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CHARACTERIZATION OF NEUTROSOPHIC NOWHERE DENSE SETS

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ABSTRACT

In this paper, the concept of neutrosophic nowhere dense set is introduced and characterizations of neutrosophic nowhere dense sets are studied.

Keywords: *Neutrosophic dense set; neutrosophic open hereditarily irresolvable; neutrosophic nowhere dense set.*

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1. INTRODUCTION AND PRELIMINARIES

The fuzzy concept has imposed a great influence in almost all branches of mathematics since the introduction of fuzzy sets by L. A. Zadeh [14]. The theory of fuzzy topological space was introduced and developed by C. L. Chang [6] and since then various notions in classical topology have been extended into the context of fuzzy topological space. The idea of "intuitionistic fuzzy set" was first published by Atanassov [1] and some research in this respect have been done by him and his colleagues [2, 3, 4]. Later, this concept was generalized to "intuitionistic L - fuzzy sets" by Atanassov and Stoeva [5]. The concept of nowhere dense set in intuitionistic fuzzy topological space introduced by S. S. Thakur and R. Dhavaseelan in [13]. F. Smarandache introduced the important and useful concepts of neutrosophy and neutrosophic set [[11], [12]]. The concepts of neutrosophic crisp set and neutrosophic crisp topological space were introduced by A. A. Salama and S. A. Alblowi [10]. The Basic definitions and Proposition related to neutrosophic topological spaces was introduced and discussed by Dhavaseelan et al. [8].

In this paper, we introduce the concept of neutrosophic nowhere dense set and study its fundamental properties. Her we mention some well-known notions which will be used in what follows.

Definition 1.1: Let T,I,F be real standard or non standard subsets of $]0^{-},1^{+}[$, with $sup_T = t_{sup}, inf_T = t_{inf}$ $sup_I = i_{sup}, inf_I = i_{inf}$ $sup_F = f_{sup}, inf_F = f_{inf}$ $n-sup = t_{sup} + i_{sup} + f_{sup}$ $n-inf = t_{inf} + i_{inf} + f_{inf}$. T,I,F are neutrosophic components.

Definition 1.2: Let X be a nonempty fixed set. A neutrosophic set (briefly NS) A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ which represents the degree of membership function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) respectively of each element $x \in X$ to the set A.

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Remark 1.1:

- A neutrosophic set $A=\{\langle x,\mu_A(x),\sigma_A(x),\gamma_A(x)\rangle : x\in X\}$ can be identified to an ordered triple $\langle \mu_A,\sigma_A,\gamma_A\rangle$ in $]0^-,1^+[$ on X.
- For the sake of simplicity, we shall use the symbol $A=\langle \mu_A,\sigma_A,\gamma_A\rangle$ for the neutrosophic set $A=\{\langle x,\mu_A(x),\sigma_A(x),\gamma_A(x)\rangle : x\in X\}$.

Definition 1.3: Let X be a nonempty set and the neutrosophic sets A and B in the form $A=\{\langle x,\mu_A(x),\sigma_A(x),\gamma_A(x)\rangle : x\in X\}$, $B=\{\langle x,\mu_B(x),\sigma_B(x),\gamma_B(x)\rangle : x\in X\}$. Then

- $A\subseteq B$ iff $\mu_A(x)\le\mu_B(x)$, $\sigma_A(x)\le\sigma_B(x)$ and $\gamma_A(x)\ge\gamma_B(x)$ for all $x\in X$;
- $A=B$ iff $A\subseteq B$ and $B\subseteq A$;
- $\overline{A}=\{\langle x,\gamma_A(x),\sigma_A(x),\mu_A(x)\rangle : x\in X\}$; [Complement of A]
- $A\cap B=\{\langle x,\mu_A(x)\wedge\mu_B(x),\sigma_A(x)\wedge\sigma_B(x),\gamma_A(x)\vee\gamma_B(x)\rangle : x\in X\}$;
- $A\cup B=\{\langle x,\mu_A(x)\vee\mu_B(x),\sigma_A(x)\vee\sigma_B(x),\gamma_A(x)\wedge\gamma_B(x)\rangle : x\in X\}$;
- $[\,]A=\{\langle x,\mu_A(x),\sigma_A(x),1-\mu_A(x)\rangle : x\in X\}$;
- $\langle\rangle A=\{\langle x,1-\gamma_A(x),\sigma_A(x),\gamma_A(x)\rangle : x\in X\}$.

Definition 1.4: Let $\{A_i : i\in J\}$ be an arbitrary family of neutrosophic sets in X. Then

- $\bigcap A_i=\{\langle x,\wedge\mu_{A_i}(x),\wedge\sigma_{A_i}(x),\vee\gamma_{A_i}(x)\rangle : x\in X\}$;
- $\bigcup A_i=\{\langle x,\vee\mu_{A_i}(x),\vee\sigma_{A_i}(x),\wedge\gamma_{A_i}(x)\rangle : x\in X\}$.

Since our main purpose is to construct the tools for developing neutrosophic topological spaces, we must introduce the neutrosophic sets 0_N and 1_N in X as follows:

Definition 1.5: $0_N=\{\langle x,0,0,1\rangle : x\in X\}$ and $1_N=\{\langle x,1,1,0\rangle : x\in X\}$.

Definition 1.6: [8] A neutrosophic topology (briefly NT) on a nonempty set X is a family T of neutrosophic sets in X satisfying the following axioms:

- $0_N,1_N\in T$,
- $G_1\cap G_2\in T$ for any $G_1,G_2\in T$,
- $\cup G_i\in T$ for arbitrary family $\{G_i \mid i\in\Lambda\}\subseteq T$.

In this case the ordered pair (X,T) or simply X is called a neutrosophic topological space (briefly NTS(X)) and each neutrosophic set in T is called a neutrosophic open set (briefly NOS). The complement $\overline{A}$ of a NOS A in X is called a neutrosophic closed set (briefly NCS) in X.

Definition 1.7: [8] Let A be a neutrosophic set in a $NTS(X)$. Then

$Nint(A)=\bigcup\{G \mid G$ is an NOS in X and $G\subseteq A\}$ is called the neutrosophic interior of A;

$Ncl(A)=\bigcap\{G \mid G$ is an NCS in X and $G\supseteq A\}$ is called the neutrosophic closure of A.

Definition 1.8: Let X be a nonempty set. If r,t,s be real standard or non standard subsets of $]0^-,1^+[$ then the neutrosophic set $x_{r,t,s}$ is called a neutrosophic point(briefly NP)in X given by

$$x_{r,t,s}(x_p)=\begin{cases}(r,t,s), & if\ x=x_p\\ (0,0,1), & if\ x\ne x_p\end{cases}$$

for $x_p \in X$ is called the support of $x_{r,t,s}$, where r denotes the degree of membership value, t denotes the degree of indeterminacy and s is the degree of non-membership value of $x_{r,t,s}$.

2. NEUTROSOPHIC NOWHERE DENSE SETS

Definition 2.1: A neutrosophic set A in NTS (X,T) is called neutrosophic dense if there exists no neutrosophic closed set B in (X,T) such that $A \subset B \subset 1_N$

Definition 2.2: A neutrosophic set A in NTS (X,T) is called neutrosophic nowhere dense set if there exists no NOS, U in (X,T) such that $U \subset Ncl(A)$. That is $NintNcl(A) = 0_N$.

Example 2.1: Let $X = \{a,b,c\}$. Define the neutrosophic sets A, B and C as follows:
$A = \langle x, (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.5}), (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.5}), (\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.5}) \rangle$ $B = \langle x, (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.6}), (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.6}), (\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.3}) \rangle$, and
$C = \langle x, (\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.4}), (\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.4})(\frac{a}{0.7}, \frac{b}{0.7}, \frac{c}{0.4}) \rangle$. Then the family $T = \{0_N, 1_N, A\}$ is an NT on X. Thus, (X,T) is an Neutrosophic Topology .
Now $NintNcl(\overline{A}) = 0_N, NintNcl(\overline{B}) = 0_N, NintNcl(C) = 0_N, NintNcl(B) = 1_N \neq 0_N, NintNcl(\overline{C}) = 1_N \neq 0_N$. and $\overline{A}, \overline{B}$ and C are neutrosophic nowhere dense sets in (X,T). B and $\overline{C}$ are not neutrosophic nowhere dense set in (X,T).

Proposition 2.1: Let A be a neutrosophic set. If A is a neutrosophic closed set in (X,T) with $Nint(A) = 0_N$, then A is a neutrosophic nowhere dense set in (X,T).

Proof: Let A be a neutrosophic closed set in (X,T). Then $Ncl(A) = A$. Now $Nint(Ncl(A)) = Nint(A) = 0_N$ and hence A is a neutrosophic nowhere dense set A in (X,T).

Proposition 2.2: Let A be a neutrosophic set. If A is a neutrosophic nowhere dense set in (X,T), then $Nint(A) = 0_N$.

Proof: Let A be a neutrosophic nowhere dense set in (X,T). Now $A \subseteq Ncl(A)$ implies that $Nint(A) \subseteq NintNcl(A) = 0_N$. Hence we have $Nint(A) = 0_N$.

The converse of Proposition 2.2, need not be true as shown in Example 2.2.

Example 2.2: Let $X = \{a,b,c\}$. Define the neutrosophic sets
$A = \langle x, (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.5}), (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.5}), (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.5}) \rangle$ $B = \langle x, (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}), (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}), (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.6}) \rangle$,
$C = \langle x, (\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5}), (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.4}) \rangle$,and $D = \langle x, (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}), (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}), (\frac{a}{0.7}, \frac{b}{0.5}, \frac{c}{0.5}) \rangle$. Clearly $T = \{0_N, 1_N, A, B, C\}$ is an NT on X. Thus (X,T) is an NT. Now $Nint(D) = 0_N$,

where as $NintNcl(D) = B \neq 0_N$. Also $Ncl(D) = \overline{C} \neq D$. Hence D is not a neutrosophic nowhere dense set in (X,T). Also D is not a neutrosophic closed set in (X,T).

Remark 2.1: The complement of a neutrosophic nowhere dense set need not be a neutrosophic nowhere dense set. See Example 2.3.

Example 2.3: Let $X = \{a,b,c\}$. Define the neutrosophic sets $A = \langle x, (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.5}), (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.5}), (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.5}) \rangle$ and $B = \langle x, (\frac{a}{0.4}, \frac{b}{0.3}, \frac{c}{0.5}), (\frac{a}{0.4}, \frac{b}{0.3}, \frac{c}{0.5}), (\frac{a}{0.6}, \frac{b}{0.7}, \frac{c}{0.5}) \rangle$. Clearly $T = \{0_N, 1_N, A\}$ is a neutrosophic topology on X. Thus (X,T) is an NT. Now B is a neutrosophic nowhere dense set in (X,T) where as $\overline{B}$ is not a neutrosophic nowhere dense set, since $NintNcl(\overline{B}) = Nint(1_N) = 1_N \neq 0_N$.

Proposition 2.3: If A is a neutrosophic dense, NOS in (X,T), such that $B \subseteq \overline{A}$, then B is a neutrosophic nowhere dense set in (X,T).

Proof: Let A be a neutrosophic open set in (X,T) such that $Ncl(A) = 1_N$. Now $B \subseteq \overline{A}$ implies that $Ncl(B) \subseteq Ncl(\overline{A}) = \overline{A}$. Then we have $NintNcl(B) \subseteq Nint(\overline{A}) = \overline{Ncl(A)} = 0_N$ and hence $NintNcl(B) = 0_N$. Therefore B is a neutrosophic nowhere dense sets in (X,T).

Proposition 2.4: If A is a neutrosophic closed set in (X,T), then A is a neutrosophic nowhere dense set in (X,T) if and only if $Nint(A) = 0_N$.

Proof: Let A be a neutrosophic closed set in (X,T), with $Nint(A) = 0_N$. Then by Proposition 2.1, A is a neutrosophic nowhere dense set in (X,T). Conversely, let A be a neutrosophic nowhere dense set in (X,T). Then $NintNcl(A) = 0_N$ which implies that $Nint(A) = 0_N$. Since A is a neutrosophic closed, $Ncl(A) = A$.
The proofs of following propositions are obvious.

Definition 2.3: Let A be a neutrosophic set. The NT, (X,T) is called neutrosophic open hereditarily irresolvable if $NintNcl(A) \neq 0_N$, then $Nint(A) \neq 0_N$ for any non-zero neutrosophic set in (X,T)

Proposition 2.5: If (X,T) is a neutrosophic open hereditarily irresolvable space, any non zero neutrosophic set A with $Nint(A) = 0_N$ is a neutrosophic nowhere dense set in (X,T).

Proof: Let A be a non zero neutrosophic set in a neutrosophic open hereditarily irresolvable space (X,T) with $Nint(A) = 0_N$. Suppose that $NintNcl(A) \neq 0_N$. Since (X,T) is neutrosophic open hereditarily irresolvable space, $Nint(A) \neq 0_N$, which is contradiction to $Nint(A) = 0_N$. Hence we must have $NintNcl(A) = 0_N$ and therefore A is a neutrosophic nowhere dense set in (X,T).

Proposition 2.6: If A is a neutrosophic nowhere dense set in (X,T), then $\overline{A}$ is a neutrosophic dense set in (X,T).

Proof: Let A be a neutrosophic nowhere dense set in (X,T). Then by Proposition 2.2, we have, $Nint(A) = 0_N$.

Now $Ncl(\overline{A}) = \overline{Nint(A)} = 1_N$. Therefore $\overline{A}$ is a neutrosophic dense set in (X,T).

Proposition 2.7: If A is an intuitionsitic fuzzy dense and neutrosophic open set in (X,T), then $\overline{A}$ is a neutrosophic nowhere dense set in (X,T).

Proof: Let A be a neutrosophic open set in (X,T) such that $Ncl(A) = 1_N$.
Now $NintNcl(\overline{A}) = \overline{NclNint(A)} = \overline{Ncl(A)} = 0_N$. Hence $\overline{A}$ is a neutrosophic nowhere dense set in (X,T).

Proposition 2.8: If A is a neutrosophic nowhere dense set in (X,T), then $Ncl(A)$ is a neutrosophic nowhere dense set in (X,T).

Proof: Let $Ncl(A)=B$. Now $NintNcl(B)=NintNcl(Ncl(A))=NintNcl(A)=0_N$. Hence $B=Ncl(A)$ is a neutrosophic nowhere dense set in (X,T).

Proposition 2.9: If A is a neutrosophic nowhere dense set in (X,T), then $\overline{Ncl(A)}$ is a neutrosophic dense set in (X,T).

Proof: By Proposition 2.8, we have $Ncl(A)$ is a neutrosophic nowhere dense set in (X,T). By Proposition 2.6, we have $\overline{Ncl(A)}$ is a neutrosophic dense set in (X,T).

Proposition 2.10: Let A be a neutrosophic dense set in a neutrosophic topological space (X,T). If B is a neutrosophic set in (X,T), then B is a neutrosophic nowhere dense set in (X,T) if and only if $A\cap B$ is a neutrosophic nowhere dense set in (X,T).

Proof: Let B be a neutrosophic nowhere dense set in (X,T).
Now
$NintNcl(A\cap B)=Nint(Ncl(A))\cap Nint(Ncl(B))=Nint(1_N)\cap Nint(Ncl(B))=NintNcl(B)=0_N$.
Therefore $A\cap B$ is a neutrosophic nowhere dense set in (X,T).

Conversely, let $A\cap B$ be a neutrosophic dense set in (X,T). Then $NintNcl(A\cap B)=0_N$ implies that $Nint(Ncl(A))\cap Nint(Ncl(B))=0_N$. Hence $Nint(1_N)\cap Nint(Ncl(B))=0_N$ and therefore $Nint(Ncl(B))=0_N$ which means that B is a neutrosophic nowhere dense set in (X,T).

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