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SOLVING FUZZY SEQUENCING PROBLEM USING OCTAGONAL FUZZY NUMBER

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ABSTRACT

In this paper, we have proposed a method to solve fuzzy sequencing problem using octagonal fuzzy numbers. The optimal solution for the completion of the task and idle time for each machine is obtained by solving corresponding sequencing problem using a modified algorithm. A numerical example is given for illustrating the modified algorithm.

Keywords: *Fuzzy Arithmetic, Octagonal Fuzzy Number, Fuzzy sequencing problem, Fuzzy Optimal sequencing.*

1. INTRODUCTION

A sequencing problem is to decide the order in which required tasks are to be done so as to minimize the total elapsed time taken for all the tasks. In operations research, Johnson's rule [4] has given a method of scheduling jobs in two work centers. Its primary objective is to find an optimal sequence of jobs to reduce makespan (the total amount of time it takes to complete all jobs). It also reduces the amount of idle time between the two work centers. Furthermore, the method finds the shortest makespan in the case of three work centers if additional constraints are met. But in reality, it is observed that the processing times during performance of the job are imprecise.

The concept of fuzzy sets was proposed by Zadeh [9]. The impreciseness occurring in scheduling problems are categorized as fuzzy-scheduling problems. Chen *et al.* [3] has described a fuzzy production system for multi-objective scheduling. A Heuristic algorithm is proposed to find the optimal sequences of different single objectives as the production rules, and the fuzzy min-operator with non-linear membership function as the test criterion.

Adamopoulos *et al.* [1] have considered a fuzzy approach to a single machine scheduling problem. The scheduling criteria are the common due date, the total earliness and tardiness and the controllable duration of the jobs' processing times. Their aim is to determine the length of the processing times, to sequence the jobs in the machine and, finally, to determine the common due date in a near optimal way. Amalore Arumica *et al.* [2] have considered three-machine, n-jobs flow shop scheduling problem in which the uncertainties involved in processing time, setup time and transportation time are studied using octagonal fuzzy numbers.

Yager's ranking method [8] is one of the robust ranking techniques which is used to solve fuzzy sequencing problems involving fuzzy numbers. In this paper ranking of octagonal fuzzy numbers as in [6] is used to compare two octagonal fuzzy numbers.

The optimal solution and idle time for each machine is obtained by solving corresponding sequencing problems using the modified method. Finally, to illustrate the modified method, a numerical example is solved and results are presented.

This paper is scheduled as follows: *In Section 2:* Basic definitions and perliminary results on octagonal fuzzy numbers are given. *In Section 3:* Ranking of octagonal fuzzy numbers is given, In *Section 4:* the Modified Algorithm for solving fuzzy sequencing problem is explained, In *Section 5:* Numerical example illustrating the algorithm is given and *In Section 6*: Conclusion is given.

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2. OCTAGONAL FUZZY NUMBERS: BASIC DEFINITIONS

For the sake of completeness we recall from [6], the required definitions and results.

Definition 2.1: An octagonal fuzzy number denoted by $\tilde{A}_\omega$ is defined to be the ordered quadruple $\tilde{A}_\omega = (l_1(r), s_1(t), s_2(t), l_2(r))$, for $r \in [0, k]$, and $t \in [k, \omega]$ where

(i) $l_1(r)$ is a bounded right continuous non decreasing function over $[0, \omega_1]$, $[0 \le \omega_1 \le k]$
(ii) $s_1(t)$ is a bounded right continuous non decreasing function over $[k, \omega_2]$, $[k \le \omega_2 \le \omega]$
(iii) $s_2(t)$ is bounded right continuous non increasing function over $[k, \omega_2]$, $[k \le \omega_2 \le \omega]$
(iv) $l_2(r)$ is bounded right continuous non increasing function over $[0, \omega_1]$. $[0 \le \omega_1 \le k]$

Remark 2.1: If $\omega = 1$, then the above-defined number is called a normal octagonal fuzzy number.

The octagonal numbers we consider for our study is a subclass of the octagonal fuzzy numbers (Definition 2.1) defined as follows:

Definition 2.2: A fuzzy number $\tilde{A}$ is a normal octagonal fuzzy number denoted by $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ where $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8$ are real numbers and its membership function $\mu_{\tilde{A}}(x)$ is given below

$$\mu_{\tilde{A}}(x) = \begin{cases} k\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \le x \le a_2 \\ k, & a_2 \le x \le a_3 \\ k + (1-k)\left(\frac{x-a_3}{a_4-a_3}\right), & a_3 \le x \le a_4 \\ 1, & a_4 \le x \le a_5 \\ k + (1-k)\left(\frac{a_6-x}{a_6-a_5}\right), & a_5 \le x \le a_6 \\ k, & a_6 \le x \le a_7 \\ k\left(\frac{a_8-x}{a_8-a_7}\right), & a_7 \le x \le a_8 \\ 0, & Otherwise \end{cases}$$

where $0 < k < 1$.

Remark 2.2: If $k = 0$, the octagonal fuzzy number reduces to the trapezoidal number (a_3, a_4, a_5, a_6) and if $k=1$, it reduces to the trapezoidal number (a_1, a_4, a_5, a_8).

Remark 2.3: Membership functions $\mu_{\tilde{A}}(x)$ are continuous functions.

Definition 2.3: If $\tilde{A}$ be an octagonal fuzzy number, then the α-cut of $\tilde{A}$ is

$$[\tilde{A}]_\alpha = \{x / \tilde{A}(x) \ge \alpha\}$$
$$= \begin{cases} [l_1(\alpha), l_2(\alpha)] \text{ for } \alpha \in (0, k] \\ [s_1(\alpha), s_2(\alpha)] \text{ for } \alpha \in (k, 1] \end{cases}$$

The octagonal fuzzy number is convex as their α-cuts are convex sets in the classical sense.

Working Rule I:
Using interval arithmetic given by Kaufmann A, [5] we obtain α-cuts, $\alpha \in (0, 1]$, addition and subtraction of two octagonal fuzzy numbers is obtained as follows:

a) α-cut of an octagonal fuzzy number: To find the α-cut of a normal octagonal fuzzy number $\tilde{A} \approx (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8; k, 1)$ given in Definition 2.3 (i.e. $\omega = 1$), for $\alpha \in (0, 1]$,

$$[\tilde{A}]_\alpha = \begin{cases} \left[a_1 + \left(\frac{\alpha}{k}\right)(a_2 - a_1),\ a_8 - \left(\frac{\alpha}{k}\right)(a_8 - a_7)\right] & \text{for } \alpha \in (0, k] \\ \left[a_3 + \left(\frac{\alpha-k}{1-k}\right)(a_4 - a_3),\ a_6 - \left(\frac{\alpha-k}{1-k}\right)(a_6 - a_5)\right] & \text{for } \alpha \in (k, 1] \end{cases}$$

b) Addition on octagonal fuzzy numbers: Let $\tilde{A} \approx (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8; k, 1)$ and $\tilde{B} \approx (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8; k, 1)$ be two octagonal fuzzy numbers. To calculate addition of fuzzy numbers $\tilde{A}$ and $\tilde{B}$ we first add the α–cuts of $\tilde{A}$ and $\tilde{B}$ using interval arithmetic.

$$[\tilde{A}]_\alpha + [\tilde{B}]_\alpha = \begin{cases} [a_1 + \frac{\alpha}{k}(a_2 - a_1),\ a_8 - \frac{\alpha}{k}(a_8 - a_7)] + [b_1 + \frac{\alpha}{k}(b_2 - b_1),\ b_8 - \frac{\alpha}{k}(b_8 - b_7)] & for \quad \alpha \in (0, k] \\ [a_3 + \left(\frac{\alpha-k}{1-k}\right)(a_4 - a_3),\ a_6 - \left(\frac{\alpha-k}{1-k}\right)(a_6 - a_5)] + [b_3 + \left(\frac{\alpha-k}{1-k}\right)(b_4 - b_3),\ b_6 - \left(\frac{\alpha-k}{1-k}\right)(b_6 - b_5)] & for \quad \alpha \in (k, 1] \end{cases}$$

c) Subtraction on octagonal fuzzy numbers: Let $\tilde{A} \approx (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8; k, 1)$ and $\tilde{B} \approx (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8; k, 1)$ be two octagonal fuzzy numbers. To calculate subtraction of fuzzy numbers $\tilde{A}$ and $\tilde{B}$ we first subtract the *α–cuts* of $\tilde{A}$ and $\tilde{B}$ using interval arithmetic.

$$[\tilde{A}]_\alpha - [\tilde{B}]_\alpha = \begin{cases} \left[(a_1 + \frac{\alpha}{k}(a_2 - a_1)) - (b_8 - \frac{\alpha}{k}(b_8 - b_7), (a_8 - \frac{\alpha}{k}(a_8 - a_7)) - (b_1 + \frac{\alpha}{k}(b_2 - b_1)\right], \ for \alpha \in [0, k] \\ \left[(a_3 + \left(\frac{\alpha - k}{1-k}\right)(a_4 - a_3)) - (b_6 - \left(\frac{\alpha - k}{1-k}\right)(b_6 - b_5), (a_6 - \left(\frac{\alpha - k}{1-k}\right)(a_6 - a_5)) - (b_3 + \left(\frac{\alpha - k}{1-k}\right)(b_4 - b_3)\right], \ for \alpha \in [k, 1] \end{cases}$$

3. RANKING OF OCTAGONAL FUZZY NUMBERS

In this paper, ranking of octagonal fuzzy numbers introduced in [6] is used to solve fuzzy Sequencing problem, wherein all the entries are octagonal fuzzy numbers.

Definition 3.1: A measure of normal octagonal fuzzy number $\tilde{A}$ is a function $M_\alpha: R(I) \rightarrow R^+$ which assigns a non-negative real number $M_\alpha(\tilde{A})$ that expresses the measure of $\tilde{A}$.

Definition 3.2: Let $\tilde{A}$ be a normal octagonal fuzzy number. The value $M_0^{Oct}(\tilde{A})$, called the measure of $\tilde{A}$ is calculated as follows:

$$M_0^{Oct}(\tilde{A}) = \frac{1}{2}\int_0^k (l_1(r) + l_2(r))dr + \frac{1}{2}\int_k^1 (s_1(t) + s_2(t))dt \text{ where } 0<k<1$$

$$= \frac{1}{4}[(a_1 + a_2 + a_7 + a_8)k + (a_3 + a_4 + a_5 + a_6)(1-k)] \tag{3.1}$$

Remark 3.1: If k=0.5, $M_0^{Oct}(\tilde{A}) = \frac{1}{8}(a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8)$

Remark 3.2: If

$$a_1 + a_2 + a_7 + a_8 = a_3 + a_4 + a_5 + a_6 \tag{3.2}$$

we would get the measure of an octagonal number same for any value of k ($0<k<1$)

Remark 3.3: If $\tilde{A}$ and $\tilde{B}$ are two normal octagonal fuzzy numbers, then we have:

1. $\tilde{A} \preccurlyeq \tilde{B} \Leftrightarrow M_0^{Oct}(\tilde{A}) \leq M_0^{Oct}(\tilde{B})$
2. $\tilde{A} \approx \tilde{B} \Leftrightarrow M_0^{Oct}(\tilde{A}) = M_0^{Oct}(\tilde{B})$
3. $\tilde{A} \succcurlyeq \tilde{B} \Leftrightarrow M_0^{Oct}(\tilde{A}) \geq M_0^{Oct}(\tilde{B})$

Definition 3.3: Let $\{\tilde{a}_i, i = 1, 2, 3,\ldots,n\}$be a set of octagonal fuzzy numbers. If $M_0^{Oct}(\tilde{a}_k) \leq M_0^{Oct}(\tilde{a}_i)$, for all i, then the fuzzy number $\tilde{a}_k$ is the minimum of $\{\tilde{a}_i, i = 1,2,3,\ldots,n\}$.

Definition 3.4: Let $\{\tilde{a}_i, i = 1, 2, 3,\ldots,n\}$ be a set of octagonal fuzzy numbers. If $M_0^{Oct}(\tilde{a}_t) \geq M_0^{Oct}(\tilde{a}_i)$, for all i, then the fuzzy number $\tilde{a}_t$ is the maximum of $\{\tilde{a}_i, i = 1,2,3,\ldots,n\}$.

4. ALGORITHM FOR SOLVING FUZZY SEQUENCING PROBLEM

4.1 Processing n jobs on two machines:

Let $a_{11}, a_{21}, \ldots\ldots\ldots\ldots\ldots.., a_{n1}$ be processing times of "n" jobs on Machine A_1 and let $a_{12}, a_{22}, \ldots\ldots\ldots\ldots\ldots.., a_{n2}$ processing times of "n" jobs on Machine A_2.

Step-1: Find $Min\ (a_{i1}, a_{i2}), i = 1, 2, \ldots\ldots..n$.

Step-2: (i) If we get minimum on machine A say a_{i1} then process the i^{th} first.

(ii) If we get minimum on machine B say a_{m2} then process the m^{th} job last.

(iii) If there is a tie, i.e., $a_{i1} = a_{m2}$ process the i^{th} job first and m^{th} job last.

Step-3: Cancel the jobs already assigned and repeat the steps 1 to 2 until all the jobs have been assigned.

4.2 Processing "n" jobs on three machines

Let A_1, A_2, A_3 be the three machines, and each job be processed in the order A_1, A_2, A_3. This problem can be converted into a two machine problem if any one of the following conditions is satisfied.

Condition (1) $Min\ a_{i1} \geq Max\ a_{i2}$ for $i = 1,2, \ldots\ldots\ldots\ldots n$

(or)

Condition (2) $Min\ a_{i3} \leq Max\ a_{i2}$ for $i = 1,2, \ldots\ldots\ldots.\ldots n$.

If the above condition is not satisfied, the method fails.

If one of the condition is satisfied, we introduce two dummy machines H and K such that,

$H_i = a_{i1} + a_{i2}$, for $i = 1, 2, \ldots\ldots\ldots\ldots n$.

$K_i = a_{i2} + a_{i3}$, for $i = 1, 2, \ldots\ldots\ldots...n$.

Now proceed to determine the optimal sequence using 4.1.

4.3 Processing "n" jobs on "m" machines

Let $A_1, A_2, \dots \dots \dots .. A_m$ be the m machines, each job is processed in the order $A_1, A_2, \dots \dots \dots .. A_m$. This problem can be converted into a two machine problem if any one of the following conditions is satisfied.

Let $a_{i1}, a_{i2}, \dots \dots \dots .. a_{im}$ be the processing times on machines $A_1,\ A_2, \dots \dots \dots .. A_m$ respectively.

Condition (1) $\quad Min\ a_{i1} \geq Max\ a_{ij}$ for $i = 1,2, \dots \dots \dots \dots\ n, j = 2, \dots \dots ., m-1.$

(or)

Condition(2) $\quad Min\ a_{im} \leq Max\ a_{ij}$ for $i = 1,2, \dots \dots \dots \dots .. n, j = 2, \dots \dots , m-1.$

If the above condition is not satisfied, the method fails.

If one of the condition is satisfied, we introduce two dummy machines H and K such that,
$H_i = a_{i1} + a_{ij}$, for $i = 1,2, \dots \dots \dots \dots\ n, j = 2, \dots \dots ., m-1.$
$K_i = a_{i2} +\ a_{ij}$, for $i = 1,2, \dots \dots \dots \dots\ n, j = 2, \dots \dots ., m-1.$
Where H_i and K_i are processing times for job i on machines H and K respectively.

Now proceed to determine the optimal sequence using 4.1.

5. NUMERICAL EXAMPLE

The following example is taken from the paper "Fuzzy Sequencing Problem: A Noval Approach" by *M. Shanmugasundari* **[7]**.

Example 5.1: The fuzzy sequencing problem with nine jobs through two machines has been considered where the order is AB. Here, processing time of each machine for each job is Octagonal Fuzzy Number. Consider the following sequencing problem [7] given below:

Table-1: Quantitative table

Task	*Machine A*	*Machine B*
I	(0,0.2,1,2,3,3.5,4,5)	(15,16.3,17,18,20,20.5,21,22)
II	(9,10,11,12,14,14.5,15,16)	(9.5,10.4,11.2,12,13,13.5,14,15)
III	(7.2,8,9,10,12,12.5,13,14)	(14.6,15.5,16,17,20,20.5,21,22)
IV	(17.9,18.7,19.1,20,22,22.5,23,24)	(7.2,8,9,10,12,12.5,13,14)
V	(15,16.3,17,18,20,20.5,21,22)	(5,6.3,7,8,10,10.5,11,12)
VI	(9.5,10.4,11.2,12,13,13.5,14,15)	(17.9,18.7,19.1,20,22,22.5,23,24)
VII	(14.6,15.5,16,17,20,20.5,21,22)	(5,6.3,7,8,10,10.5,11,12)
VIII	(9,10,11,12,14,14.5,15,16)	(17.9,18.7,19.1,20,22,22.5,23,24)
IX	(7.2,8,9,10,12,12.5,13,14)	(22,23,24,25,28,28.5,29,30)

Solution: Using the measure given by (3.1) we obtain the following crisp values which will enable us to compare two octagonal fuzzy numbers.

Table-2: Converted Quantitative table

Task	**Machine A**	
I	(0,0.2,1,2,3,3.5,4,5)	2.3525
II	(9,10,11,12,14,14.5,15,16)	12.7625
III	(7.2,8,9,10,12,12.5,13,14)	10.7775
IV	(17.9,18.7,19.1,20,22,22.5,23,24)	20.9
V	(15,16.3,17,18,20,20.5,21,22)	18.785
VI	(9.5,10.4,11.2,12,13,13.5,14,15)	12.365
VII	(14.6,15.5,16,17,20,20.5,21,22)	18.345
VIII	(9,10,11,12,14,14.5,15,16)	12.7625
IX	(7.2,8,9,10,12,12.5,13,14)	10.7775
	Machine B	

I	(15,16.3,17,18,20,20.5,21,22)	18.785
II	(9.5,10.4,11.2,12,13,13.5,14,15)	12.365
III	(14.6,15.5,16,17,20,20.5,21,22)	18.345
IV	(7.2,8,9,10,12,12.5,13,14)	10.7775
V	(5,6.3,7,8,10,10.5,11,12)	8.785
VI	(17.9,18.7,19.1,20,22,22.5,23,24)	20.9
VII	(5,6.3,7,8,10,10.5,11,12)	8.785
$VIII$	(17.9,18.7,19.1,20,22,22.5,23,24)	20.9
IX	(22,23,24,25,28,28.5,29,30)	26.2625

Table-3: Fuzzy optimal sequence

I	III	IX	VI	$VIII$	II	IV	VII	V

Table-4: Calculation table

$Task$	Machine A		
	$(Time\ In)$	$(Time\ Out)$	$Idle\ Time$
I	(0,0,0,0,0,0,0,0)	(0,0.2,1,2,3,3.5,4,5)	
III	(0,0.2, 1,2,3,3.5,4,5)	(7.2,8.2,10,12,15,15.5,17,19)	
IX	(7.2,8.2,10,12,15,15.5,17,19)	(16.2,18.2,21,24,29,30,32,35)	-
VI	(16.2,18.2,21,24,29,30,32,35)	(25.7,28.6,32.2,36,42,43.5,46,50)	-
$VIII$	(25.7,28.6,32.2,36,42,43.5,46,50)	(32.9,36.6,41.2,46,54,56,59,64)	-
II	(32.9,36.6,41.2,46,54,56,59,64)	(41.9,46.6,52.2,58,68,70.5,74,80)	-
IV	(41.9,46.6,52.2,58,68,70.5,74,80)	(59.8,65.3,71.3,78,90,92.5,97,104)	-
VII	(59.8,65.3,71.3,78,90,92.5,97,104)	(74.4,80.8,87.3,95,110,113,118,126)	-
V	(74.4,80.8,87.3,95,110,113,118,126)	(89.4,97.1,104.3,113,130,133.5,139,148)	(-33.9,-15.6,-3.1,10,47,60.7,72.9,90.6)

$Task$	Machine B		
	$(Time\ In)$	$(Time\ Out)$	$Idle\ Time$
I	(0,0.2,1,2,3,3.5,4,5)	(15,16.5,18,20,23,24,25,27)	(0,0.2,1,2,3,3.5,4,5)
III	(15,16.5,18,20,23,24,25,27)	(29.6,32,34,37,43,44.5,46,49)	-
IX	(29.6,32,34,37,43,44.5,46,49)	(51.6,55,58,62,71,73,75,79)	-
VI	(51.6,55,58,62,71,73,75,79)	(69.5,73.7,77.1,82,93,95.5,98,103)	-
$VIII$	(69.5,73.7,77.1,82,93,95.5,98,103)	(87.4,92.4,96.2,102,115,118,121,127)	-
II	(87.4,92.4,96.2,102,115,118,121,127)	(96.9,102.8,107.4,114,128,131.5,135,142)	-
IV	(96.9,102.8,107.4,114,128,131.5,135,142)	(104.1,110.8,116.4,124,140,144,148, 156)	-
VII	(104.1,110.8,116.4,124,140,144,148,156)	(109.1,117.1,123.4,132,150,154.5,159,168)	-
V	(109.1,117.1,123.4,132,150,154.5,159,168)	(114.1,123.4,130.4,140,160,165,170, 180)	(114.1,123.4,130.4,140,160,165, 170,180)

Total elapsed time = (114.1, 123.4, 130.4, 140, 160, 165, 170, 180) hours;
M_0^{oct} ((114.1, 123.4, 130.4, 140, 160, 165, 170, 180)) = 148.2575 hours.

Idle time on Machine A = $(-33.9, -15.6, -3.1, 10, 47, 60.7, 72.9, 90.6)$ hours;
M_0^{oct} $((-33.9, -15.6, -3.1, 10, 47, 60.7, 72.9, 90.6)$ = 28.605 hours.

Idle time on Machine B = (0,0.2,1,2,3,3.5,4,5) hours;
M_0^{oct} ((0, 0.2, 1,2, 3, 3.5, 4, 5) = 2.3525 hours.

CONCLUSION

In this paper, we have solved fuzzy sequencing problem using octagonal fuzzy numbers. The numerical example in [7] is solved using octagonal fuzzy numbers. Also it can be seen that the solution obtained gives the minimum elapsed time for completion of the jobs is much lesser when done using octagonal fuzzy numbers than when it is done using trapezoidal fuzzy numbers [7].

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