

REVIEW OF MINIMUM SPANNING TREE IN AN UNDIRECTED GRAPH

S. N. BANASODE¹ AND Y. M. UMATHAR^{*2}

¹Department of Mathematics,
R. L. Science Institute Belagavi, Karnataka, India.

²Department of Mathematics,
K. L. E. Technological University, Hubballi, Karnataka, India.

(Received On: 14-07-18; Revised & Accepted On: 29-08-18)

ABSTRACT

A Minimum spanning tree of a weighted graph is a tree having all vertices of a given graph with weight less than or equal to the weight of all such possible spanning trees from the given graph. In this paper we propose new method and corresponding algorithm to construct a minimum spanning tree of a weighted graph. This new approach is based on **Edge Elimination Method**. In this method we eliminate an edge with highest weight in each of the smallest cycle of the weighted graph until no cycle is remained in the graph with care that graph is not disconnected at any stage. Once such a graph (tree) is obtained it is the required minimum spanning tree in an undirected weighted graph.

Keywords: Graphs, Weighted graph, Tree, minimum spanning tree

AMS Subject Classification: 05C50, 05C69.

1. INTRODUCTION

A spanning tree of a connected graph is sub graph that is a tree and connects all vertices of a tree. When weights are assigned to the edges of a graph, graph is said to be weighted graph. Then minimum spanning tree (MST) is obtained from the graph, which is spanning tree with weight less than or equal to the weight of every other possible spanning tree [3]. The problem of finding MST in an undirected graph is one of the combinatorial optimization problem. [11]. The problem of obtaining MST has been studied well and many efficient algorithms have been developed by many researchers [1, 2]. Kruskal and Prims methods are the most commonly used methods for finding MST of a weighted graph.

In this paper we consider a weighted graph and will present a simple procedure that helps to find minimum spanning tree. This method will help in some network applications.

2. APPLICATION OF MINIMUM SPANNING TREE

In Computer Science Engineering and Electronic & Communication Engineering, image processing is one of most important and fast growing branch, wherein process of analysis and manipulation of digitized image takes place, especially in order to improve the quality of an image. It is a method by which the information from the image is extracted. Using MST concept image processing method can be improved. MST provides the calculation of alignment of the picture.

3. SOME EXISTING ALGORITHMS FOR THE CONSTRUCTION OF MST

Kruskal Algorithm and Prims Algorithm are the two fundamental Algorithms for the construction of Minimum Spanning Tree. In addition to this, authors [1, 2, 3] have given an efficient algorithm for the construction of MST. For the detail method readers are requested to refer the same.

Corresponding Author: Y. M. Umathar^{*2}

²Department of Mathematics, K. L. E. Technological University, Hubballi, Karnataka, India.

4. NEW PROPOSED ALGORITHM FOR THE CONSTRUCTION OF MST

Algorithm:

Input: Weighted graph
 Output: Minimum Spanning Tree

Step-1: Start

Step-2: Find all possible cycles of smallest lengths (cycles that does not contain any cycle inside to itself) in the graph. Arrange the edges of each cycle in non decreasing order according to their weights

Step-3: Eliminate an edge from each cycle of the graph with highest weight. Care is taken that graph is not disconnected

Step-4: Continue the process of elimination of edges of highest weight from each of the cycles obtained in the step 3 using step 2, until no cycle remains in the graph. So that there are exactly (n-1) edges

5. GRAPHICAL ILLUSTRATION OF THE ALGORITHM

Step-1: The method is explained by considering weighted graph shown in **figure. 1**. Graph is having nine points and weights are assigned to each edge. It is assumed that, graph is undirected. We wish to find minimum spanning tree.

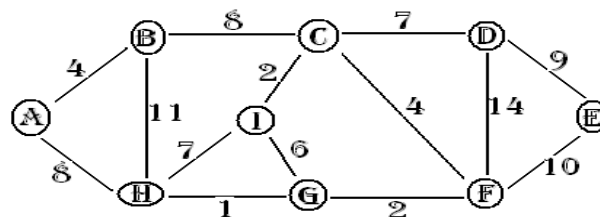


Figure-1: Weighted graph

Step-2: All possible cycles of smallest lengths in the graph are noted. Edges of each cycle are in non decreasing order according to their weights. Edge with highest weight in each cycle is circled.

Cycle 01: ABHA, Edges AB – 4, AH – 8,

HB -11

Cycle 02: BCIHB, Edges IC – 2, HI – 7, BC – 8,

BH -11

Cycle 03: HIGH, Edges HG – 1, IG – 6,

HI - 7

Cycles 04: CIGFC, Edges CI – 2, GF – 2, CF – 4,

IG - 6

Cycle 05: CDFC, Edges CF – 4, CD – 7,

DF - 14

Cycle 06: DEFD, Edges DE – 9, EF – 10,

FD - 14

The weighted graph with circled edges in each cycle is shown in figure 2. Some weights have been double Trees since they are the highest weights from two cycles.

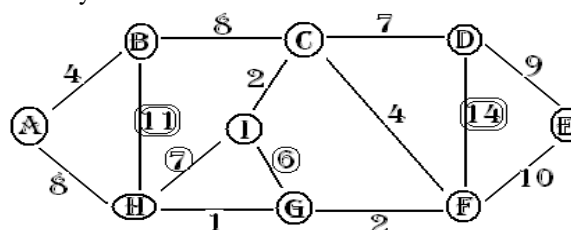


Figure-2: Graph with Trees edges

Step-3: circled edges are eliminated from the graph. With care that graph is not disconnected. After elimination of the circled edges the resulting graph is shown **Figure 3**.

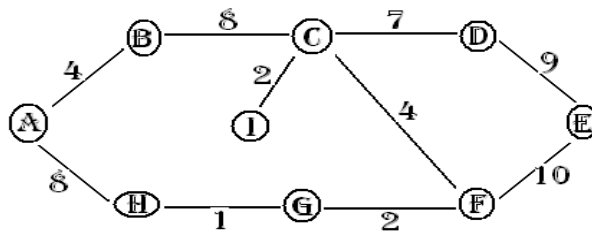


Figure-3: Graph after step 3

Step-4: Process of elimination of edges of highest weight is continued from each of the cycles obtained in the step 3 using step 2, until no cycle remains in the graph. So that there are exactly (n-1) edges

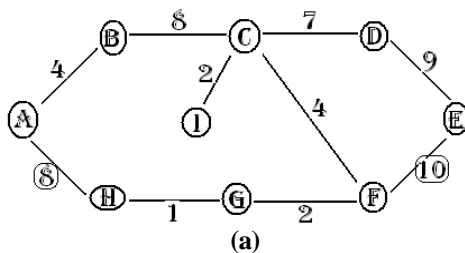
The resulting graph after step 4 is in **figure 4**

Cycle 07: ABCFGHA, HG – 1, GF – 2, FC – 4, AB – 4, BC – 8, AH - 8

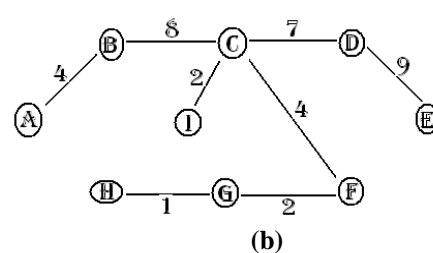
Cycle 08: CDEFC, CF – 4, CD – 7, DE – 9,

(Edges **AH** and **BC** are of equal weight, that is **8**, eliminating an edge **AH**)

EF - 10



(a)
Graph with **AH** is Trees

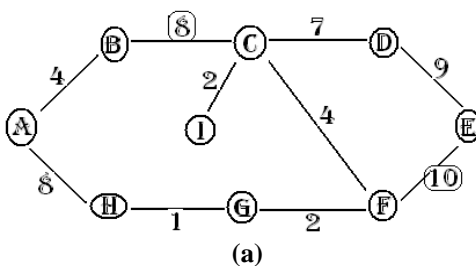


(b)
Graph after elimination of **AH**

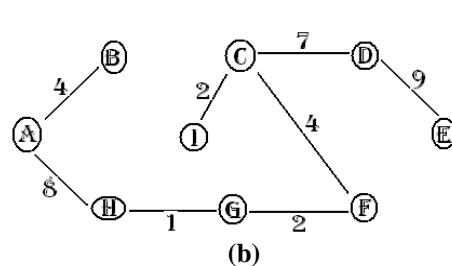
Figure-4

Note: If there are more than one edge with highest weight and equal weight then any one is eliminated. Corresponding to each edge there exists a minimum spanning tree.

After the elimination of an edge **BC**, The resulting graph is shown in **figure 5**



(a)
Graph with **BC** is Trees



(b)
Graph after elimination of **BC**

Figure 5

The graphs in the figure 4 and in the figure 5 after elimination of an edge **AH** and edge **BC** respectively are the required Minimum Spanning Trees for the weighted graph given in the **figure 1**

6. CONCLUSION

1. Finding Minimum Spanning Tree using an “**Edge Elimination Method**” is an alternative method which is convenient and efficient.
2. All possible number of spanning trees can be obtained simultaneously.

7. REFERENCES

- [1]. Abishek Kumar, Gaurav Kumar, *An Efficient Method to Construct Minimum Spanning Tree*. International Journal of latest technology in engineering, management and applied sciences, IV(X), 48-51, 2015.
- [2]. Ardhendu Mandal, Jayanta Dutta, *An Efficient Technique to Construct a Minimum Spanning Tree*, International Journal of Advanced Research in Computer Science and Software Engineering, 2(10), 93-97, 2012.
- [3]. D. Vijayaalakshir, R Kalaivani, *Minimum Cost of Spanning Tree Using Matrix Algorithm*, International Journal of Scientific research publication, 4(9), 1-5, 2014.
- [4]. Graham, R L, Hell, Pavol, *On the History of Minimum Spanning Tree Problem*, Annals of History of Computing, 7(1), 44-57, 1985.

Source of support: Nil, Conflict of interest: None Declared.

[Copy right © 2018. This is an Open Access article distributed under the terms of the International Journal of Mathematical Archive (IJMA), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.]