)(u, u_2) = \max((v_1 \otimes v'_1)^2(u, u_2), (v_1 \otimes \mu'_1)^2(u, u_2)) \quad (4.8)\]

Again if \((u_1, v_1) \in E_1 \) and \((u_2, v_2) \in E_2 \) then,
\[
\begin{align*}
(v_2 \otimes v'_2)(u_1, u_2)(v_1, w) &= \min((v_1 \otimes v'_1)^2(u_1, w), (v_1 \otimes \mu'_1)^2(v_1, w)) \\
&= \min((v_1 \otimes v'_1)^2(u_1, v_1), (v_1 \otimes \mu'_1)^2(v_1, v_1)) \\
&= \min((v_1 \otimes v'_1)^2(u_1, v_1), (v_1 \otimes \mu'_1)^2(v_1, v_1)) \\
&= \min((v_1 \otimes v'_1)^2(u_1, v_1), (v_1 \otimes \mu'_1)^2(v_1, v_1)) \quad (4.9) \\
\end{align*}
\]

Similarly we have
\[
(v_2 \otimes v'_2)(u_1, u_2)(v_1, v_2) = \max((v_1 \otimes v'_1)^2(u_1, v_2), (v_1 \otimes \mu'_1)^2(v_1, v_2)) \quad (4.10)\]

From the equations (4.7), (4.8), (4.9), (4.10), (4.11) and (4.12) it follows that \(G_1 \otimes G_2\) is also strong IFGST.

This completes the theorem.

**Theorem 4.4:** Let \(G_1 = [V_1, E_1]\) and \(G_2 = [V_2, E_2]\) be any two IFGSTs. If \(G_1\) and \(G_2\) are strong IFGST, then \(G_1 \times G_2\) is also strong IFGST.

**Proof:** Let \(G_1\) and \(G_2\) are strong IFGST then for all \((u_1, v_1) \in E_1\) and \((u_2, v_2) \in E_2\) we have,
\[
\begin{align*}
\mu_2(u_1, v_1) &= \min(\mu_1^2(u_1), \mu_2^2(v_1)) \\
v_2(u_1, v_1) &= \max(v_1^2(u_1), v_1^2(v_1)) \\
\mu_2(u_2, v_2) &= \min(\mu_2^2(u_2), (\mu_2^2(v_2))) \\
v_2(u_2, v_2) &= \max(v_2^2(u_2), v_2^2(v_2)) \\
\end{align*}
\]

If \(u \in V_1\) and \((u_2, v_2) \in E_2\) then,
\[
\begin{align*}
(\mu_2 \times \mu'_2)(u, u_2)(u, v_2) &= \min(\mu_2^2(u), \mu'_2(u_2, v_2)) \\
&= \min(\mu_2^2(u), \min(\mu'_2^2(u_2), (\mu'_2(v_2)) \\
&= \min(\mu_2^2(u), \min(\mu'_2^2(u_2), (\mu'_2(v_2)) \\
&= \min((\mu_2 \times \mu'_2)^2(u, u_2), (\mu_2 \times \mu'_2)^2(v_2)) \quad (4.13) \\
\end{align*}
\]

Similarly we have
\[
(v_2 \times v'_2)(u, u_2)(u, v_2) = \max((v_1 \times v'_1)^2(u_2, v_2), (v_1 \times \mu'_1)^2(u_2, v_2)) \quad (4.14)\]

Again if \((u_1, v_1) \in E_1\) and \(w \in V_2\) then,
\[
\begin{align*}
(\mu_2 \times \mu'_2)(u_1, w)(v_1, w) &= \min(\mu_2(u_1, v_1), (\mu'_2)^2(w)) \\
&= \min(\mu_2(u_1, v_1), (\mu'_2)^2(w)) \\
&= \min(\mu_2(u_1, v_1), (\mu'_2)^2(w)) \\
&= \min((\mu_2 \times \mu'_2)^2(u_1, w), (\mu_2 \times \mu'_2)^2(v_1, w)) \quad (4.15) \\
\end{align*}
\]

Similarly we have
\[
(v_2 \times v'_2)(u_1, w)(v_1, w) = \max((v_1 \times v'_1)^2(u_1, w), (v_1 \times \mu'_1)^2(v_1, w)) \quad (4.16)\]

From the equations (4.13),(4.14),(4.15) and (4.16), \(G_1 \times G_2\) is also strong IFGST.

This completes the proof.

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Theorem 4.5: Let $G_1 = [V_1, E_1]$ and $G_2 = [V_2, E_2]$ be any two IFGSTs. If $G_1$ and $G_2$ are strong IFGST, then $G_1 \circ G_2$ is also strong IFGST.

Proof: Let $G_1$ and $G_2$ are strong IFGST then for all $(u_1, v_1) \in E_1$ and $(u_2, v_2) \in E_2$ we have,

$\mu_2(u_1, v_1) = \min(\mu_2^1(u_1), \mu_2^2(v_1))$

$\nu_2(u_1, v_1) = \max(\nu_2^1(u_1), \nu_2^2(v_1))$

$\mu_2(u_2, v_2) = \min((\mu_2^1(u_2), (\mu_2^2(v_2)))$

$\nu_2(u_2, v_2) = \max((\nu_2^1(u_2), (\nu_2^2(v_2)))$

If $u \in V_1$ and $(u_2, v_2) \in E_2$ then,

$(\mu_2 \circ \mu_2)(u, u_2)(u_2, v_2) = \min(\mu_2^1(u), \mu_2^2(u_2), (\mu_2^1(u_2), (\mu_2^2(v_2)))$

$= \min(\mu_2^1(u), \mu_2^2(u_2), (\mu_2^1(u_2), (\mu_2^2(v_2)))$

Similarly we have

$(\nu_2 \circ \nu_2)(u, u_2)(u_2, v_2) = \max((\nu_2^1(u), (\nu_2^1(u_2), (\nu_2^1(u)), (\nu_2^1(v_2))))$

$(4.17)$

$(4.18)$

If $(u_1, v_1) \in E_1$ and $w \in V_2$ then,

$(\mu_2 \circ \mu_2)(u_1, w)(v_1, w) = \min((\mu_2^1(u_1), (\mu_2^1(v_1), (\mu_2^1(v_1)), (\mu_2^1(w))))$

$= \min((\mu_2^1(u_1), (\mu_2^1(v_1), (\mu_2^1(v_1)), (\mu_2^1(w))))$

Similarly we have

$(\nu_2 \circ \nu_2)(u_1, w)(v_1, w) = \max((\nu_2^1(u_1), (\nu_2^1(v_1), (\nu_2^1(w))))$

$(4.19)$

$(4.20)$

Again if $(u_1, v_1) \in E_1$, $(u_2, v_2) \in E_2$ and $u_2 \neq v_2$ then,

$(\mu_2 \circ \mu_2)(u_1, u_2)(v_1, v_2) = \min((\mu_2^1(u_1), (\mu_2^1(v_1), (\mu_2^1(v_1)), (\mu_2^1(u_2), (\mu_2^1(v_2))))$

$= \min((\mu_2^1(u_1), (\mu_2^1(v_1), (\mu_2^1(v_1)), (\mu_2^1(u_2), (\mu_2^1(v_2))$

Similarly we have

$(\nu_2 \circ \nu_2)(u_1, u_2)(v_1, v_2) = \max((\nu_2^1(u_1), (\nu_2^1(v_1), (\nu_2^1(u_2), (\nu_2^1(v_2))))$

$(4.21)$

$(4.22)$

From the equations $(4.17),(4.18),(4.19),(4.20),(4.21)$ and $(4.22)$, $G_1 \circ G_2$ is also strong IFGST.

This completes the proof.

5. CONCLUSION

In this paper, we have defined some new operations direct product, semi-strong product, strong product, cartesian product and composition on IFGST. Also we have defined the concept of strong intuitionistic fuzzy graphs of second type and established some of their properties. In future we will study some more properties and applications of IFGST.

REFERENCES

