INTERVAL VALUED SIGNED NEUTROSOPHIC GRAPH

SUDHAKAR.V.J, MOHAMED ALI. A
Department of Mathematics, Islamiah College, Vaniyambadi.
E-mail: vjsvec1@gmail.com

VINOTH.D
Department of Mathematics, Islamiah College, Vaniyambadi.
E-mail: vinov002@gmail.com

(Received On: 17-08-18; Revised & Accepted On: 04-09-18)

ABSTRACT

The notion of interval valued neutrosophic sets is a generalization of fuzzy sets, intuitionistic fuzzy sets, interval valued fuzzy sets, interval valued intuitionistic fuzzy sets and single valued neutrosophic sets. We apply for the first time the concept of interval valued neutrosophic sets, an instance of neutrosophic sets, to the graph theory. We introduce certain types of interval valued signed neutrosophic graphs (IVNG) and investigate some of their properties with proof and example.

Keywords: Interval valued neutrosophic set, interval valued neutrosophic graph, strong interval valued neutrosophic graph, constant interval valued neutrosophic graph, complete interval valued neutrosophic graph, degree of interval valued neutrosophic graph, interval valued signed neutrosophic graph.

1. INTRODUCTION

Zadeh introduced interval-valued fuzzy set theory which is an extension of fuzzy set theory. Membership degrees in an interval-valued fuzzy set are intervals rather than numbers and uncertainty is reflected by length of interval membership degree. Zhan et al. applied the concept of interval-valued fuzzy sets to algebraic structures. For representing vagueness and uncertainty Atanassov proposed an extension of fuzzy sets by adding nonmembership function in the definition is called intuitionistic fuzzy sets. The concept of intuitionistic fuzzy sets is more meaningful and inventive due to the presence of degree of truth, indeterminacy M. Akram and M. Sitara and falsity membership. The intuitionistic fuzzy sets have more describing possibilities as compared to fuzzy sets. The hesitation margin of an intuitionistic fuzzy set is its uncertainty by default and sum of truth-membership degree and falsity-membership degree does not exceeds unity. In many phenomena like information fusion, uncertainty and indeterminacy is doubtlessly quantified. Smarandache proposed the idea of neutrosophic sets, he mingled tricomponent logic, non-standard analysis, and philosph- ophy. It is a branch of philosophy which studies the origin, nature and scope of neutralities as well as their interactions with different ideational spectra. For convenient and advantageous usage of neutrosophic sets in science and engineering, Wang et al. proposed the notion of single-valued neutrosophic (SVNS) sets, whose three independent components have values in standard unit interval [0, 1]. Neutrosophic set theory being a generalization of fuzzy set theory and intuitionistic fuzzy set theory is more practical, advantageous and applicable in various fields, including medical diagnosis, control theory, topology, decision-making problems and in many more real-life problems. Wang et al. proposed the notion of interval-valued neutrosophic sets, which is more precise and flexible than the single-valued neutrosophic sets. An interval-valued neutrosophic set is a generalization of the notion of single-valued neutrosophic set, in which three independent components (t, i, f) are intervals which are subsets of standard unit interval [0, 1]. On the basis of Zadehs fuzzy relations Kaufmann proposed fuzzy graph. Rosenfeld discussed fuzzy analogue of various graph-theoretic ideas. Later on, Bhat-tacharya gave some remarks on fuzzy graphs, and some operations on fuzzy graphs were introduced by Mordeson and Peng. Mordeson-son and Nair presented a valuable contribution on fuzzy graphs as well as fuzzy hypergraphs. Mathew and Sunita discussed arcs and strong cycles in fuzzy graphs. On the other hand, Dinesh and Ramakrishnan defined fuzzy graph structures and discussed their properties. Akram and Akmal proposed the notion of bipolar fuzzy graph structures. Akram et al. [1, 2, 3, 4] have introduced several concepts on interval-valued fuzzy graphs and interval-valued neutrosophic graphs. Akram and Shahzadi introduced the notion of neutrosophic soft graphs with applications. Recently, Akram and Nasir [5, 6] considered interval-valued neutrosophic graphs. In this research article, we introduce certain notions of interval-valued neutrosophic signed graph and establish some of properties. We elaborate the concepts of interval-valued neutrosophic signed graph with examples.
2. PRELIMINARIES

2.1. Definition: Let $X$ be a space of points (objects) with generic elements in $X$ denoted by $x$; then the neutrosophic set $A$ is object of the form

$$A = \{ < x : T_A(x), I_A(x), F_A(x) > | x \in X \}$$

Where the functions $T, I, F : X \rightarrow [0, 1]$ define respectively the a truth-membership function, indeterminacy-membership function and falsity-membership function of the element $x \in X$ to the set $A$ with the condition

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+.$$ 

The functions $T_A(x); I_A(x)$ and $F_A(x)$ are real standard or non standard subsets of $[0, 1]^+$. 

2.2. Definition: Let $X$ be a space of points (options) with generic elements in $X$ denoted by $x$. A single valued neutrosophic set $A$ (SVNS) is characterized by truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$, falsity-membership function $F_A(x)$. For each point $x \in X$, $T_A(x), I_A(x), F_A(x) \in \{0, 1\}$. A (SVNS) can be written as

$$A = \{ < x : T_A(x), I_A(x), F_A(x) > | x \in X \}.$$ 

2.3. Definition: A fuzzy graph $G(\sigma, \mu)$ is said to be a fuzzy signed graph if there is a mapping $\eta : E \rightarrow \{+, -\}$ such that each signed to $\{+, -\}$ or all nodes edges assigned to $\{+, -\}$ when we assign $\{+, -\}$ to each of the following nodes called vertex signed fuzzy graph. For assignment of sign to any edge we follow some rule, according to the problems or relations between the objects we define some $\alpha$-after it we take an $\alpha$-cut for the set of edges, then we assign positive or negative to the edges appear in $\alpha$-cut set alternate sign for those which are not in $\alpha$-cut set.

In the following fuzzy graph shown in figure, we assume $\alpha = 0.4$, thus $\alpha$-cut set for edge set contain only two edges $v_1v_2$ and $v_1v_4$. So we assign positive sign to these edges and for remaining we assigned it by negative sign.

![Figure-2.1: Fuzzy graph and its signed graph](image-url)

2.4. Definition: Sum of the all membership values of all incident positive edge to $v$ is known as positive degree of any vertex $v$, i.e.,

$$\deg^+[\sigma(v)] = \sum_{\{\mu^+(v,v_i)\in E\}} \mu^+(v,v_i)$$

Similarly by negative degree we mean

$$\deg^-[\sigma(v)] = \sum_{\{\mu^-(v,v_i)\in E\}} \mu^-(v,v_i)$$

And sign degree of any vertex $v$ is difference between

$$sdeg(v) = |\deg^+[\sigma(v)] - \deg^-[\sigma(v)]|$$

2.4.1. Example: A example to calculate sign degree of all notes of a fuzzy signed graph.

![Figure-2.2: Signed fuzzy graph](image-url)
Sign degree of vertices for signed fuzzy graph as shown in figure are
\[\text{sdeg}(v_i) = |(0.5 + 0.6) - (0.2 + 0.3)| = |1.1 - 0.5| = 0.6,\]
\[\text{sdeg}(v_2) = |(0.5 + 0.7) - 0| = |1.2 - 0| = 1.2,\]
\[\text{sdeg}(v_3) = |(0.6 + 0.7) - 0| = |1.3 - 0| = 1.3\]
\[\text{sdeg}(v_4) = |0 - (0.2 + 0.3)| = |0 - 0.5| = 0.5,\]
\[\text{sdeg}(v_5) = |0 - (0.2 + 0.2)| = |0 - 0.4| = 0.4\]

2.5. Definition
Let \(A = (T_A, I_A, F_A)\) and \(B = (T_B, I_B, F_B)\) be single valued neutrosophic sets on a set \(X\). If \(A = (T_A, I_A, F_A)\) is a single valued neutrosophic relation on a set \(X\), then
\[T_B(x,y) = \min (T_A(x), T_A(y))\]
\[I_B(x,y) = \max (I_A(x), I_A(y))\]
\[F_B(x,y) = \min (F_A(x), F_A(y))\]

A single valued neutrosophic relation \(A\) on \(X\) is called symmetric if
\[T_A(x,y) = T_A(y,x)\]
\[I_A(x,y) = I_A(y,x)\]
\[F_A(x,y) = F_A(y,x)\]

2.6. Definition: A single valued neutrosophic graph (SVN-graph) with underlying set \(V\) is defined to be a pair \(G = (A, B)\) where
i. The functions \(T_A: V \rightarrow [0,1], I_A: V \rightarrow [0,1], F_A: V \rightarrow [0,1]\) denote degree of truth-membership, degree of indeterminacy-membership, degree of falsity-membership of the element \(v_i \in V\) respectively and
\[0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3\]
for all \(v_i \in V, 1(2, \ldots, n)\).
ii. The functions \(T_B: E \leq V \times V \rightarrow [0,1], I_B: E \leq V \times V \rightarrow [0,1], T_B: E \leq V \times V \rightarrow [0,1]\) are defined by
\[T_B(v_i, v_j) = \min [T_A(v_i), T_A(v_j)]\]
\[I_B(v_i, v_j) = \max [I_A(v_i), I_A(v_j)]\]
\[F_B(v_i, v_j) = \max [F_A(v_i), F_A(v_j)]\]

Denotes the degree of truth-membership, indeterminacy-membership, and falsity-membership of the edge \((v_i, v_j) \in E\) respectively, where
\[0 \leq T_B(v_i, v_j) + I_B(v_i, v_j) + F_B(v_i, v_j) \leq 3\]
for all \((v_i, v_j) \in E, i, j = 1, 2, \ldots, n\). We call \(A\) the single valued neutrosophic vertex on \(V, B\) the single valued neutrosophic edge set of \(E\). Note that \(B\) is a symmetric single valued neutrosophic relation on \(A\). We use the notation for an element of \(E\). Thus \(G = (A, B)\) is a single valued neutrosophic graph of \(G^* = (V, E)\) if
\[T_B: E \leq V \times V \rightarrow [0,1], I_B: E \leq V \times V \rightarrow [0,1]\]
\[F_B: E \leq V \times V \rightarrow [0,1]\]

Denote by \([A, B]\) and \([A, B]^{*}\) the single valued neutrosophic subset of \(V\) and let \(B\) a single valued neutrosophic subset of \(E\) denoted by

<table>
<thead>
<tr>
<th>(E)</th>
<th>(v_1)</th>
<th>(v_2)</th>
<th>(v_3)</th>
<th>(v_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_A)</td>
<td>0.4</td>
<td>0.5</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>(I_A)</td>
<td>0.2</td>
<td>0.2</td>
<td>0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>(F_A)</td>
<td>0.5</td>
<td>0.3</td>
<td>0.6</td>
<td>0.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(E)</th>
<th>(v_1v_2)</th>
<th>(v_2v_3)</th>
<th>(v_3v_4)</th>
<th>(v_4v_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_B)</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>(I_B)</td>
<td>0.2</td>
<td>0.5</td>
<td>0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>(F_B)</td>
<td>0.5</td>
<td>0.6</td>
<td>0.6</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Figure-2.3: Single valued neutrosophic graph

i. \((v_1, 0.4, 0.2, 0.5)\) is a single valued neutrosophic vertex or SVN-vertex.

ii. \((v_1v_2, 0.4, 0.2, 0.5)\) is a single valued neutrosophic edge or SVN-edge.

iii. \((v_1, 0.4, 0.2, 0.5)\) and \((v_2, 0.5, 0.2, 0.3)\) are single valued neutrosophic adjacent vertices.

iv. \((v_1v_2, 0.4, 0.2, 0.5)\) and \((v_1v_4, 0.1, 0.4, 0.5)\) are single valued neutrosophic adjacent edge.

2.7. Definition: A single valued neutrosophic graph \(G\) is said to be a single valued signed neutrosophic graph if 
\[
\sigma: E(G) \rightarrow \{+1, -1\}
\]
is a function associated from \(E(G)\) of \(G\) such that each edge signed to \(\{+1, -1\}\). We assign \(E(G) \rightarrow \{+1, -1\}\) on the comparison basis of its truth-membership, indeterminacy-membership and falsity-membership values, we assign it positive and in reverse case we assign it native and in case of equality we keep it unsigned.

2.8. Definition: For a single value neutrosophic graph any vertex is said to be positive or negative signed if 
\[
\sigma: V(G) \rightarrow \{+1, -1\}
\]
is positive or negative, where \(\sigma\) is a function associated from \(V(G)\) on the comparison basis of truth-membership, indeterminacy-membership and falsity-membership values of \(V\), similar as edge sign.

A single valued neutrosophic graph is said to be positive if all the edges gets positive sign or only even number of edges have negative sign, basically, sign of SVSNG is determined by the product of the signs of all edges. Similarly, a SVSNG is said to be negative signed if odd number of edges of SVSNG are negative.

2.9. Definition: Let \(X\) be a space of points (objects) with generic elements in \(X\) denoted by \(x\). An interval valued neutrosophic set (for short IVNS) \(A\) in \(X\) is characterized by truth-membership function \(T_A(x)\), indeterminacy-membership function \(I_A(x)\) and falsity-membership function \(F_A(x)\). For each point \(x\) in \(X\), we have that
\[
T_A(x) = [T_A^L(x), T_A^U(x)],
I_A(x) = [I_A^L(x), I_A^U(x)],
F_A(x) = [F_A^L(x), F_A^U(x)]
\]
\[
0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3.
\]

2.10. Definition: A single valued neutrosophic graph is said to be balanced if every cycle of the graph have even number of negative sign edges or all positive signed edges. We say SVSNG is completely balanced if \(\sum_{i=1}^{n} T_i = \sum_{i=1}^{n} I_i = \sum_{i=1}^{n} F_i\) for all edges of \(G\).

Figure-2.4: Balanced single valued signed neutrosophic graph

2.11. Definition: Let \(G = (A, B)\) be a single valued neutrosophic graph, then the degree of any vertex \(v\) is sum of degree of truth-membership, degree of indeterminacy-membership and degree of falsity-membership of all those edges which are incident on vertex \(v\) denoted by \(d(v) = (d_T(v), d_I(v), d_F(v))\) where 
\[
d_T(v) = \sum_{u \neq v} T_B(u, v)
\]
denotes the degree of truth-membership vertex.

\[
d_I(v) = \sum_{u \neq v} I_B(u, v)
\]
denotes the degree of indeterminacy-membership vertex.

\[
d_F(v) = \sum_{u \neq v} F_B(u, v)
\]
denotes the degree of falsity-membership vertex.
2.12. Definition: Let $X$ be a space of points (objects) with generic elements in $X$ denoted by $x$. An interval neutrosophic set (for short IVNS) $A$ in $X$ is characterized by truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$ and falsity-membership function $F_A(x)$. For each point $x$ in $X$, we have that

$$T_A(x) = [T_{AL}(x), T_{AU}(x)],$$
$$I_A(x) = [I_{AL}(x), I_{AU}(x)],$$
$$F_A(x) = [F_{AL}(x), F_{AU}(x)] \subseteq [0,1]$$

and

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3.$$

2.13. Definition: An interval valued neutrosophic graph of a graph $G^* = (V,E)$ we mean a pair $G = (A,B)$ where $A = [T_{AL},T_{AU},I_{AL},I_{AU},F_{AL},F_{AU}] >$ is an interval valued neutrosophic set on $V$ and

$$B = [T_{BL},T_{BU},I_{BL},I_{BU},F_{BL},F_{BU}] >$$

is an interval valued neutrosophic set on $E$ satisfies the following condition:

i. $V = \{v_1,v_2,...,v_n\}$ such that $T_{AL}: V \rightarrow [0,1], T_{AU}: V \rightarrow [0,1], I_{AL}: V \rightarrow [0,1], I_{AU}: V \rightarrow [0,1]$ and $F_{AL}: V \rightarrow [0,1], F_{AU}: V \rightarrow [0,1]$ denote the degree of truth-membership, degree of indeterminacy-membership and degree of falsity-membership of the element $x \in V$, respectively, and

$$0 \leq T_{AL}(v_i) + I_{AL}(v_i) + F_{AL}(v_i) \leq 3$$

for all $v_i \in V (i = 1,2, ..., n)$. We call $A$ the IVN vertex set of $V$. $B$ the IVN edge set of $E$, repectively. Note that $B$ is a symmetric IVN relation on $A$. We use the notation $(v_i, v_j)$ for an element of $E$. Thus, $G = (A,B)$ is an IVNG of $G^* = (V,E)$

$$T_{BL}(v_i,v_j) \leq \min[T_{AL}(v_i),T_{AL}(v_j)]$$
$$T_{BU}(v_i,v_j) \leq \min[T_{AU}(v_i),T_{AU}(v_j)]$$
$$I_{BL}(v_i,v_j) \geq \min[I_{AL}(v_i),I_{AL}(v_j)]$$
$$I_{BU}(v_i,v_j) \geq \min[I_{AU}(v_i),I_{AU}(v_j)]$$
$$F_{BL}(v_i,v_j) \geq \min[F_{AL}(v_i),F_{AL}(v_j)]$$
$$F_{BU}(v_i,v_j) \geq \min[F_{AU}(v_i),F_{AU}(v_j)]$$

Denotes the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $(v_i, v_j) \in E$ respectively, where

$$0 \leq T_{BL}(v_i, v_j) + I_{BL}(v_i, v_j) + F_{BL}(v_i, v_j) \leq 3$$

For all $(v_i, v_j) \in E (i,j = 1,2, ..., n)$. We call $A$ the IVN vertex set of $V$, $B$ the IVN edge set of $E$, repectively. Note that $B$ is a symmetric IVN relation on $A$. We use the notation $(v_i, v_j)$ for an element of $E$. Thus, $G = (A,B)$ is an IVNG of $G^* = (V,E)$

$$T_{BL}(v_i,v_j) \leq \min[T_{AL}(v_i),T_{AL}(v_j)]$$
$$T_{BU}(v_i,v_j) \leq \min[T_{AU}(v_i),T_{AU}(v_j)]$$
$$I_{BL}(v_i,v_j) \geq \min[I_{AL}(v_i),I_{AL}(v_j)]$$
$$I_{BU}(v_i,v_j) \geq \min[I_{AU}(v_i),I_{AU}(v_j)]$$
$$F_{BL}(v_i,v_j) \geq \min[F_{AL}(v_i),F_{AL}(v_j)]$$
$$F_{BU}(v_i,v_j) \geq \min[F_{AU}(v_i),F_{AU}(v_j)]$$

for all $(v_i, v_j) \in E$.

2.13.1. Example: Let consider $G = (A,B)$ of $G^* = (V,E)$ where $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{v_1v_2, v_2v_3, v_3v_4, v_4v_1\}$.

![Figure-2.5: Interval valued neutrosophic graph](image-url)
3. INTERVAL VALUED SIGNED NEUTROSOPHIC GRAPH

Throughout this paper, we approach some sign for interval valued neutrosophic graph. i.e. It assigns some positive or negative sign.

3.14. Definition: Let $T_A(x) = [T_A^L(x), T_A^U(x)]$, $I_A(x) = [I_A^L(x), I_A^U(x)]$ and $F_A(x) = [F_A^L(x), F_A^U(x)]$ be the interval valued truth-membership, interval valued indeterminacy-membership and interval valued falsity-membership and the relation is $T_B^L(x, y) \leq \min(T_A^L(x), T_A^L(y))$, $T_B^U(x, y) \leq \min(T_A^U(x), T_A^U(y))$, $I_B^L(x, y) \geq \max(I_A^L(x), I_A^L(y))$, $I_B^U(x, y) \geq \max(I_A^U(x), I_A^U(y))$, $F_B^L(x, y) \geq \max(F_A^L(x), F_A^L(y))$, $F_B^U(x, y) \geq \max(F_A^U(x), F_A^U(y))$.

Then interval valued signed neutrosophic graph (IVSNG) $A$ is $S_A(G) = (S(G^L), S(G^U))$ where $S(G^L)$ is sign of $G^L_B$ and $S(G^U) = $ sign of $G^U_B$.

The condition for sign is as follows:

If $T_B^L$ is greater than the both $I_B^L, F_B^L$ then it assigns positive sign otherwise negative sign. Similarly $T_B^U$ is greater than the both $I_B^U, F_B^U$ then it assigns positive sign otherwise negative sign.

3.14.1. Example: Let consider $G = (A, B)$ of $G^* = (V, E)$ where $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{v_1v_2, v_2v_3, v_3v_4, v_4v_1\}$ has the values are given in figure 3.1 then the interval valued signed neutrosophic graph is as follows.

We rewrite here as $L$-values and $U$-values separately for convenience.

![Figure-3.1: Interval valued signed neutrosophic graph](image-url)

$S_N(G)(v_1, v_2) = (+, -)$

i.e., $L$-values has positive sign and $U$- values negative sign. In this way

$S_N(G)(v_2, v_3) = (-, -)$

$S_N(G)(v_3, v_4) = (-, -)$

$S_N(G)(v_4, v_1) = (+, -)$.

3.15. Definition: An IVSNG is said to be $L$-balanced IVSNG if every cycle of $L$-values have even number of negative signed edges or all positive signed edges. An IVSNG is said to be $U$-balanced IVSNG if every cycle of $U$-values have even number of negative signed edges or all positive signed edges.
3.15.1. Example

![Figure-3.2: L-balanced IVSNG and U-balanced IVSNG](image)

3.16. Definition: If an IVSNG’s edge has alternative signs of $L$-values and $U$-values then it is called as self-balanced.

3.16.1. Example: In the figure 3.2, $S_N(G)(v_1, v_2)$ and $S_N(G)(v_4, v_1)$ are self-balanced and $S_N(G)(v_2, v_3)$ and $S_N(G)(v_3, v_4)$ are not self-balanced.

3.17. Definition: An IVSNG’s $L$-values is positive if every even length cycles of $L$-values having all negative signed edges.

3.18. Theorem: An IVSNG’s $L$-values is positive if every even length cycles of $L$-values having all negative signed edges.

Proof: If all edges contains negative sign in even length cycle then the product of edges sign is always positive. Hence it is always a positive signed graph.

Note: It is also true for also $U$-values.

3.19. Theorem: Odd length having all negative signed is always a negative signed graphs.

3.20. Theorem: If an IVNG is a complete IVNG then the IVSNG’s sign will be always negative. But the converse is need not be true.

Proof: Consider a complete IVNG. By the definition of complete IVNG is all the $L$-values $U$-values are equal to 1. i.e.,

$T_L = I_L = F_L = 1$ and $T_U = I_U = F_U = 1$.

Since the condition of signs $L$-values have positive sign when $T_L > I_L + F_L$ and otherwise negative.

Similarly $U$-values have positive sign when $T_U > I_U + F_U$ and otherwise negative.

In complete IVNG,

$T_L \leq I_L + F_L$ and $T_U \leq I_U + F_U$.

All its $L$-values and $U$-values have negative signs.

Hence, if there is a complete IVNG then their IVSNG is always negative.

But the converse is need not be true because it is not possible every negative signed IVNG be complete IVNG.

Hence the proof.
3. CONCLUSION

In this we have introduced Interval valued signed neutrosophic graph graph and balanced interval valued signed neutrosophic graph. In future research we can extend this concept to some more neutrosophic graphs and derive their properties.

REFERENCES