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ESTIMATION OF THE MARSHALL–OLKIN EXTENDED WEIBULL DISTRIBUTION PARAMETERS UNDER ADAPTIVE CENSORING SCHEMES

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ABSTRACT

In this paper, we discuss the estimation of the Marshall–Olkin Extended Weibull (MOEW) distribution under complete censoring data and adaptive type-II progressive censoring scheme by using the maximum likelihood method. We discuss the interval estimation for parameters of the MOEW, where we use this length of confidence interval in comparison. The optimal censoring scheme has been suggested by using two different optimality criteria. We provide an application to schemes of real data which illustrates the usefulness of the model in censoring.

Keywords: *Marshall–Olkin Extended Weibull Distribution, adaptive type-II progressive Censoring, Maximum Likelihood Estimation, Interval Estimation, Monte Carlo.*

1. INTRODUCTION

The need for extended forms of the Weibull distribution arises in many applied areas, and the Weibull distribution is one of the most popular distributions in analyzing lifetime data. For some extended forms of the Weibull distribution and applications referred to Cordeiro et al. (2010) and Silva et al. (2010). Cordeiro and Lemonte (2013) used the Marshall–Olkin family to generating the Marshall–Olkin Extended Weibull distribution (MOEW). A random variable x has MOEW distribution with parameters α, γ and θ, say if its cumulative distribution function (cdf), probability density function (pdf) and the quantile function are given as follows:
the cdf is

$$F(x;\alpha,\gamma,\theta) = \frac{1-e^{-\gamma x^{\theta}}}{1-\bar{\alpha}e^{-\gamma x^{\theta}}}, \qquad (1)$$

and the pdf is

$$f(x;\alpha,\gamma,\theta) = \frac{\alpha\gamma\theta x^{\theta-1}e^{-\gamma x^{\theta}}}{\left(1-\bar{\alpha}e^{-\gamma x^{\theta}}\right)^{2}}, \qquad (2)$$

and the quantile function of the MOEW is

$$x = \gamma^{-1/\theta}\left[\ln\left(\frac{1-\bar{\alpha}u}{1-u}\right)\right]^{1/\theta}, \qquad 0<u<1. \qquad (3)$$

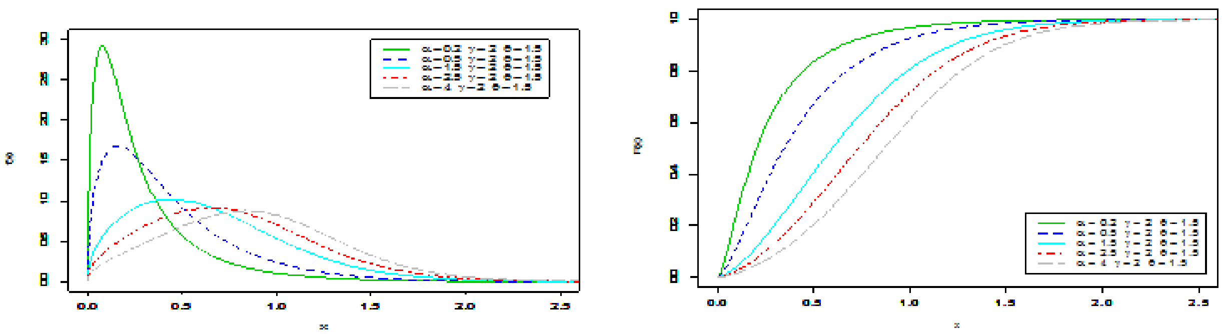

Figure-1: Plot of MOEW distribution

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Cordeiro and Lemonte (2013) discussed the MOEW model is better than the Marshall–Olkin Extended exponential (MOEE), Weibull and exponential models based on the likelihood ratio statistics and the MOEW distribution has the lowest AIC, BIC and HQIC values among all fitted model, so it could be chosen as the better model of censoring. Kundu and Pradhan (2009) discussed the two most common censoring schemes are termed as type-I and type-II censoring schemes. A progressive type-II censoring scheme introduced by Ng *et al.* (2004). Kim and Han (2009) discussed, progressively type-II censored sampling is an important method of obtaining data in lifetime studies, to more information see Balakrishnan (2007) and more application see Almetwaly and Almongy (2018). While, Ng *et al.* (2009) suggested an adaptive type-II progressive censoring scheme (AT-II PCS) in which the effective number of failures m is fixed in advance and the progressive censoring scheme $R_1 \dots R_m$ is provided, but the values of the R_i may be change accordingly during the experiment, but the experimenter consider provides a time T.

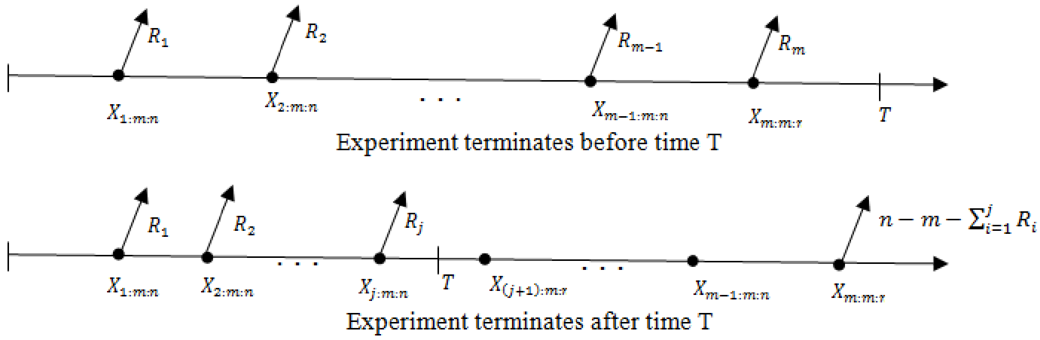

Figure-2: Schematic representation of adaptive type-II progressive censoring

$$\begin{cases} \text{if } (x_m \le T) \rightarrow \text{ the experiment stops at this time(Progressive)} \\ \text{if } (x_m > T) \rightarrow \text{ adapt the number of items progressively with removed items(Adaptive)} \end{cases}.$$

To more example, see Ye *et al.* (2014), Sobhi and Soliman (2016) and Hemmati and Khorram (2017). For instance the book by Balakrishnan and Aggrawalla (2000), and an excellent review article by Balakrishnan *et al.* (2007).

2. THE MAXIMUM LIKELIHOOD ESTIMATION METHOD

Based on the observed sample $x_1 < \cdots < x_m$ from an adaptive Type-II progressive censoring scheme, $R_1, \dots, R_m$, the likelihood function of MOEW distribution can be written as

$$L = A(\alpha\gamma\theta)^m e^{-\gamma \sum_{i=1}^{m} x_i{}^\theta} \prod_{i=1}^{m} \left[\frac{x_i{}^{\theta-1}}{\left(1-\bar{\alpha}e^{-\gamma x_i{}^\theta}\right)^2} \left(\frac{\alpha e^{-\gamma x_i{}^\theta}}{1-\bar{\alpha}e^{-\gamma x_i{}^\theta}} \right)^{R_i} \right] \left(\frac{\alpha e^{-\gamma x_m{}^\theta}}{1-\bar{\alpha}e^{-\gamma x_m{}^\theta}} \right)^{n-m-\sum_{i=1}^{j} R_i}, \tag{4}$$

where A is a constant which doesn't depend on parameters.

The natural logarithm of the maximum likelihood function equation can be obtained as follows:

$$\begin{aligned} \ln L = \ln A + m\ln(\alpha\gamma\theta) &- \gamma \sum_{i=1}^{m} x_i^\theta + (\theta-1)\sum_{i=1}^{m} \ln(x_i) - 2\sum_{i=1}^{m} \ln\left(1-\bar{\alpha}e^{-\gamma x_i{}^\theta}\right) \\ &+ \sum_{i=1}^{m} R_i \left(\ln(\alpha) - \gamma x_i{}^\theta - \ln\left(1-\bar{\alpha}e^{-\gamma x_i{}^\theta}\right) \right) \\ &+ \left(n-m-\sum_{i=1}^{j} R_i \right)\left(\ln(\alpha) - \gamma x_m{}^\theta - \ln\left(1-\bar{\alpha}e^{-\gamma x_m{}^\theta}\right) \right), \end{aligned} \tag{5}$$

respectively. To obtain the normal equations for the unknown parameters, we partially differentiate the equation (5) partially with respect to the parameters α, γ and θ, then equate them to zero. The estimators $\hat{\alpha}, \hat{\gamma}$ and $\hat{\theta}$ can be obtained as the solution of the following equations.

$$\frac{\partial \ln L}{\partial \alpha} = \frac{m}{\alpha} - 2\sum_{i=1}^{m} \frac{e^{-\gamma x_i{}^\theta}}{1-\bar{\alpha}e^{-\gamma x_i{}^\theta}} + \sum_{i=1}^{m} R_i \left(\frac{1}{\alpha} - \frac{e^{-\gamma x_i{}^\theta}}{1-\bar{\alpha}e^{-\gamma x_i{}^\theta}} \right) + \left(n-m-\sum_{i=1}^{j} R_i \right)\left(\frac{1}{\alpha} - \frac{e^{-\gamma x_m{}^\theta}}{1-\bar{\alpha}e^{-\gamma x_m{}^\theta}} \right), \tag{6}$$

$$\begin{aligned} \frac{\partial \ln L}{\partial \gamma} = \frac{m}{\gamma} - \sum_{i=1}^{m} x_i^\theta + 2\bar{\alpha}\sum_{i=1}^{m} \frac{x_i{}^\theta e^{-\gamma x_i{}^\theta}}{1-\bar{\alpha}e^{-\gamma x_i{}^\theta}} + \bar{\alpha}\sum_{i=1}^{m} R_i \left(x_i{}^\theta - \frac{x_i{}^\theta e^{-\gamma x_i{}^\theta}}{1-\bar{\alpha}e^{-\gamma x_i{}^\theta}} \right) \\ + \left(n-m-\sum_{i=1}^{j} R_i \right)\left(x_m{}^\theta - \frac{x_m{}^\theta e^{-\gamma x_m{}^\theta}}{1-\bar{\alpha}e^{-\gamma x_m{}^\theta}} \right), \end{aligned} \tag{7}$$

$$\frac{\partial \ln L}{\partial \theta}=\frac{m}{\theta}+-\gamma\sum_{i=1}^{m}x_i^{\theta}\ln x_i+\sum_{i=1}^{m}\ln(x_i)\ -2\bar{\alpha}\gamma\sum_{i=1}^{m}\frac{{x_i}^{\theta}e^{-\gamma {x_i}^{\theta}}\ln(x_i)}{1-\bar{\alpha}e^{-\gamma {x_i}^{\theta}}}$$
$$+\sum_{i=1}^{m}R_i\left(\gamma {x_i}^{\theta}\ln(x_i)\ -\bar{\alpha}\gamma\sum_{i=1}^{m}\frac{{x_i}^{\theta}e^{-\gamma {x_i}^{\theta}}\ln(x_i)}{1-\bar{\alpha}e^{-\gamma {x_i}^{\theta}}}\right)$$
$$+\left(n-m-\sum_{i=1}^{j}R_i\right)\left(\gamma {x_m}^{\theta}\ln(x_m)\ -\bar{\alpha}\gamma\frac{{x_m}^{\theta}e^{-\gamma {x_m}^{\theta}}\ln(x_m)}{1-\bar{\alpha}e^{-\gamma {x_m}^{\theta}}}\right). \tag{8}$$

But the three equation has to be performed numerically using a nonlinear optimization algorithm by use conjugate gradients (CG) method can be successful in much larger optimization problems.

3. ASYMPTOTIC CONFIDENCE INTERVAL

In this section, we propose the asymptotic confidence intervals using methods of estimation. Keeping this in mind, we may propose the asymptotic confidence intervals using ML method can be used to construct unknown confidence intervals for the parameters. $I(\hat{\alpha},\hat{\gamma},\hat{\theta})$ is the observed inverse Fishers information matrix of MOEW distribution and is define as:

$$I(\hat{\alpha},\hat{\gamma},\hat{\theta})=\begin{bmatrix}-L''_{\alpha} & -L''_{\alpha\gamma} & -L''_{\alpha\theta}\\ -L''_{\gamma\alpha} & -L''_{\gamma} & -L''_{\gamma\theta}\\ -L''_{\theta\alpha} & -L''_{\theta\gamma} & -L''_{\theta}\end{bmatrix}^{-1}=\begin{vmatrix}I_{\hat{\alpha}} & I_{\hat{\alpha}\hat{\gamma}} & I_{\hat{\alpha}\hat{\theta}}\\ I_{\hat{\gamma}\hat{\alpha}} & I_{\hat{\gamma}} & I_{\hat{\gamma}\hat{\theta}}\\ I_{\hat{\theta}\hat{\alpha}} & I_{\hat{\theta}\hat{\gamma}} & I_{\hat{\theta}}\end{vmatrix} \tag{9}$$

And then compensation for the value of each parameter estimated for each methods and get the matrix mathematically through simulation package. An approximate 95% two side confidence intervals for (α,γ and θ) are respectively

$$\hat{\alpha}\pm Z_{0.025}\sqrt{I_{\hat{\alpha}}}\ ,\qquad \hat{\gamma}\pm Z_{0.025}\sqrt{I_{\hat{\gamma}}}\ ,\qquad \hat{\theta}\pm Z_{0.025}\sqrt{I_{\hat{\theta}}}$$

where

$$L''_{\alpha}=\frac{\partial^2\ln L}{\partial\alpha^2}=\frac{-m}{\alpha^2}+2\sum_{i=1}^{m}\big(W(x_i,\alpha,\gamma,\theta)\big)^2+\sum_{i=1}^{m}R_i\left(\frac{-1}{\alpha^2}+\big(W(x_i,\alpha,\gamma,\theta)\big)^2\right)$$
$$+\left(n-m-\sum_{i=1}^{j}R_i\right)\left(\frac{-1}{\alpha^2}+\big(W(x_m,\alpha,\gamma,\theta)\big)^2\right),$$

$$L''_{\gamma}=\frac{\partial^2\ln L}{\partial\gamma^2}=\frac{-m}{\gamma^2}+2\bar{\alpha}\sum_{i=1}^{m}{x_i}^{2\theta}W(x_i,\alpha,\gamma,\theta)\big(1+\bar{\alpha}\,W(x_i,\alpha,\gamma,\theta)\big)$$
$$+\bar{\alpha}\sum_{i=1}^{m}R_i\left({x_i}^{2\theta}W(x_i,\alpha,\gamma,\theta)\big(1+\bar{\alpha}\,W(x_i,\alpha,\gamma,\theta)\big)\right)$$
$$+\left(n-m-\sum_{i=1}^{j}R_i\right)\left({x_m}^{2\theta}W(x_m,\alpha,\gamma,\theta)\big(1+\bar{\alpha}\,W(x_m,\alpha,\gamma,\theta)\big)\right),$$

$$L''_{\theta}=\frac{\partial^2\ln L}{\partial\theta^2}=\frac{-m}{\theta^2}\pm\gamma\sum_{i=1}^{m}x_i^{\theta}(\ln x_i)^2-2\bar{\alpha}\gamma\sum_{i=1}^{m}{x_i}^{\theta}W(x_i,\alpha,\gamma,\theta)\left(1-\gamma {x_i}^{\theta}-\gamma\bar{\alpha}{x_i}^{\theta}W(x_i,\alpha,\gamma,\theta)\right)(\ln x_i)^2$$
$$+\sum_{i=1}^{m}R_i\left(\gamma {x_i}^{\theta}(\ln(x_i)\,)^2-\bar{\alpha}\gamma\sum_{i=1}^{m}{x_i}^{\theta}W(x_i,\alpha,\gamma,\theta)\left(1-\gamma {x_i}^{\theta}-\gamma\bar{\alpha}{x_i}^{\theta}W(x_i,\alpha,\gamma,\theta)\right)(\ln x_i)^2\right)$$
$$+\left(n-m-\sum_{i=1}^{j}R_i\right)\left(\gamma {x_m}^{\theta}(\ln(x_m)\,)^2\right.$$
$$\left.-\bar{\alpha}\gamma\sum_{i=1}^{m}{x_m}^{\theta}W(x_m,\alpha,\gamma,\theta)\left(1-\gamma {x_m}^{\theta}-\gamma\bar{\alpha}{x_m}^{\theta}W(x_m,\alpha,\gamma,\theta)\right)(\ln(x_m))^2\right).$$

Where $W(x_i,\alpha,\gamma,\theta)=\frac{e^{-\gamma {x_i}^{\theta}}}{1-\bar{\alpha}e^{-\gamma {x_i}^{\theta}}}$

4. SIMULATION STUDY

In this section; Monte Carlo simulation is done for comparison between censoring schemes based on maximum likelihood estimation methods under adaptive type-II progressive censoring scheme. For estimating the parameters of MOEW Distribution in life time by R language.

Simulation Algorithm Monte Carlo experiments were carried out based on the following data- generated form MOEW distribution by use the equation (3), where x are distributed as MOEW for different shape parameters $(\alpha, \gamma, \theta) = (0.5, 2, 1.5)$ and $(1.5, 2, 1.5)$, and for different sample size $n = 30, 50, 70$ and 100, different ratio of effective sample sizes $r = \frac{m}{n}$, different of time for experiment terminates and set of different samples schemes, where

- Scheme 1: $R_1 = R_2 = \cdots = R_{m-1} = 0, and\ R_m = n - m.$
- Scheme 2: $R_1 = n - m\ and\ R_2 = R_3 = \cdots = R_m = 0.$
- Scheme 3: $R_1 = R_2 = \cdots = R_{m-1} = 1, and\ R_m = n - 2m - 1.$

We could define the best scheme as the scheme which minimizes the mean squared error (MSE), the bias of estimation and the length of confidence interval (Length.CI) of the estimator.

Some graphs of MSE in the case of complete sample and different schemes are plotted and attached them as following:

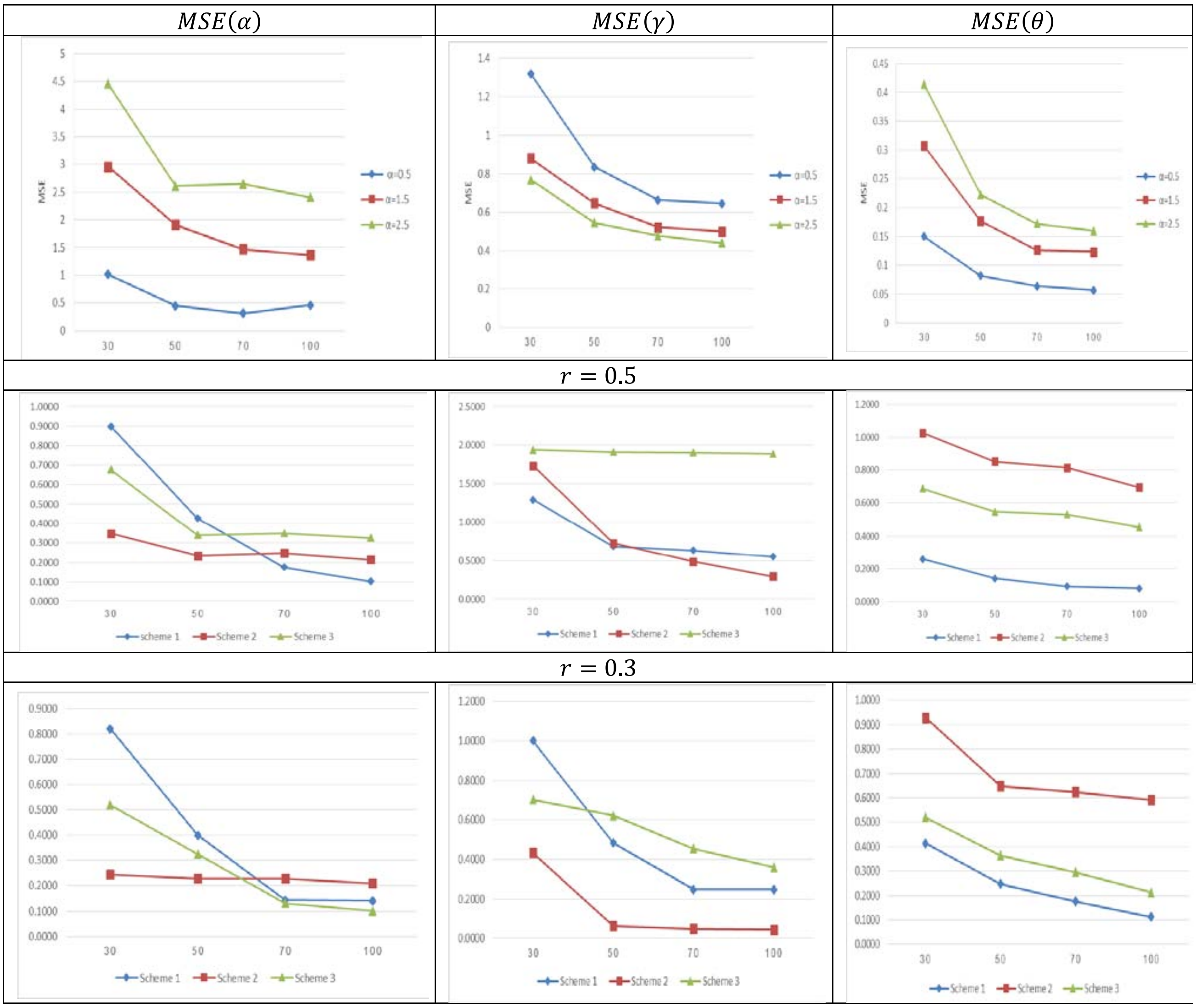

Figure-3: MSE of the estimates for different parameters with variation of sample size (n)

Table-1: Estimation to adaptive type-II progressive censoring scheme when $\alpha = 0.5$ and $T = 0.25$

		Scheme 1			Scheme 2			Scheme 3		
$r = 0.3$										
n		Bias	MSE	Length.CI	Bias	MSE	Length.CI	Bias	MSE	Length.CI
30	$\hat{\alpha}$	0.0725	0.8198	3.5396	-0.4682	0.2452	0.6328	-0.0923	0.5201	2.8053
	$\hat{\gamma}$	0.1884	1.0001	3.8519	0.1925	0.4340	2.4709	0.3111	0.7021	3.0513
	$\hat{\theta}$	0.2949	0.4129	2.2392	0.8600	0.9277	1.7009	0.4559	0.5197	2.1904
50	$\hat{\alpha}$	0.0714	0.4004	2.4659	-0.4769	0.2283	0.1111	-0.1185	0.3248	2.1865
	$\hat{\gamma}$	0.1672	0.4841	2.6489	0.1447	0.0630	0.8045	0.3049	0.6224	3.9792
	$\hat{\theta}$	0.1811	0.2471	1.8157	0.7347	0.6483	1.2921	0.3725	0.3635	1.8595
70	$\hat{\alpha}$	-0.0073	0.1432	1.4840	-0.4780	0.2290	0.0889	-0.1715	0.1311	1.2507
	$\hat{\gamma}$	0.0655	0.2477	1.9351	0.1442	0.0485	0.6529	0.2538	0.4541	2.4482
	$\hat{\theta}$	0.1578	0.1754	1.5214	0.7306	0.6246	1.1816	0.3621	0.2954	1.5899
100	$\hat{\alpha}$	0.1200	0.1407	2.6965	-0.4796	0.2102	0.0401	-0.0665	0.1003	2.6481
	$\hat{\gamma}$	0.2234	0.2410	3.4524	0.1769	0.0435	0.5846	0.4270	0.3591	3.8763
	$\hat{\theta}$	0.0881	0.1127	1.2705	0.7147	0.5913	1.1133	0.2969	0.2125	1.3831
$r = 0.5$										
n		Bias	MSE	Length.CI	Bias	MSE	Length.CI	Bias	MSE	Length.CI
30	$\hat{\alpha}$	0.2299	0.8988	3.6073	-0.3827	0.3486	1.7635	-0.0776	0.6754	3.2087
	$\hat{\gamma}$	0.3532	1.2874	4.2289	0.4846	1.7305	4.7964	0.7362	1.9392	4.6360
	$\hat{\theta}$	0.1683	0.2623	1.8970	0.8861	1.0268	1.9277	0.6301	0.6879	2.1152
50	$\hat{\alpha}$	0.1236	0.4270	2.5166	-0.4313	0.2343	0.8614	-0.1639	0.3406	2.1966
	$\hat{\gamma}$	0.1899	0.6879	3.1665	0.3594	0.7288	3.0370	0.7294	1.9143	4.6109
	$\hat{\theta}$	0.0993	0.1430	1.4310	0.8387	0.8532	1.5179	0.5897	0.5463	1.7473
70	$\hat{\alpha}$	0.0635	0.1759	1.6259	-0.4403	0.2473	0.9066	-0.1913	0.3497	2.1947
	$\hat{\gamma}$	0.1707	0.6352	3.0533	0.3161	0.4897	2.4483	0.6912	1.9071	4.7928
	$\hat{\theta}$	0.0909	0.0948	1.1538	0.8414	0.8152	1.2845	0.6116	0.5288	1.5427
100	$\hat{\alpha}$	0.1663	0.1034	2.7051	-0.4525	0.2148	0.3922	-0.1899	0.3252	2.4465
	$\hat{\gamma}$	0.2660	0.5524	3.4675	0.2679	0.2961	1.8577	0.6855	1.8873	4.6692
	$\hat{\theta}$	0.0620	0.0825	1.1000	0.7858	0.6947	1.0895	0.5799	0.4545	1.3484

Table-2: Estimation to adaptive type-II progressive censoring scheme when $\alpha = 1.5$ and $T = 0.25$

		Scheme 1			Scheme 2			Scheme 3		
$r = 0.3$										
n		Bias	MSE	Length.CI	Bias	MSE	Length.CI	Bias	MSE	Length.CI
30	$\hat{\alpha}$	-0.0711	3.7200	5.1361	-1.2094	2.2831	3.5526	-0.2053	3.6787	5.0173
	$\hat{\gamma}$	0.4418	2.5740	4.6054	1.8546	8.7640	7.1510	0.7443	2.9200	4.5838
	$\hat{\theta}$	0.4122	0.7374	2.9545	1.4534	2.8316	3.3260	0.5382	0.8673	2.9808
50	$\hat{\alpha}$	0.3394	3.0248	6.6899	-1.1426	2.2095	4.1206	0.2754	3.0566	6.7712
	$\hat{\gamma}$	0.5044	2.1481	5.3970	2.0539	8.1458	8.3449	0.9044	2.8602	5.6048
	$\hat{\theta}$	0.2251	0.3616	2.1870	1.3158	2.3262	3.0247	0.3453	0.4698	2.3220
70	$\hat{\alpha}$	0.2179	1.9189	5.3651	-1.2205	2.1729	3.2419	0.1387	2.0464	5.5840
	$\hat{\gamma}$	0.3661	1.5671	4.6950	1.9194	7.2128	7.3674	0.8240	2.3889	5.1286
	$\hat{\theta}$	0.1838	0.2298	1.7365	1.3087	2.1486	2.5891	0.3196	0.3561	1.9764
100	$\hat{\alpha}$	0.2722	1.4260	4.5601	-1.2944	2.0301	2.3353	0.1427	1.7564	5.1675
	$\hat{\gamma}$	0.3135	0.9819	3.6866	1.7821	6.2087	6.8300	0.7737	1.7347	4.1805
	$\hat{\theta}$	0.0822	0.1167	1.3002	1.1917	1.7363	2.2054	0.2689	0.2735	1.7590
$r = 0.5$										
n		Bias	MSE	Length.CI	Bias	MSE	Length.CI	Bias	MSE	Length.CI
30	$\hat{\alpha}$	0.2303	3.9292	5.8688	-0.1596	3.9655	6.7248	0.2453	3.1506	6.1337
	$\hat{\gamma}$	0.2711	1.8301	5.1981	3.7326	9.9878	9.6509	0.9554	3.1310	5.8413
	$\hat{\theta}$	0.2420	0.3494	2.1150	1.0813	2.0255	3.6290	0.3920	0.4785	2.2351
50	$\hat{\alpha}$	0.5381	3.3694	6.8828	0.3103	3.4028	7.1317	0.4222	2.9978	6.5856
	$\hat{\gamma}$	0.2445	1.6048	4.8749	4.7392	8.7783	10.6101	0.8992	2.7290	5.4350
	$\hat{\theta}$	0.1189	0.1780	1.5876	0.8367	1.3282	3.1083	0.3057	0.3427	1.9581
70	$\hat{\alpha}$	0.4413	2.9409	6.4992	0.3467	3.1980	6.8806	0.4377	2.5710	6.0498
	$\hat{\gamma}$	0.2184	1.5311	4.7767	5.0097	7.9412	10.2600	0.8582	2.1604	4.6800
	$\hat{\theta}$	0.1251	0.1689	1.5356	0.7765	1.0506	2.6238	0.2368	0.1986	1.4809
100	$\hat{\alpha}$	0.3552	2.0831	5.4865	0.0710	1.7041	5.1122	0.2482	1.3961	4.5306
	$\hat{\gamma}$	0.2063	1.0429	3.9226	4.7983	6.6704	9.3195	0.7708	1.3382	3.3830
	$\hat{\theta}$	0.1031	0.1237	1.3188	0.6797	0.7682	2.1703	0.1939	0.1237	1.1509

Table-3: Estimation to adaptive type-II progressive censoring scheme when $\alpha = 1.5$ and $T = 0.5$

		Scheme 1			Scheme 2			Scheme 3		
$r = 0.3$										
n		Bias	MSE	Length.CI	Bias	MSE	Length.CI	Bias	MSE	Length.CI
30	$\hat{\alpha}$	-0.0711	2.7200	5.1361	-1.2094	2.6833	3.5526	-0.3892	3.8384	5.2657
	$\hat{\gamma}$	0.4418	2.5740	4.6054	1.8530	8.7566	7.1495	0.9526	3.3868	4.7703
	$\hat{\theta}$	0.4122	0.7374	2.9545	1.4560	2.8525	3.3567	0.6152	1.0383	3.1858
50	$\hat{\alpha}$	0.3394	2.0248	6.6899	-1.1427	2.4095	4.1206	0.3652	3.2732	6.9496
	$\hat{\gamma}$	0.5044	2.0148	5.3970	2.0524	8.3380	8.3435	1.2711	3.0693	6.0167
	$\hat{\theta}$	0.2251	0.3616	2.1870	1.3167	2.3350	3.0413	0.4125	0.5811	2.5140
70	$\hat{\alpha}$	0.2179	1.9189	5.3651	-1.2205	2.1729	3.2419	0.2211	2.0885	5.6012
	$\hat{\gamma}$	0.3661	1.5671	4.6950	1.9194	7.2128	7.3674	1.1837	3.0186	4.9880
	$\hat{\theta}$	0.1838	0.2298	1.7365	1.3087	2.1486	2.5891	0.3690	0.3967	2.0018
100	$\hat{\alpha}$	0.2722	1.4260	4.5601	-1.2944	2.0301	2.3353	0.1964	1.5479	4.8184
	$\hat{\gamma}$	0.3135	0.9819	3.6866	1.7821	6.2087	6.8300	1.1071	2.3042	4.0730
	$\hat{\theta}$	0.0822	0.1167	1.3002	1.1917	1.7363	2.2054	0.2827	0.2424	1.5810
$r = 0.5$										
n		Bias	MSE	Length.CI	Bias	MSE	Length.CI	Bias	MSE	Length.CI
30	$\hat{\alpha}$	0.2303	3.2922	5.8688	-0.1597	3.2966	6.7250	0.8494	6.6622	7.7856
	$\hat{\gamma}$	0.2711	1.8301	5.1981	3.7320	9.9849	9.6523	2.8957	3.3645	7.8239
	$\hat{\theta}$	0.2420	0.3494	2.1150	1.0819	2.0304	3.6369	0.6675	1.0445	3.0355
50	$\hat{\alpha}$	0.5381	3.0694	6.8828	0.3102	3.0403	7.1319	1.4136	5.9821	7.8281
	$\hat{\gamma}$	0.2445	1.6048	4.8749	4.7394	8.7795	10.6095	3.2252	2.9459	6.8570
	$\hat{\theta}$	0.1189	0.1780	1.5876	0.8377	1.3367	3.1250	0.4936	0.6159	2.3931
70	$\hat{\alpha}$	0.4413	2.9409	6.4992	0.3467	2.9805	6.8806	1.5042	5.8379	7.4158
	$\hat{\gamma}$	0.2184	1.5311	4.7767	5.0097	7.9412	10.2600	3.4152	2.3568	6.4359
	$\hat{\theta}$	0.1251	0.1689	1.5356	0.7765	1.0506	2.6238	0.4438	0.4403	1.9345
100	$\hat{\alpha}$	0.3552	2.0831	5.4865	0.0710	1.7041	5.1122	0.9201	2.5499	5.1187
	$\hat{\gamma}$	0.2063	1.0429	3.9226	4.7983	6.6704	9.3195	3.1451	1.8334	4.7106
	$\hat{\theta}$	0.1031	0.1237	1.3188	0.6797	0.7682	2.1703	0.4109	0.2999	1.4197

We estimate the parameters of MOEW distribution under adaptive type-II progressive censoring scheme. If we increase the parameter $\boldsymbol{\alpha}$ of MOEW distribution, it lead to increase MSE for the parameter $\boldsymbol{\theta}$ of MOEW distribution and decrease MSE for the parameter $\boldsymbol{\gamma}$ of MOEW distribution, while the vales of Bias are extent around zero, where we note the extent of the gap that was positive or negative, but in a small scale, as for the length of confidence interval will be in solidarity with the previous calibrations. If we increased the sample size, it lead to decrease MSE, Bias and the length of confidence interval for the parameters of MOEW.

We note how different the efficiency of each schemes and this is a strong evidence of the need to follow the censoring schemes, and also shows how different the impact of both progressive censoring schemes and adaptive censoring schemes and shows this by changing the amount of the time, which set for the terminates experiment. We note that the lower the value of the effective sample ratio in the censoring schemes, the better the efficiency in the second and third schemes and that through the above mentioned measurements, and this is illustrated by the previous tables and graphs.

Under difference T, we note that there is stability in the numerical and practical results of schemes 1 and 2. Under much differences that have been taken into consideration, we note the efficiency and quality of the Scheme 3 whenever the time limit for the experiment low. In addition, we note that the higher the time allotted to the experiment, the greater the variance in the schemes.

5. APPLICATION OF REAL DATA

We present the numerical results of the parameter estimation of MOEW under progressive censoring scheme of two cases of real data.

Case-I: Mahmoud *et al.* (2016) discussed the real data set of sample size 62 observed failure times to studied estimation for the new Weibull-Pareto distribution (NWPD) based on progressive Type-II censored sample. The data set is represented the strength data measured in GPA, for single carbon fibers and impregnated 1000 carbon fiber tows.

Case-II: Mead (2016) discussed estimation of BEBXII distribution, where analyzed a real data set on the active repair times (hours) for an airborne communication transceiver.

We computed the Kolmogorov-Smirnov (KS) distance between the empirical and the fitted NWPD functions.

Table-4: Goodness of Fit Data

One-sample Kolmogorov-Smirnov test (D)		
	Case-I	Case-II
D	0.0663	0.1086
p-value	0.1749	0.1274

Table-5: Estimation of coefficient and stander error for Complete Data

	Case-I	Case-II
$\hat{\alpha}$	0.6687334 (0.8007)	0.0277 (0.0507)
$\hat{\gamma}$	0.3426 (0.3314)	0.0074 (0.0140)
$\hat{\theta}$	2.3375 (0.5768)	1.6176 (0.2436)
AIC	117.9363	189.2471

The two sets of data were compared with the Kolmogorov-Smirnov test and AIC, resulting in the case-I in data being more suitable for MOEW distribution.

Table-6: Estimation for adaptive type-II progressive censoring scheme of real data

	$r\sim 0.3$			$r = 0.5$		
$T = 0.75$						
	scheme I	scheme 2	scheme 3	scheme I	scheme 2	scheme 3
$\hat{\alpha}$	20.0935 (42.4841)	12.5167 (21.5401)	23.8566 (50.8867)	0.0074 (0.7459)	6.3792 (21.7498)	0.3518 (1.1825)
$\hat{\gamma}$	3.3311 (2.1184)	7.9297 (1.9692)	4.0440 (2.1507)	0.0047 (0.4763)	3.4924 (3.4495)	0.2971 (0.8462)
$\hat{\theta}$	1.7252 (1.0328)	2.4746 (1.0695)	1.8358 (1.0927)	2.5642 (0.5682)	2.0857 (1.7952)	2.9362 (0.8891)
AIC	60.9762	8.35372	51.7119	91.9728	10.4944	70.11778
$T = 1.1$						
$\hat{\alpha}$	24.3804 (76.9209)	12.4913 (21.4310)	29.8779 (68.4074)	0.0130 (0.6878)	7.1286 (31.1262)	7.7420 (2.2186)
$\hat{\gamma}$	3.5143 (3.0977)	7.9283 (1.9675)	4.2970 (2.2648)	0.0083 (0.4384)	3.6034 (4.4182)	2.4189 (3.0962)
$\hat{\theta}$	1.6430 (1.4259)	2.4757 (1.0673)	1.7527 (1.1190)	2.5669 (0.5462)	2.0299 (2.2224)	1.9703 (5.7563)
AIC	60.9518	8.35371	51.4347	91.9756	10.4930	55.1964

In practice, the standard deviation of the estimates and the use of the AIC, these will be used to judge of schemes and obtain the best schemes. Where there is a slight difference between the schemes in terms of the time set for the experiment, due to the extent of the low dispersion of data and convergence values from each other. We note that the preference of the scheme 2 in all cases followed by the scheme 3.

6. CONCLUSION

In this paper, we discussed the parameters estimation problem of the MOEW distribution under on adaptive type-II progressive censoring schemes. The performances of the maximum likelihood estimators of the MOEW are also quite satisfactory. The performance of the different estimator’s optimal censoring schemes is compared based on simulation study to determine the optimal censoring schemes by using MSE, the bias and the length of confidence interval. The application of real data set is used to show how the schemes work in practice. The findings of this paper will be useful to researchers and statisticians which such types of things were required and also in cases where we have different samples size to analyses and exclusively where MOEW distribution is used.

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