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OBSERVATIONS ON THE HYPERBOLA $8x^2 - 5y^2 = 27$

SHREEMATHI ADIGA[1*], N. ANUSHEELA[2] AND M. A. GOPALAN[3]

[1]Assistant Professor, Department of Mathematics,
Government First Grade College, Koteshwara,
Kundapura Taluk, Udupi – 576 222, Karnataka, India.

[2]Assistant Professor, Department of Mathematics,
Government Arts College, Udhagamandalam, The Nilgiris-643 002, India.

[3]Professor, Department of Mathematics,
Shrimati Indira Gandhi College, Trichy-620 002, Tamil Nadu, India.

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ABSTRACT

This paper concerns with the problem of finding non-zero distinct integer solutions to the Pell-like equation represented by $8x^2 - 5y^2 = 27$. A few interesting relations among its solutions are presented. Also, knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are presented. Employing the solutions, a special Pythagorean triangle is constructed.

***Keywords**: Binary quadratic, Hyperbola, Parabola, Pell equation, Integral solutions.*

***2010 Mathematics Subject Classification:** 11D09.*

INTRODUCTION

The diophantine equation offer an field for research due to their variety [1-3]. In particular, the binary quadratic diophantine equations of the form $ax^2 - by^2 = N,\ (a, b, N \neq 0)$ are rich in variety and have been analyzed by many mathematicians for their respective integer solutions for particular values of a, b and N. In this context, one may refer [4-16].

This communication concerns with the problem of obtaining non-zero distinct integer solutions to the binary quadratic equation representing hyperbola given by $8x^2 - 5y^2 = 27$. A few interesting relations among its solutions are presented. Knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are presented. Employing the solutions, a special Pythagorean triangle is constructed.

METHOD OF ANALYSIS

The binary quadratic equation representing hyperbola is given by

$$8x^2 - 5y^2 = 27 \tag{1}$$

Taking $x = X + 5T\ ,\ y = X + 8T$ (2)

In (1), it simplifies the equation

$$X^2 = 40T^2 + 9 \tag{3}$$

Corresponding Author: Shreemathi Adiga[1*],
[1]Assistant Professor, Department of Mathematics,
Government First Grade College, Koteshwara, Kundapura Taluk, Udupi – 576 222, Karnataka, India.

The smallest positive integer solution (T_0, X_0) of (3) is

$$T_0 = 1\ ,\ X_0 = 7$$

To obtain the other solutions of (3), consider the pellian equation

$$X^2 = 40T^2 + 1 \qquad (4)$$

whose smallest positive integer solution is

$$\tilde{T}_0 = 3,\ \tilde{X}_0 = 19$$

The general solution $(\tilde{T}_n, \tilde{X}_n)$ of (4) is given by

$$\tilde{X}_n + \sqrt{40}\tilde{T}_n = \left(19 + 3\sqrt{40}\right)^{n+1}, n = 0,1,2.... \qquad (5)$$

Since irrational roots occur in pairs, we have

$$\tilde{X}_n - \sqrt{40}\tilde{T}_n = \left(19 - 3\sqrt{40}\right)^{n+1}, n = 0,1,2.... \qquad (6)$$

From (5) and (6), solving for $\tilde{X}_n, \tilde{T}_n$, we have

$$\tilde{X}_n = \frac{1}{2}\left[\left(19 + 3\sqrt{40}\right)^{n+1} + \left(19 - 3\sqrt{40}\right)^{n+1}\right] = \frac{1}{2} f_n$$

$$\tilde{T}_n = \frac{1}{2\sqrt{40}}\left[\left(19 + 3\sqrt{40}\right)^{n+1} - \left(19 - 3\sqrt{40}\right)^{n+1}\right] = \frac{1}{2\sqrt{40}} g_n$$

Applying Brahmagupta lemma between the solutions (T_0, X_0) and $(\tilde{T}_n, \tilde{X}_n)$, the general solution (T_{n+1}, X_{n+1}) of (3) is found to be

$$\Rightarrow T_{n+1} = \frac{7}{2\sqrt{40}} g_n + \frac{1}{2} f_n \qquad (7)$$

$$\Rightarrow X_{n+1} = \frac{7}{2} f_n + \frac{\sqrt{40}}{2} g_n \qquad (8)$$

Using (7) and (8) in (2) we have

$$x_{n+1} = X_{n+1} + 5T_{n+1} = 6 f_n + \frac{75}{2\sqrt{40}} g_n \qquad (9)$$

$$y_{n+1} = X_{n+1} + 8T_{n+1} = \frac{15}{2} f_n + \frac{48}{\sqrt{40}} g_n \qquad (10)$$

Thus, (9) and (10) represent the integer solutions of the hyperbola (1).

A few numerical examples are given in the Table: 1 below:

Table-1: Numerical Examples

n	x_{n+1}	y_{n+1}
-1	12	15
0	453	573
1	17202	21759
2	653223	826269
3	24805272	31376463
4	941947113	1191479325

Recurrence relations for x and y are:

$$x_{n+3} - 38x_{n+2} + x_{n+1} = 0\ ,\ n = -1,0,1.....$$

$$y_{n+3} - 38y_{n+2} + y_{n+1} = 0\ ,\ n = -1,0,1.....$$

- A few interesting relations among the solutions are given below:
  - $38x_{n+2}-x_{n+1}-x_{n+3}=0$
  - $x_{n+2}-19x_{n+1}-15y_{n+1}=0$
  - $x_{n+3}-721x_{n+1}-570y_{n+1}=0$
  - $y_{n+2}-24x_{n+1}-19y_{n+1}=0$
  - $y_{n+3}-912x_{n+1}-721y_{n+1}=0$
  - $19x_{n+2}-x_{n+1}-15y_{n+2}=0$
  - $x_{n+3}-x_{n+1}-30y_{n+2}=0$
  - $19y_{n+3}-24x_{n+1}-721y_{n+2}=0$
  - $721x_{n+2}-19x_{n+1}-15y_{n+3}=0$
  - $721x_{n+3}-x_{n+1}-570y_{n+3}=0$
  - $19x_{n+3}-721x_{n+2}-15y_{n+1}=0$
  - $19y_{n+2}-24x_{n+2}-y_{n+1}=0$
  - $y_{n+3}-48x_{n+2}-y_{n+1}=0$
  - $x_{n+3}-19x_{n+2}-15y_{n+2}=0$
  - $y_{n+3}-24x_{n+2}-19y_{n+2}=0$
  - $19y_{n+2}+24x_{n+2}-y_{n+3}=0$
  - $721y_{n+2}-24x_{n+3}-19y_{n+1}=0$
  - $721y_{n+3}-912x_{n+3}-y_{n+1}=0$
  - $x_{n+1}-721x_{n+3}+570y_{n+3}=0$
  - $38y_{n+2}-y_{n+3}-y_{n+1}=0$
  - $x_{n+2}-19x_{n+3}-15y_{n+3}=0$
  - $y_{n+2}+24x_{n+3}-19y_{n+3}=0$

- Each of the following expressions represent a cubical integer.
  - $\frac{1}{405}\left[5730x_{3n+3}-150x_{3n+4}+3\left(5730x_{n+1}-150x_{n+2}\right)\right]$
  - $\frac{1}{3078}\left[43518x_{3n+3}-30x_{3n+5}+3\left(43518x_{n+1}-30x_{n+3}\right)\right]$
  - $\frac{1}{27}\left[192x_{3n+3}-150y_{3n+3}+3\left(192x_{n+1}-150y_{n+1}\right)\right]$
  - $\frac{1}{513}\left[7248x_{3n+3}-150y_{3n+4}+3\left(7248x_{n+1}-150y_{n+2}\right)\right]$
  - $\frac{1}{6489}\left[91744x_{3n+3}-50y_{3n+5}+3\left(91744x_{n+1}-50y_{n+3}\right)\right]$
  - $\frac{1}{405}\left[217590x_{3n+4}-5730x_{3n+5}+3\left(217590x_{n+2}-5730x_{n+3}\right)\right]$
  - $\frac{1}{513}\left[192x_{3n+4}-5730y_{3n+3}+3\left(192x_{n+2}-5730y_{n+1}\right)\right]$
  - $\frac{1}{27}\left[7248x_{3n+4}-5730y_{3n+4}+3\left(7248x_{n+2}-5730y_{n+2}\right)\right]$

- $\frac{1}{513}\left[275232x_{3n+4}-5730y_{3n+5}+3\left(275232x_{n+2}-5730y_{n+3}\right)\right]$
- $\frac{1}{6489}\left[64x_{3n+5}-72530y_{3n+3}+3\left(64x_{n+3}-72530y_{n+1}\right)\right]$
- $\frac{1}{27}\left[275232x_{3n+5}-217590y_{3n+5}+3\left(275232x_{n+3}-217590y_{n+1}\right)\right]$
- $\frac{1}{513}\left[7248x_{3n+5}-217590y_{3n+4}+3\left(7248x_{n+3}-217590y_{n+2}\right)\right]$
- $\frac{1}{162}\left[48y_{3n+4}-1812y_{3n+3}+3\left(48y_{n+2}-1812y_{n+1}\right)\right]$
- $\frac{1}{6156}\left[48y_{3n+5}-68808y_{3n+3}+3\left(48y_{n+3}-68808y_{n+1}\right)\right]$
- $\frac{1}{648}\left[7248y_{3n+5}-275232y_{3n+4}+3\left(7248y_{n+3}-275232y_{n+2}\right)\right]$

➢ Each of the following expressions represent bi-quadratic integer:

- $\frac{1}{405^2}\left[2320650x_{4n+4}-60750x_{4n+5}+4\left(5730x_{n+1}-150x_{n+2}\right)^2-328050\right]$
- $\frac{1}{3078^2}\left[133948404x_{4n+4}-92340x_{4n+6}+4\left(43518x_{n+1}-30x_{n+3}\right)^2-18948168\right]$
- $\frac{1}{27^2}\left[5184x_{4n+4}-4050y_{4n+4}+4\left(192x_{n+1}-150y_{n+2}\right)^2-1458\right]$
- $\frac{1}{513^2}\left[3718224x_{4n+4}-76950y_{4n+5}+4\left(7248x_{n+1}-150y_{n+2}\right)^2-526338\right]$
- $\frac{1}{6489^2}\left[595326816x_{4n+4}-324450y_{4n+6}+4\left(91744x_{n+1}-50y_{n+3}\right)^2-84214242\right]$
- $\frac{1}{405^2}\left[88123950x_{4n+5}-2320650x_{4n+6}+4\left(217590x_{n+2}-5730x_{n+3}\right)^2-328050\right]$
- $\frac{1}{513^2}\left[98496x_{4n+5}-2939490y_{4n+4}+4\left(192x_{n+2}-5730y_{n+1}\right)^2-526338\right]$
- $\frac{1}{27^2}\left[195696x_{4n+5}-154710y_{4n+5}+4\left(7248x_{n+2}-5730y_{n+2}\right)^2-1458\right]$
- $\frac{1}{513^2}\left[141194016x_{4n+5}-2939490y_{4n+6}+4\left(275232x_{n+2}-5730y_{n+3}\right)^2-526338\right]$
- $\frac{1}{6489^2}\left[415296x_{4n+6}-470647170y_{4n+4}+4\left(64x_{n+3}-72530y_{n+1}\right)^2-84214242\right]$
- $\frac{1}{27^2}\left[7431264x_{4n+6}-5874930y_{4n+6}+4\left(275232x_{n+3}-217590y_{n+3}\right)^2-1458\right]$
- $\frac{1}{162^2}\left[7776y_{4n+5}-293544y_{4n+4}+4\left(48y_{n+2}-1812y_{n+1}\right)^2-52488\right]$
- $\frac{1}{6156^2}\left[295488y_{4n+6}-423582048y_{4n+4}+4\left(48y_{n+3}-68808y_{n+1}\right)^2-75792672\right]$
- $\frac{1}{513^2}\left[3718224x_{4n+6}-111623670y_{4n+5}+4\left(7248x_{n+3}-217590y_{n+2}\right)^2-526338\right]$
- $\frac{1}{648^2}\left[4696704y_{4n+6}-178350336y_{4n+5}+4\left(7248y_{n+3}-275232y_{n+2}\right)^2-839808\right]$

- $\frac{1}{513^2}\left[141194016x_{4n+5}-2939490y_{4n+6}+4\left(275232x_{n+2}-5730y_{n+3}\right)^2-526338\right]$

➢ Each of the following expressions represents Nasty number:

- $\frac{1}{405}\left[4860+34380x_{2n+2}-150x_{2n+3}\right]$
- $\frac{1}{3078}\left[36936+261108x_{2n+2}-180x_{2n+4}\right]$
- $\frac{1}{27}\left[324+1152x_{2n+2}-900y_{2n+2}\right]$
- $\frac{1}{513}\left[6156+43488x_{2n+2}-900y_{2n+3}\right]$
- $\frac{1}{6489}\left[77868+550464x_{2n+2}-300y_{2n+4}\right]$
- $\frac{1}{405}\left[4860+1305540x_{2n+3}-34380x_{2n+4}\right]$
- $\frac{1}{513}\left[6156+1152x_{2n+3}-34380y_{2n+2}\right]$
- $\frac{1}{27}\left[324+43488x_{2n+3}-34380y_{2n+3}\right]$
- $\frac{1}{513}\left[6156+1651392x_{2n+3}-34380y_{2n+4}\right]$
- $\frac{1}{6489}\left[77868+384x_{2n+4}-435180y_{2n+2}\right]$
- $\frac{1}{27}\left[324+1651392x_{2n+4}-1305540y_{2n+4}\right]$
- $\frac{1}{513}\left[6156+43488x_{2n+4}-1305540y_{2n+3}\right]$
- $\frac{1}{162}\left[1944+288y_{2n+3}-10872y_{2n+2}\right]$
- $\frac{1}{6156}\left[73872+288y_{2n+4}-412848y_{2n+2}\right]$
- $\frac{1}{648}\left[7776+43488y_{2n+4}-1651392y_{2n+3}\right]$

➢ Each of the following expressions represents quintic integer

- $\frac{1}{405^3}\left[\begin{array}{l}\left(939863250x_{5n+5}-24603750x_{5n+6}\right)+5\left(5730x_{n+1}-150x_{n+2}\right)^3\\-5\left(939863250x_{n+1}-24603750x_{n+2}\right)\end{array}\right]$
- $\frac{1}{3078^3}\left[\begin{array}{l}\left(412293187500x_{5n+5}-284222520x_{5n+7}\right)+5\left(43518x_{n+1}-30x_{n+3}\right)^3\\-5\left(412293187500x_{n+1}-284222520x_{n+3}\right)\end{array}\right]$
- $\frac{1}{27^3}\left[\begin{array}{l}\left(139968x_{5n+5}-109350y_{5n+5}\right)+5\left(192x_{n+1}-150y_{n+1}\right)^3\\-5\left(139968x_{n+1}-109350y_{n+1}\right)\end{array}\right]$
- $\frac{1}{513^3}\left[\begin{array}{l}\left(1907448912x_{5n+5}-39475350y_{5n+6}\right)+5\left(7248x_{n+1}-150y_{n+2}\right)^3\\-5\left(1907448912x_{n+1}-39475350y_{n+2}\right)\end{array}\right]$

- $\frac{1}{6489^3}\begin{bmatrix}(3863075709000x_{5n+5} - 2105356050y_{5n+7}) + 5(91744x_{n+1} - 50y_{n+3})^3 \\ -5(3863075709000x_{n+1} - 2105356050y_{n+3})\end{bmatrix}$
- $\frac{1}{405^3}\begin{bmatrix}(35690199750x_{5n+6} - 939863250x_{5n+7}) + 5(217590x_{n+2} - 5730x_{n+3})^3 \\ -5(35690199750x_{n+2} - 939863250x_{n+3})\end{bmatrix}$
- $\frac{1}{513^3}\begin{bmatrix}(50528448x_{5n+6} - 1507958370y_{5n+5}) + 5(192x_{n+2} - 5730y_{n+1})^3 \\ -5(50528448x_{n+2} - 1507958370y_{n+1})\end{bmatrix}$
- $\frac{1}{27^3}\begin{bmatrix}(5283792x_{5n+6} - 4177170y_{5n+6}) + 5(7248x_{n+2} - 5730y_{n+2})^3 \\ -5(5283792x_{n+2} - 4177170y_{n+2})\end{bmatrix}$
- $\frac{1}{513^3}\begin{bmatrix}(72432530210x_{5n+6} - 1507958370y_{5n+7}) + 5(275232x_{n+2} - 5730y_{n+3})^3 \\ -5(72432530210x_{n+2} - 1507958370y_{n+3})\end{bmatrix}$
- $\frac{1}{6489^3}\begin{bmatrix}(2694855744x_{5n+7} - 3054029486000y_{5n+5}) + 5(64x_{n+3} - 72530y_{n+1})^3 \\ -5(2694855744x_{n+3} - 3054029486000y_{n+1})\end{bmatrix}$
- $\frac{1}{27^3}\begin{bmatrix}(200644128x_{5n+7} - 158623110y_{5n+7}) + 5(275232x_{n+3} - 217590y_{n+3})^3 \\ -5(200644128x_{n+3} - 158623110y_{n+3})\end{bmatrix}$
- $\frac{1}{513^3}\begin{bmatrix}(1907448912x_{5n+7} - 57262942710y_{5n+6}) + 5(7248x_{n+3} - 217590y_{n=2})^3 \\ -5(1907448912x_{n+3} - 57262942710y_{n+2})\end{bmatrix}$
- $\frac{1}{162^3}\begin{bmatrix}(1259712y_{5n+6} - 47554128y_{5n+5}) + 5(48y_{n+2} - 1812y_{n+1})^3 \\ -5(1259712y_{n+2} - 47554128y_{n+1})\end{bmatrix}$
- $\frac{1}{6156^3}\begin{bmatrix}(1819024128y_{5n+7} - 2607571087000y_{5n+5}) + 5(48y_{n+3} - 68808y_{n+1})^3 \\ -5(1819024128y_{n+3} - 2607571087000y_{n+1})\end{bmatrix}$
- $\frac{1}{6483^3}\begin{bmatrix}(3043464192y_{5n+7} - 115571017700y_{5n+6}) + 5(7248y_{n+3} - 275232y_{n+2})^3 \\ -5(3043464192y_{n+3} - 115571017700y_{n+2})\end{bmatrix}$

REMARKABLE OBSERVATIONS

1. Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of hyperbola which are presented in Table: 2 below:

Table-2: Hyperbolas

S.No	Hyperbolas	(X_n, Y_n)
1	$40X_n^2 - Y_n^2 = 26244000$	$[(5730x_{n+1} - 150x_{n+2}), (960x_{n+2} - 36240x_{n+1})]$
2	$40X_n^2 - Y_n^2 = 1515853440$	$[(43518x_{n+1} - 30x_{n+3}), (192x_{n+3} - 275232x_{n+1})]$
3	$40X_n^2 - Y_n^2 = 116640$	$[(192x_{n+1} - 150y_{n+1}), (960y_{n+1} - 1200x_{n+1})]$
4	$40X_n^2 - Y_n^2 = 42107040$	$[(7248x_{n+1} - 150y_{n+2}), (960y_{n+2} - 45840x_{n+1})]$
5	$40X_n^2 - Y_n^2 = 6737139360$	$[(91744x_{n+1} - 50y_{n+3}), (320y_{n+3} - 580240x_{n+1})]$
6	$40X_n^2 - Y_n^2 = 26244000$	$[(217590x_{n+2} - 5730x_{n+3}), (36240x_{n+3} - 1376160x_{n+2})]$
7	$40X_n^2 - Y_n^2 = 42107040$	$[(192x_{n+2} - 5730y_{n+1}), (36240y_{n+1} - 1200x_{n+2})]$
8	$40X_n^2 - Y_n^2 = 116640$	$[(7248x_{n+2} - 5730y_{n+2}), (36240y_{n+2} - 45840x_{n+2})]$

9	$40X_n^2-Y_n^2=42107040$	$\left[\left(275232x_{n+2}-5730y_{n+3}\right),\left(36240y_{n+3}-1740720x_{n+2}\right)\right]$
10	$40X_n^2-Y_n^2=6737139360$	$\left[\left(64x_{n+3}-72530y_{n+1}\right),\left(458720y_{n+1}-400x_{n+3}\right)\right]$
11	$40X_n^2-Y_n^2=116640$	$\left[\left(275232x_{n+3}-217590y_{n+3}\right),\left(1376160y_{n+3}-1740720x_{n+3}\right)\right]$
12	$40X_n^2-Y_n^2=42107040$	$\left[\left(7248x_{n+3}-217590y_{n+2}\right),\left(1376160y_{n+3}-45840x_{n+3}\right)\right]$
13	$40X_n^2-Y_n^2=4199040$	$\left[\left(48y_{n+2}-1812y_{n+1}\right),\left(11460y_{n+1}-300y_{n+2}\right)\right]$
14	$40X_n^2-Y_n^2=6063413760$	$\left[\left(48y_{n+3}-68808y_{n+1}\right),\left(435180y_{n+1}-300y_{n+3}\right)\right]$
15	$40X_n^2-Y_n^2=67184640$	$\left[\left(7248y_{n+3}-275232y_{n+2}\right),\left(1740720y_{n+2}-45840y_{n+3}\right)\right]$

2. Employing linear combination among the solutions for other choices of parabola which are presented in Table 3 below:

Table-3: Parabolas

S.No	Parabolas	$\left(X_n,Y_n\right)$
1	$16200X_n-Y_n^2=26244000$	$\left[\left(810+5730x_{2n+2}-150x_{2n+3}\right),\left(960x_{n+2}-36240x_{n+1}\right)\right]$
2	$123120X_n-Y_n^2=1515853440$	$\left[\left(6156+43518x_{2n+2}-30x_{2n+4}\right),\left(192x_{n+3}-275232x_{n+1}\right)\right]$
3	$1080X_n-Y_n^2=116640$	$\left[\left(54+192x_{2n+2}-150y_{2n+2}\right),\left(960y_{n+1}-1200x_{n+1}\right)\right]$
4	$20520X_n-Y_n^2=42107040$	$\left[\left(1026+7248x_{2n+2}-150y_{2n+3}\right),\left(960y_{n+2}-45840x_{n+1}\right)\right]$
5	$259560X_n-Y_n^2=6737139360$	$\left[\left(12978+91744x_{2n+2}-50y_{2n+4}\right),\left(320y_{n+3}-580240x_{n+1}\right)\right]$
6	$16200X_n-Y_n^2=26244000$	$\left[\left(810+217590x_{2n+3}-5730x_{2n+4}\right),\left(36240x_{n+3}-1376160x_{n+2}\right)\right]$
7	$20520X_n-Y_n^2=42107040$	$\left[\left(1026+192x_{2n+3}-5730y_{2n+2}\right),\left(36240y_{n+1}-1200x_{n+2}\right)\right]$
8	$1080X_n-Y_n^2=116640$	$\left[\left(54+7248x_{2n+3}-5730y_{2n+3}\right),\left(36240y_{n+2}-45840x_{n+2}\right)\right]$
9	$20520X_n-Y_n^2=42107040$	$\left[\left(1026+275232x_{2n+3}-5730y_{2n+4}\right),\left(36240y_{n+3}-1740720x_{n+2}\right)\right]$
10	$259560X_n-Y_n^2=6737139360$	$\left[\left(12978+64x_{2n+4}-72530y_{2n+2}\right),\left(458720y_{n+1}-400x_{n+3}\right)\right]$
11	$1080X_n-Y_n^2=116640$	$\left[\left(54+275232x_{2n+4}-217590y_{2n+4}\right),\left(1376160y_{n+3}-1740720x_{n+3}\right)\right]$

12	$20520X_n - Y_n^2 = 42107040$	$\left[\begin{array}{l}(1026+7248x_{2n+4}-217590y_{2n+3}),\\ (1376160y_{n+3}-45840x_{n+3})\end{array}\right]$
13	$6480X_n - Y_n^2 = 4199040$	$\left[\begin{array}{l}(324+48y_{2n+3}-1812y_{2n+2}),\\ (11460y_{n+1}-300y_{n+2})\end{array}\right]$
14	$246240X_n - Y_n^2 = 6063413760$	$\left[\begin{array}{l}(12312+48y_{2n+4}-68808y_{2n+2}),\\ (435180y_{n+1}-300y_{n+3})\end{array}\right]$
15	$25920X_n - Y_n^2 = 67184640$	$\left[\begin{array}{l}(1296+7248y_{2n+4}-275232y_{2n+3}),\\ (1740720y_{n+2}-45840y_{n+3})\end{array}\right]$

3. GENERATORS OF THE PYTHAGOREAN TRIANGLE

Let p, q be the non-zero distinct integers such that $p = x_{n+1} + y_{n+1}$, $q = x_{n+1}$

Note that $p > q > 0$. Treat p, q as the generators of the Pythagorean triangle $T(X, Y, Z)$

$$X = 2pq \ , \ Y = p^2 - q^2 \ , \ Z = p^2 + q^2 , \ p > q > 0$$

Let A,P represent the area and perimeter of T

Then the following interesting relations are observed.

- $5X - 4Y - Z = 27$
- $4Z - 9Y + \dfrac{20A}{P} = 27$
- $\dfrac{2A}{P} = x_{n+1} y_{n+1}$

Suppose p, q be the non-zero distinct integers such that

$$p = x_{n+1} + y_{n+1} \ , \ q = y_{n+1}$$

In this case, the corresponding $T(X, Y, Z)$ Pythagorean satisfies the relation

$$5Y - 16X + 11Z = 54$$

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