

## SEPARATION AXIOMS THROUGH SEMI\* REGULAR OPEN SETS

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### ABSTRACT

*In this paper, we introduce semi\*regular-Ti( $i = 0, 1, 2$ ) spaces using semi\*regular open sets and investigate their properties. We give characterizations for these spaces. We study the relationship among themselves and with known separation axioms. We further study relationships among themselves and with the already existing concepts.*

**Key Words:** Semi\*regular closed sets, Semi\*regular open sets, Semi\*regular  $T_i(i=0.1.2)$  spaces,

### 1. INTRODUCTION

S.G.Crossely and S.K.Hildebrand [7] defined semi closure of sets and irresolute functions. S.Maheswari and Prasad [3] *et.al* defined and studied the separation axioms, namely semi-Ti,  $i=0, 1, 2$ . Recently S.Pious Missier and A.Robert [9] introduced separation axioms, namely Semi\* $\alpha$ -Ti  $i = 0, 1, 2$  in a topological space. Here we study semi\*r-Regular, s\*r-Regular, semi\*r-normal and s\*r-Normal spaces using semi\*regular open sets and investigate their properties.

### 2. PRELIMINARIES

**Definition 2.1:** A subset  $A$  of a Topological space  $(X, \tau)$  is called a Semi\*regular open set (briefly s\*r-open) if there exists a regular open set  $U$  in  $X$  such that  $U \subseteq A \subseteq Cl^*(U)$ . The Class of all Semi\*regular open sets in  $(X, \tau)$  is denoted by  $S^*RO(X, \tau)$  or simply  $S^*RO(X)$ .

**Theorem 2.2** For a subset  $A$  of a topological space  $(X, \tau)$  the following are equivalent.

- (i)  $A$  is Semi\* regular open
- (ii)  $A = Cl^*(r\text{-int}(A))$
- (iii)  $Cl^*(A) = Cl^*(r\text{-int}(A))$
- (iv)  $Cl^*(A) = Cl^*(A \cap \text{int}(cl(A)))$

**Theorem 2.3:** Arbitrary Union of Semi\*regular open sets in  $X$  is Semi\*regular open set in  $X$ .

**Theorem 2.4:** If  $A$  is Semi\*regular open in  $X$ , then  $A$  can be expressed as  $A = U \cup B$  where (i)  $U$  is regular open in  $X$  (ii)  $B$  is nowhere dense in  $X$  (iii)  $U \cap B = \emptyset$ .

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**Theorem 2.5:**

- (i) Every Semi\*regular open set is Semi\* $\alpha$ -open.
- (ii) Every Semi\*regular open set is Semi\*pre-open.
- (iii) Every Semi\*regular open set is Semi\*open.
- (iv) Every Semi\* regular open set is Semi open.
- (v) Every Semi\*regular open set is Semi  $\alpha$ -open.
- (vi) Every Semi\*regular open set is Semi pre-open.
- (vii) Every Semi\*regular open set is regular generalized open set.
- (viii) Every Semi\*regular open set is generalized pre regular open set.
- (ix) Every Semi\*regular open is regular weakly generalized open set.

**Theorem 2.6:** If A is any subset of a topological space X, then the following statements hold:

- (i)  $s^*rCl(A)$  is semi\*regular closed set in X. In fact, it is the smallest semi\*regular closed in X containing A.
- (ii) A is semi\*regular closed if and only if  $s^*rCl(A)=A$ .

### 3. Semi\*regular- $T_0$ Spaces

**Definition 3.1:** A space X is said to be semi\*regular- $T_0$  if whenever x and y are distinct points in X there is a semi\*regular open set in X containing one of x and y but not the other.

**Theorem 4.2:**

- (i) Every semi\*regular- $T_0$  space is semi\* $\alpha$ - $T_0$ .
- (ii) Every semi\*regular- $T_0$  space is semi\*Pre- $T_0$ .
- (iii) Every semi\*regular- $T_0$  space is semi\*- $T_0$ .
- (iv) Every semi\*regular- $T_0$  space is semi- $T_0$ .
- (v) Every semi\*regular- $T_0$  space is semi  $\alpha$ - $T_0$ .

**Proof:**

- (i) Suppose X is a semi\*regular- $T_0$  space. Let x and y be two distinct points in X. Since X is semi\*regular- $T_0$  space, there exists a semi\*regular open set U containing one of x and y but not the other. By Theorem 3.5(i), U is a semi\* $\alpha$ -open set. Hence X is semi\* $\alpha$ - $T_0$ .
- (ii) Suppose X is a semi\*regular- $T_0$  space. Let x and y be two distinct points in X. Since X is semi\*regular- $T_0$  space, there exists a semi\*regular open set U containing one of x and y but not the other. By Theorem 2.5(ii), U is a semi\*preopen set. Hence X is semi\*Pre- $T_0$ .
- (iii) (iii),(iv)&(v) follows from the fact that every semi\*regular open set is semi\*open, semiopen and semi  $\alpha$ -open.

**Remark 3.3:** The converse of each of the statements of the above theorem is not true as given by the following examples.

**Example 3.4:**

- (i) Let  $X = \{a, b, c, d\}$   $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$ . Clearly X is semi\* $\alpha$ - $T_0$  but not semi\*regular- $T_0$ .
- (ii) Let  $X = \{a, b, c, d\}$   $\tau = \{\emptyset, \{a\}, \{b, c, d\}, X\}$  Here X is semi\*pre- $T_0$  but not semi\*regular- $T_0$ .

**Theorem 3.5:** A space X is a semi\*regular- $T_0$  space if and only if the semi\*regular closure of distinct points are distinct.

**Proof:** Let x and y be two distinct points of a semi\*regular- $T_0$  space X.

Then by definition 4.1, there exist a semi\*regular open set U containing one of x and y but not the other. If  $x \in U$  and  $y \notin U$  then U is a semi\*regular open set containing x that does not intersect {y}.

By Theorem 3.6,  $x \notin s^*rCl(\{y\})$ . But  $x \in s^*rCl(\{x\})$ , so we get  $s^*rCl(\{x\}) \neq s^*rCl(\{y\})$ .

Similarly we can prove the case when  $y \in U$  and  $x \notin U$ . Thus the semi\*regular closure of x and y are distinct. On the other hand suppose the semi\*regular closure of distinct points are different. Let x and y be two distinct points of X. Then  $s^*rCl(\{x\}) \neq s^*rCl(\{y\})$ . Hence there exist a point z in X such that z lies in only one of the two sets  $s^*rCl(\{x\})$  and  $s^*rCl(\{y\})$ , say  $s^*rCl(\{x\})$ .

If  $x \in s^*rCl(\{y\})$  then  $z \in s^*rCl(\{x\}) \subseteq s^*rCl(\{y\})$  which implies  $z \in s^*rCl(\{y\})$  which is a contradiction to  $z \notin s^*rCl(\{y\})$ . Therefore  $x \notin s^*rCl(\{y\})$ . Hence  $X \setminus s^*rCl(\{y\})$  is a semi\*regular open set containing x but not y. Therefore X is semi\*regular- $T_0$ .

**Theorem 3.6:** Let  $f: X \rightarrow Y$  be a bijection. Then the following are true

- (i) If  $f$  is semi\*regular open and  $X$  is  $T_0$ , then  $Y$  is semi\*regular- $T_0$
- (ii) If  $f$  is pre-semi\*regular open and  $X$  is semi\*regular- $T_0$ , then  $Y$  is semi\*regular- $T_0$
- (iii) If  $f$  is semi\*r-continuous and  $Y$  is  $T_0$ , then  $X$  is semi\*regular- $T_0$
- (iv) If  $f$  is semi\*r-irresolute and  $Y$  is semi\*regular- $T_0$ , then  $X$  is semi\*regular- $T_0$ .

**Proof:**

- (i) Suppose  $f$  is semi\*regular open and  $X$  is  $T_0$ . Let  $y_1, y_2 \in Y$ . Since  $f$  is a bijection, there exist  $x_1, x_2$  in  $X$  such that  $f(x_1) = y_1$  and  $f(x_2) = y_2$  with  $x_1 \neq x_2$ . Since  $X$  is  $T_0$ , there exists an open set  $U$  in  $X$  containing one of  $x_1$  and  $x_2$  but not the other. Since  $f$  is semi\*regular open bijection,
- (ii)  $f(U)$  is a semi\*regular open set in  $Y$  containing  $y_1$  or  $y_2$  but not the other. Thus  $Y$  is semi\*regular- $T_0$ .
- (iii) Let  $f$  be semi\*regular open and  $X$  be a semi\*regular- $T_0$  space. Let  $y_1, y_2 \in Y$ . Since  $f$  is a bijection, there exist  $x_1, x_2$  in  $X$  such that  $f(x_1) = y_1$  and  $f(x_2) = y_2$  with  $x_1 \neq x_2$ . Since  $X$  is semi\*regular- $T_0$ , there exists a semi\*regular open set  $U$  in  $X$  containing one of  $x_1$  and  $x_2$  but not the other. Since  $f$  is pre-semi\*regular open bijection,  $f(U)$  is a semi\*regular open set in  $Y$  containing  $y_1$  or  $y_2$  but not the other. Thus  $Y$  is semi\*regular- $T_0$ .
- (iv) Suppose  $f: X \rightarrow Y$  is semi\*r-continuous and  $Y$  is  $T_0$ . Let  $x_1$  and  $x_2$  in  $X$  with  $x_1 \neq x_2$ . Let  $y_1 = f(x_1)$  and  $y_2 = f(x_2)$ . Since  $f$  is one to one,  $y_1 \neq y_2$ . Since  $Y$  is  $T_0$ , there exists an open set  $V$  in  $Y$  containing  $y_1$  or  $y_2$  but not the other. Since  $f$  is semi\*r-continuous,  $f^{-1}(V)$  is a semi\*regular open set containing one of  $x_1$  and  $x_2$  but not the other. Thus  $X$  is semi\*regular- $T_0$ .
- (v) Let  $f: X \rightarrow Y$  be semi\*regular-irresolute and  $Y$  be semi\*regular- $T_0$ . Let  $x_1, x_2 \in X$  with  $x_1 \neq x_2$ . Let  $y_1 = f(x_1)$  and  $y_2 = f(x_2)$ . Since  $f$  is one to one,  $y_1 \neq y_2$ . Since  $Y$  is semi\*regular- $T_0$ , there exists a semi\*regular open set  $V$  in  $Y$  containing  $y_1$  or  $y_2$  but not the other. Since  $f$  is semi\*r-irresolute  $f^{-1}(V)$  is semi\*regular open set in  $X$  containing one of  $x_1$  and  $x_2$  but not the other. This proves  $X$  is semi\*regular- $T_0$ .

#### 4. Semi\*regular- $T_1$ Spaces

**Definition 4.1:** A space  $X$  is said to be semi\*regular- $T_1$  if whenever  $x$  and  $y$  are distinct points in  $X$ , there are semi\*regular open sets containing each but not the other.

**Theorem 4.2:**

- (i) Every semi\*regular- $T_1$  space is semi\* $\alpha$ - $T_1$ .
- (ii) Every semi\*regular- $T_1$  space is semi\*Pre- $T_1$ .
- (iii) Every semi\*regular- $T_1$  space is semi\*- $T_1$ .
- (iv) Every semi\*regular- $T_1$  space is semi- $T_1$ .
- (v) Every semi\*regular- $T_1$  space is semi  $\alpha$ - $T_1$ .

**Proof:**

- (i) Suppose  $X$  is a semi\*regular- $T_1$  space. Let  $x$  and  $y$  be two distinct points in  $X$ . Since  $X$  is semi\*regular- $T_1$ , there exist semi\*regular open sets  $U$  and  $V$  with  $x \in U$  but  $y \notin U$  and  $y \in V$  but  $x \notin V$ . By Theorem 3.5(i),  $U$  and  $V$  are semi\* $\alpha$ -open sets. Hence  $X$  is semi\* $\alpha$ - $T_1$ .
- (ii) Let  $X$  be semi\*regular- $T_1$  space. Let  $x$  and  $y$  be two distinct points in  $X$ . Since  $X$  is semi\*regular- $T_1$ , there are semi\*regular open sets  $U$  and  $V$  with  $x \in U$  but  $y \notin U$  and  $y \in V$  but  $x \notin V$ . By Theorem 3.5(ii),  $U$  and  $V$  are semi\*pre-open sets. Hence  $X$  is semi\*pre- $T_1$ .
- (iii) (iv) & (v) follows from the fact that every semi\*regular open set is semi\*open, semiopen and semi  $\alpha$ -open.

**Remark 4.3:** The converse of the above statements are not true as shown in the following examples.

**Example 4.4:**

- (i) Let  $X = \{a, b, c, d\}$   $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$ . Here  $X$  is semi\* $\alpha$ - $T_1$  but not semi\*regular- $T_1$ .
- (ii) Let  $X = \{a, b, c, d\}$   $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$ . Here  $X$  is semi\*- $T_1$  but not semi\*regular- $T_1$ .

**Theorem 4.5:** For a topological space  $X$  the following are equivalent:

- (i)  $X$  is a semi\*regular- $T_1$  space.
- (ii) Each one point set in  $X$  is semi\*regular closed in  $X$ .
- (iii) Each subset of  $X$  is the intersection of semi\*regular open sets containing it.
- (iv) The intersection of all semi\*regular open sets in  $X$  containing the point  $x$  equals  $\{x\}$ .

**Proof:**

(i)  $\Rightarrow$  (ii): Suppose  $X$  is semi\*regular- $T_1$  space. Let  $x \in X$ . Then for every  $y \neq x$ , there exists a semi\*regular open set  $U_y$  in  $X$  containing  $y$  but not  $x$ . Hence  $y \in U_y \subseteq X \setminus \{x\}$ . Therefore  $X \setminus \{x\} = \bigcup \{U_y : y \in X \setminus \{x\}\}$ . By Theorem 3.3,  $X \setminus \{x\}$  is semi\*regular open in  $X$ . Therefore  $\{x\}$  is semi\*regular closed.

**(ii)  $\Rightarrow$  (iii):** Let  $A \subseteq X$ . Then for each  $x \in X \setminus A$ ,  $\{x\}$  is semi\*regular closed in  $X$  and hence  $X \setminus \{x\}$  is semi\*regular open. Clearly  $A \subseteq X \setminus \{x\}$  for each  $x \in X \setminus A$ . Therefore  $A \subseteq \bigcap \{X \setminus \{x\} : x \in X \setminus A\}$ . On the other hand, if  $y \notin A$ , then  $y \in X \setminus A$  and  $y \notin X \setminus \{y\}$ . This implies  $y \notin \bigcap \{X \setminus \{x\} : x \in X \setminus A\}$ . Hence  $\bigcap \{X \setminus \{x\} : x \in X \setminus A\} \subseteq A$ . Therefore  $A = \bigcap \{X \setminus \{x\} : x \in X \setminus A\}$  which proves (iii).

**(iii)  $\Rightarrow$  (iv):** Taking  $A = \{x\}$ , by (iii)  $A = \{x\} = \bigcap \{U : U \text{ is semi*regular open and } x \in U\}$   
This proves (iv).

**(iv)  $\Rightarrow$  (i):** Let  $x, y \in X$  with  $y \neq x$ . Then  $y \notin \{x\} = \bigcap \{U : U \text{ is semi*regular open and } x \in U\}$ . Hence there exists a semi\*regular open set  $U$  containing  $x$  but not  $y$ . Similarly there exists a semi\*regular open set  $V$  containing  $y$  but not  $x$ . Thus  $X$  is semi\*regular- $T_1$ .

**Theorem 4.6:** Let  $f: X \rightarrow Y$  be a bijection.

- (i) If  $f$  is a semi\*regular-continuous and  $Y$  is  $T_1$ , then  $X$  is semi\*regular- $T_1$ .
- (ii) If  $f$  is a semi\*regular-irresolute and  $Y$  is semi\*regular- $T_1$ , then  $X$  is semi\*regular- $T_1$ .
- (iii) If  $f$  is a semi\*regular open and  $X$  is  $T_1$ , then  $Y$  is semi\*regular- $T_1$
- (iv) If  $f$  is a pre-semi\*regular open and  $X$  is semi\*regular- $T_1$ , then  $Y$  is semi\*regular- $T_1$
- (v) If  $f$  is a semi\*regular closed and  $X$  is  $T_1$ , then  $Y$  is semi\*regular- $T_1$
- (vi) If  $f$  is a pre-semi\*regular closed and  $X$  is semi\*regular- $T_1$ , then  $Y$  is semi\*regular- $T_1$

**Proof:**

- (i) Suppose  $f$  is semi\*regular-continuous bijection and  $Y$  is  $T_1$ . Let  $x_1, x_2 \in X$  with  $x_1 \neq x_2$ . Let  $y_1 = f(x_1)$  and  $y_2 = f(x_2)$ . Since  $f$  is one to one,  $y_1 \neq y_2$ . Since  $Y$  is  $T_1$ , there exist open sets  $U$  and  $V$  such that  $y_1 \in U$  but  $y_2 \notin U$  and  $y_2 \in V$  but  $y_1 \notin V$ . Since  $f$  is a bijection,  $x_1 \in f^{-1}(U)$  but  $x_2 \notin f^{-1}(U)$  and  $x_2 \in f^{-1}(V)$  but  $x_1 \notin f^{-1}(V)$ . Since  $f$  is semi\*regular-continuous,  $f^{-1}(U)$  and  $f^{-1}(V)$  are semi\*regular open sets in  $X$ . Thus  $X$  is semi\*regular- $T_1$ .
- (ii) Suppose  $f$  is semi\*regular-irresolute bijection and  $Y$  is semi\*regular- $T_1$ . Let  $x_1, x_2 \in X$  with  $x_1 \neq x_2$ . Let  $y_1 = f(x_1)$  and  $y_2 = f(x_2)$ . Since  $f$  is one to one,  $y_1 \neq y_2$ . Since  $Y$  is semi\*regular- $T_1$ , there exist semi\*regular open sets  $U$  and  $V$  such that  $y_1 \in U$  but  $y_2 \notin U$  and  $y_2 \in V$  but  $y_1 \notin V$ . Since  $f$  is a bijection,  $x_1 \in f^{-1}(U)$  but  $x_2 \notin f^{-1}(U)$  and  $x_2 \in f^{-1}(V)$  but  $x_1 \notin f^{-1}(V)$ . Since  $f$  is semi\*regular-irresolute,  $f^{-1}(U)$  and  $f^{-1}(V)$  are semi\*regular open sets in  $X$ . Thus  $X$  is semi\*regular- $T_1$ .
- (iv) Suppose  $f$  is a semi\*regular open bijection and  $X$  is  $T_1$ . Let  $y_1 \neq y_2 \in Y$ .
- (v) Since  $f$  is a bijection, there exist  $x_1, x_2 \in X$  such that  $f(x_1) = y_1$  and  $f(x_2) = y_2$  with  $x_1 \neq x_2$ . Since  $X$  is  $T_1$ , there exist open sets  $U$  and  $V$  in  $X$  such that  $x_1 \in U$  but  $x_2 \notin U$  and  $x_2 \in V$  but  $x_1 \notin V$ . Since  $f$  is semi\*regular open,  $f(U)$  and  $f(V)$  are semi\*regular open sets in  $Y$  such that  $y_1 = f(x_1) \in f(U)$  and  $y_2 = f(x_2) \in f(V)$ . Since  $f$  is a bijection,  $y_2 = f(x_2) \notin f(U)$  and  $y_1 = f(x_1) \notin f(V)$ . Thus  $Y$  is semi\*regular- $T_1$ .
- (vi) Suppose  $f$  is a pre-semi\*regular open bijection and  $X$  is semi\*regular- $T_1$ . Let  $y_1 \neq y_2 \in Y$ . Since  $f$  is a bijection, there exist  $x_1, x_2 \in X$  such that  $f(x_1) = y_1$  and  $f(x_2) = y_2$  with  $x_1 \neq x_2$ . Since  $X$  is semi\*regular- $T_1$ , there exist semi\*regular open sets  $U$  and  $V$  in  $X$  such that  $x_1 \in U$  but  $x_2 \notin U$  and  $x_2 \in V$  but  $x_1 \notin V$ . Since  $f$  is pre-semi\*regular open,  $f(U)$  and  $f(V)$  are semi\*regular open sets in  $Y$  such that  $y_1 = f(x_1) \in f(U)$  and  $y_2 = f(x_2) \in f(V)$ . Since  $f$  is bijection,  $y_2 = f(x_2) \notin f(U)$  and  $y_1 = f(x_1) \notin f(V)$ . Thus  $Y$  is semi\*regular- $T_1$ .
- (vii) Suppose  $f$  is a semi\*regular closed bijection and  $X$  is  $T_1$ . Let  $y \in Y$ . Since  $f$  is a bijection, there exists  $x \in X$  such that  $f(x) = y$ . Since  $X$  is  $T_1$ ,  $\{x\}$  is closed in  $X$ . Since  $f$  is a semi\*regular closed map,  $f(\{x\}) = \{y\}$  is semi\*regular closed. Since every singleton set in  $Y$  is semi\*regular closed, by Theorem 3.5,  $Y$  is semi\*regular- $T_1$ .
- (viii) Suppose  $f$  is a pre-semi\*regular closed bijection and  $X$  is semi\*regular- $T_1$ . Let  $y \in Y$ . Since  $f$  is bijection, there exists  $x \in X$  such that  $f(x) = y$ . Since  $X$  is semi\*regular- $T_1$ ,  $\{x\}$  is closed in  $X$ . Since  $f$  is a semi\*regular closed function,  $f(\{x\}) = \{y\}$  is semi\*regular closed. Since every singleton set in  $Y$  is semi\*regular closed, by Theorem 3.5,  $Y$  is semi\*regular- $T_1$ .

**Theorem 4.7:** Every contra-semi\*regular-continuous function from a semi\*regular-connected space onto a  $T_1$  is necessarily constant.

**Proof:** Let  $f: X \rightarrow Y$  be a contra-semi\*regular-continuous function. Let  $X$  be semi\*regular-connected and  $Y$  be  $T_1$ . Since  $Y$  is  $T_1$ , for each  $y \in Y$ ,  $\{y\}$  is closed in  $Y$ . Since  $f$  is contra-semi\*regular-continuous,  $f^{-1}(\{y\})$  is semi\*regular open in  $X$ . Therefore  $\{f^{-1}(\{y\}) : y \in Y\}$  is a collection of pairwise disjoint semi\*regular open sets in  $X$ . Since  $X$  is semi\*regular-connected,  $f^{-1}(\{y_0\}) = X$  for some unique  $y_0 \in Y$ . Hence  $f(X) = \{y_0\}$ . Thus  $f$  is a constant function.

**Theorem 4.8:** Every contra-semi\*regular-irresolute function from a semi\*regular-connected space onto a semi\*regular- $T_1$  is necessarily constant.

**Proof:** Let  $f: X \rightarrow Y$  be a contra-semi\*regular-irresolute function. Let  $X$  be semi\*regular-connected and  $Y$  be semi\*regular- $T_1$ . Since  $Y$  is semi\*regular- $T_1$ , by theorem 5.5, for each  $y \in Y$ ,  $\{y\}$  is semi\*regular closed in  $Y$ . Since  $f$  is contra-semi\*regular-irresolute,  $f^{-1}(\{y\})$  is semi\*regular open in  $X$ . Therefore  $\{f^{-1}(\{y\}): y \in Y\}$  is a collection of pairwise disjoint semi\*regular open sets in  $X$ . Since  $X$  is semi\*regular-connected,  $f^{-1}(\{y_0\}) = X$  for some unique  $y_0 \in Y$ . Hence  $f(X) = \{y_0\}$ . Thus  $f$  is a constant function.

**Theorem 4.9:** Every contra-strongly semi\*regular-irresolute function from a connected space onto a semi\*regular- $T_1$  is necessarily constant.

**Proof:** Let  $f: X \rightarrow Y$  be a contra-strongly semi\*regular-irresolute function. Let  $X$  be connected and  $Y$  be semi\*regular- $T_1$ . Since  $Y$  is semi\*regular- $T_1$ , by Theorem 5.5, for each  $y \in Y$ ,  $\{y\}$  is semi\*regular closed in  $Y$ . Since  $f$  is contra-strongly semi\*regular-irresolute,  $f^{-1}(\{y\})$  is open in  $X$ . Therefore  $\{f^{-1}(\{y\}): y \in Y\}$  is a collection of pairwise disjoint semi\*regular open sets in  $X$ . Since  $X$  is semi\*regular-connected,  $f^{-1}(\{y_0\}) = X$  for some unique  $y_0 \in Y$ . Hence  $f(X) = \{y_0\}$ . Thus  $f$  is a constant function.

## 5. Semi\*regular- $T_2$ Spaces

**Definition 5.1:** A space  $X$  is said to be semi\*regular- $T_2$  if whenever  $x$  and  $y$  are distinct points in  $X$ , there are disjoint semi\*regular open sets  $U$  and  $V$  in  $X$  containing  $x$  and  $y$  respectively.

**Theorem 5.2:**

- (i) Every semi\*regular- $T_2$  space is semi\* $\alpha$ - $T_2$ .
- (ii) Every semi\*regular- $T_2$  space is semi\*pre  $T_2$ .
- (iii) Every semi\*regular- $T_2$  space is semi\*- $T_2$ .
- (iv) Every semi\*regular- $T_2$  space is semi- $T_2$ .
- (v) Every semi\*regular- $T_2$  space is semi  $\alpha$ - $T_2$ .
- (vi) Every semi\*regular- $T_2$  space is semi-pre- $T_2$ .

**Proof:**

- (i) Suppose  $X$  is a semi\*regular- $T_2$  space. Let  $x$  and  $y$  be two distinct points in  $X$ . Since  $X$  is semi\*regular- $T_2$ , there exist disjoint semi\*regular open sets  $U$  and  $V$  such that  $x \in U$  and  $y \in V$  respectively. By Theorem 2.5(i),  $U$  and  $V$  are disjoint semi\* $\alpha$ -open sets such that  $x \in U$  and  $y \in V$  respectively. Hence  $X$  is semi\* $\alpha$ - $T_2$ .
- (ii) Let  $X$  be a semi\*regular- $T_2$  space. Let  $x$  and  $y$  be two distinct points in  $X$ . Since  $X$  is semi\*regular- $T_2$ , there exist disjoint semi\*regular open sets  $U$  and  $V$  such that  $x \in U$  and  $y \in V$  respectively. By Theorem 2.5(ii),  $U$  and  $V$  are disjoint semi\*pre-open sets such that  $x \in U$  and  $y \in V$ . Hence  $X$  is semi\*pre- $T_2$ .
- (iii) follows from the fact that every semi\*regular open set is semi\*open.
- (iv) Suppose  $X$  is a semi\*regular- $T_2$  space. Let  $x$  and  $y$  be two distinct points in  $X$ . Since  $X$  is semi\*regular- $T_2$ , there exist disjoint semi\*regular open sets  $U$  and  $V$  such that  $x \in U$  and  $y \in V$  respectively. By Theorem 2.5(iv),  $U$  and  $V$  are disjoint semi-open sets such that  $x \in U$  and  $y \in V$ . Hence  $X$  is semi- $T_2$ . (v), (vi) Follows from the fact that every semi\*regular open set is semi  $\alpha$ -open and semi pre-open.

**Theorem 5.3:** For a topological space  $X$  the following are equivalent:

- (i)  $X$  is semi\*regular- $T_2$  space.
- (ii) Let  $x \in X$ . Then for each  $y \neq x$ , there exists a semi\*regular open set  $U$  such that  $x \in U$  and  $y \notin s^*rCl(U)$
- (iii) For each  $x \in X$ ,  $\bigcap \{s^*rCl(U): U \in S^*RO(X) \text{ and } x \in U\} = \{x\}$ .

**Proof:**

**(i)  $\Rightarrow$  (ii):** Suppose  $X$  is a semi\*regular- $T_2$  space. Let  $x \in X$  and  $y \in X$  with  $y \neq x$ . Then there exist disjoint semi\*regular open sets  $U$  and  $V$  such that  $x \in U$  and  $y \in V$ . Since  $V$  is semi\* regular open,  $X \setminus V$  is semi\*regular closed and  $U \subseteq X \setminus V$ . This implies that  $s^*rCl(U) \subseteq X \setminus V$ . Since  $y \notin X \setminus V$ ,  $y \notin s^*rCl(U)$ .

**(ii)  $\Rightarrow$  (iii):** If  $y \neq x$  then there exist a semi\*regular open set  $U$  such that  $x \in U$  and  $y \notin s^*rCl(U)$ . Hence  $y \notin \bigcap \{s^*rCl(U): U \in S^*RO(X) \text{ and } x \in U\}$ . This proves (iii).

**(iii)  $\Rightarrow$  (i):** Let  $y \neq x$  in  $X$ . Then  $y \notin \bigcap \{s^*rCl(U): U \in S^*RO(X) \text{ and } x \in U\}$ . This implies that there exist a semi\*regular open set  $U$  such that  $x \in U$  and  $y \notin s^*rCl(U)$ . Then  $V = X \setminus s^*rCl(U)$  is semi\* regular open and  $y \in V$ . Now  $U \cap V = U \cap (X \setminus s^*rCl(U)) \subseteq U \cap (X \setminus U) = \emptyset$ . This proves (i).

**Theorem 5.4:** Let  $f : X \rightarrow Y$  be a bijection

- (i) If  $f$  is semi\*regular-continuous and  $Y$  is  $T_2$ , then  $X$  is semi\*regular- $T_2$ .
- (ii) If  $f$  is semi\*regular-irresolute and  $Y$  is semi\*regular- $T_2$ , then  $X$  is semi\*regular- $T_2$ .
- (iii) If  $f$  is semi\*regular open and  $X$  is  $T_2$ , then  $Y$  is semi\*regular- $T_2$ .

**Proof:**

- (i) Suppose  $f$  is semi\*regular-continuous and  $Y$  is  $T_2$ . Let  $x_1, x_2 \in X$  with  $x_1 \neq x_2$ . Let  $y_1 = f(x_1)$  and  $y_2 = f(x_2)$ . Since  $f$  is one to one  $y_1 \neq y_2$ . Since  $Y$  is  $T_2$  there exist disjoint open sets  $U$  and  $V$  containing  $y_1$  and  $y_2$  respectively. Since  $f$  is semi\*regular-continuous bijection,  $f^{-1}(U)$  and  $f^{-1}(V)$  are disjoint semi\*regular open sets in  $X$  containing  $x_1$  and  $x_2$  respectively. Thus  $X$  is semi\*regular- $T_2$ .
- (ii) Proof is similar to (i)
- (iii) Suppose  $f$  is semi\*regular open and  $X$  is  $T_2$ . Let  $y_1 \neq y_2 \in Y$ . Since  $f$  is a bijection, there exist  $x_1, x_2$  in  $X$  such that  $f(x_1) = y_1$  and  $f(x_2) = y_2$  with  $x_1 \neq x_2$ . Since  $X$  is  $T_2$ , there exist disjoint open sets  $U$  and  $V$  in  $X$  such that  $x_1 \in U$  and  $x_2 \in V$ . Since  $f$  is semi\*regular open, there exist semi\*regular open sets in  $Y$  such that  $y_1 = f(x_1) \in f(U)$  and  $y_2 = f(x_2) \in f(V)$ . Since  $f$  is a bijection,  $f(U)$  and  $f(V)$  are disjoint in  $Y$ . Thus  $Y$  is semi\*regular- $T_2$ .

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