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PAIRWISE FUZZY e -CONNECTEDNESS BETWEEN FUZZY SETS

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ABSTRACT

In this paper the concept of fuzzy e -connectedness between fuzzy sets is generalized to fuzzy bitopological spaces and some of its properties are studied.

Key words and phrases: *Fuzzy bitopological spaces, pairwise fuzzy, e - Connectedness, (i, j) -fuzzy e -clopen.*

AMS (2000) subject classification: *54A40.*

1. INTRODUCTION

The concept of fuzzy set was introduced by Zadeh [21] provided a natural foundation for building new branches in mathematics. Fuzzy sets have applications in many fields such as information [17] and control [18]. In 1968 Chang [4] introduced fuzzy topological space using fuzzy sets. Kandil [7] defined and studied the concept of fuzzy bitopological spaces as a generalization of bitopological spaces [9] in fuzzy setting. Since then many results from classical topology are being extended in both fuzzy topological and fuzzy bitopological spaces ([2], [3], [6], [7], [8], [12]-[15], [20]) and their properties were also investigated. The initiations of e -open sets in topological spaces are due to Ekici [5]. In fuzzy topology, e -open sets were introduced by Seenivasan in 2015 [16]. In 1993, Maheswari [10] introduced the concept of connectedness between fuzzy sets. In this paper the concepts of fuzzy e-connectedness between fuzzy sets generalized to fuzzy bitopological spaces and some of its properties are studied.

2. PRELIMINARIES

Let X and Y be non-empty sets. A fuzzy set λ in X is a mapping from X to the unit interval [0, 1]. The null fuzzy set 0 (resp. the whole fuzzy set 1) is the mapping from X to the unit interval [0, 1] which takes the only value 0 (resp. 1) in that interval.

The closure denoted by Cl (λ) (interior, denoted by I nt(λ)) of a fuzzy set λ of X is the intersection (union) of all fuzzy closed supersets (fuzzy open subsets, respectively) of λ [4]. For a fuzzy set λ of a fuzzy topological space X, $1 - Int(\lambda) = Cl(l - \lambda)$ and $1 - Cl(\lambda) = Int(\lambda)$. A fuzzy set λ in X is said to be quasi-coincident [11] with a fuzzy set μ in X denoted by λqμ if there exists a point x ∈ X such that $\lambda(x) + \mu(x) > 1$. If λ and μ, are two fuzzy sets of X, then $\lambda \leq \mu$ if and only if λ and 1−μ are not quasi-coincident. A fuzzy topological space (X, τ) is said to be fuzzy connected [6] if there is no proper fuzzy set in X which is both fuzzy open and fuzzy closed. A fuzzy topological space (X, τ) is said to be fuzzy connected [10] between its subsets λ and μ if and only if there is no fuzzy closed fuzzy open set δ in X such that $\lambda \leq \delta$ and $\neg(\delta q \mu)$.

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Definition 2.1: A fuzzy subset λ in an fts (X, τ) is c Called fuzzy regular open (fro, for Short) [1] if $\lambda = IntCl(\lambda)$ and regular closed if $\lambda = ClInt(\lambda)$.

Definition 2.2: [16] The fuzzy δ-interior of subset λ of X is the union of all fuzzy regular open sets contained in λ and fuzzy δ closure of subset λ of X is the intersection of all fuzzy regular closed sets containing λ.

Definition 2.3: [19] A subset is λ called fuzzy δ open if $\lambda = \delta Int(\lambda)$. The complement of fuzzy δ open set is called fuzzy δ closed (i.e., $\lambda = \delta\, Cl(\lambda)$.)

Definition 2.4:A subset λ is called fuzzy e -open [16] if $\lambda \leq IntCl_{\delta}(\lambda) \vee ClInt_{\delta}(\lambda)$. The complement of a fuzzy e -open is called fuzzy e -closed.

Definition 2.5: [16] The intersection of all fuzzy e-closed sets containing λ is called fuzzy e-closure of λ and is denoted by $feCl(\lambda)$ and the union of all fuzzy e-open sets contained λ is called fuzzy e-interior of λ and is denoted by feInt(λ) .

A system (X, τ_1, τ_2) consisting of a set X with two topologies τ_1 and τ_2 on X is called a fuzzy bitopological space [7]. A fuzzy bitopological space (X, τ_1, τ_2) is said to be pairwise fuzzy connected [11] if it has no proper fuzzy set which is both $\tau_i$fuzzy open and τ_j-fuzzy closed, i, j = l, 2, i≠j . The purpose of this paper is to introduce and study the concept of pairwise fuzzy e -connectedness between fuzzy sets in fuzzy bitopological spaces.

Throughout this paper i, j = 1, 2 where i ≠j. If P is any fuzzy topological property then τ_i- P and τ_j- P denote the property P with respect to the fuzzy topology τ_i and τ_j respectively and χA denotes the characteristic function of a subset A of X.

3. PAIRWISE FUZZY e -CONNECTEDNESS BETWEEN FUZZY SETS

Definition 3.1: A fuzzy bitopological space (X, τ_1, τ_2) is said to be pairwise fuzzy e - connected between fuzzy sets λ and μ if there is no (i, j) -fuzzy e –clopen (τ_i-fuzzy e -closed and τ_j -fuzzy e -open) set δ in X such that $\lambda \leq \delta$ and ¬(δqμ)

Remark 3.1: Pairwise fuzzy e -connectedness between fuzzy sets λ and μ is not equal to the fuzzy connectedness of (X, τ_1) and (X, τ_2) between λ and μ.

Example 3.1: Let X = {a, b, c} and let $\mu_1, \mu_2, \mu_3, \mu_4, \eta_1, \eta_1, \eta_2, \eta_3$ and be fuzzy sets on X defined as follows: μ_1(a) = 0.7, μ_1(b) = 1, μ_1(c) = 0; μ_2(a) = 0.2, μ_2 (b) = 0, μ_2 (c) = 1; μ_3 (a) = 0.7, μ_3 (b) = 1, μ_3 (c) = 1; μ_4(a) = 0.2, μ_4 (b) = 0, μ_4 (c) = 0; η_1(a) = 0, η_1(b) = 0.3, η_1(c) = 0; η_2(a) = 0, η_2(b) = 0, η_2(c) = 1; η_3 (a) = 0, η_3 (b) = 0.3, η_3(c) = 1; η_4 (a) = 0.3, η_4(b) =0, η_4(c) = 0.2. Let τ_1= {0, 1, $\mu_1, \mu_2, \mu_3, \mu_4$} and$\tau_2$ = {0, 1, η_1, η_2, η_3}be fuzzy topologies on X. Then (X, τ_1) and (X, τ_2) are fuzzy e (resp. δs and δp)-connected between the fuzzy sets μ_4 and η_4 But (X, τ_1, τ_2) is not pairwise fuzzy e (resp. δs and δp)-connected between μ_4 and η_4.

Example 3.2: Let X = {a, b}. Let fuzzy sets $\mu_5, \mu_6, \mu_7, \eta_5, \eta_6$ and η_7 be defined as follows: μ_5(a) = 0.2, μ_5 (b) = 0, μ_5 (c) = 0; μ_6(a) = 0.5, μ_6(b) = 0.5, μ_6 (c) = 0.5; μ_7(a) = 0.3, μ_7 (b) = 0.2, μ_7(c) = 0; μ_8 (a) = 0.3, μ_8(b) = 0.3, μ_8 (c) = 0.1; η_5(a) = 0.2, η_5(b) = 0.1, η_5(c) = 0; η_6 (a) = 0.6, η_6(b) = 0.6, η_6(c) = 0.6; η_7(a) = 0.5, η_7(b) = 0.4, η_7(c) = 1. Let τ_1 = {0, 1, μ_5, μ_6} and τ_2 = {0, 1, η_5, μ_6} be fuzzy topologies on X. Then the fuzzy bitopological space (X, τ_1, τ_2) is pairwise fuzzy e (resp. δs and δp)-connected between μ_7 and η_6, but neither (X, τ_1) nor (X, τ_2) are fuzzy e (resp. δs and δp)-connected between μ_7 and η_6.

Theorem 3.1: A fuzzy bitopological space (X, τ_1, τ_2) is pairwise fuzzy e - connected between fuzzy sets λ and μ if and only if there is no (i, j) -fuzzy e -clopen set δ in X such that $\lambda \leq \delta \leq 1 -$ μ.

Proof: Obvious.

Theorem 3.2: If a fuzzy bitopological space (X, τ_1, τ_2) is pairwise fuzzy e - connected between fuzzy sets λ and μ then and μ are non-empty.

Proof: Evident.

Theorem 3.3: If a fuzzy bitopological space (X, τ_1, τ_2) is pairwise fuzzy e - connected between fuzzy sets λ and μ and if $\lambda \leq \lambda_1$ and $\mu \leq \mu_1$ then (X, τ_1, τ_2) is pairwise fuzzy e -connected between λ_1 and μ_1.

Proof: Suppose the fuzzy bitopological space (X, τ_1, τ_2) is not pairwise fuzzy e -connected between the fuzzy sets λ_1 and μ_1. Then there is an (i, j) -fuzzy e - clopen set δ in X such that $\lambda_1 \leq \delta$ and $\neg(\delta q \mu_1)$. Clearly $\leq \delta$. Now we claim that $\neg(\delta q \mu)$. If $(\delta q \mu)$ then there exists a point $x \in X$ such that $\delta(x) + \mu(x) > 1$. Therefore $\delta(x) + \mu_1(x) > \delta(x) + \mu(x) > 1$ and $\delta q \mu_1$, a contradiction. Consequently, (X, τ_1, τ_2) is not pairwise fuzzy e -connected between λ and μ.

Theorem 3.4: A fuzzy bitopological space (X, τ_1, τ_2) is pairwise fuzzy e - connected between λ and μ if and only if it is pairwise fuzzy e -connected between τ_i - $eCl(\lambda)$ and τ_i-$eCl(\mu)$.

Proof:
Necessity: It follows by using Theorem 3.3.

Sufficiency: Suppose the fuzzy bitopological space (X, τ_1, τ_2) is not pairwise fuzzy e -connected between λ and μ. Then there is an (i, j) -fuzzy e -clopen set δ in X such that $\lambda \leq \delta$ and $\neg(\delta q \mu)$.
Since $\lambda \leq \delta$, τ_i- $eCl(\lambda) \leq \tau_j$ - $eCl(\delta) < \delta$ because δ is τ_i - fuzzy e -closed. Now, $\neg(\delta q \mu) \Rightarrow \delta \leq 1 - \mu$
$\Rightarrow \delta \leq \tau_j - eInt(1 - \mu)$
$\Rightarrow \delta \leq 1 - \tau_j - eCl(\mu)$
$\Rightarrow \neg(\delta q \tau_j - eCl(\mu)$.

Hence X is not pairwise fuzzy e -connected between τ_i- $eCl(\lambda)$ and τ_j- $eCl(\mu)$, a contradiction.

Theorem 3.5: Let (X, τ_1, τ_2) be a fuzzy bitopological space and let λ and μ be two fuzzy sets in X. If λqμ then (X, τ_1, τ_2) is pairwise fuzzy e -connected between λ and μ .

Proof: If δ is any (i, j) -fuzzy e -clopen set in X such that $\lambda \leq \delta$ then $\lambda q \mu \Rightarrow \delta q \mu$.

Remark 3.2: The converse of Theorem 3.5. may not be true as is shown by the next example.

Example 3.3: In Example 3.2, the fuzzy bitopological space (X, τ_1, τ_2) is pairwise connected between μ_8 and η_7 but not $\neg(\mu_8 q \eta_7)$.

Theorem 3.6: If a fuzzy bitopological space (X, τ_1, τ_2) is pairwise fuzzy e - connected neither between λ and μ_0, nor between λ and μ_1 , then it is not pairwise fuzzy e -connected between λ and $\mu_0 \cup \mu_1$.

Proof: Since X is pairwise fuzzy e -connected neither between λ and μ_0 nor between λ and μ_1, there exists (i, j) -fuzzy e -clopen fuzzy sets δ_0 and δ_1 in (X, τ_1, τ_2) such that $\lambda \leq \delta_0, \neg(\delta_0 q \mu_0)$ and $\leq \delta_1, \neg(\delta_1 q \mu_1)$. Put $\delta = \delta_0 \cap \delta_{01}$. Then δ is (i, j) -fuzzy e -clopen and $\lambda \leq \delta$. Now we claim that $\neg(\delta q(\mu_0 \cup \mu_1))$. If $\delta q(\mu_0 \cup \mu_1)$ then there exists a point $x \in X$ such that $(x) + (\mu_0 \cup \mu_1)(x) > 1$. This implies that $\delta q \mu_0$ or $\delta q \mu_1$, a contradiction. Hence X is not pairwise fuzzy e -connected between λ and $\mu_0 \cup \mu_1$.

Theorem 3.7: A fuzzy bitopological space (X, τ_1, τ_2) is pairwise fuzzy e - connected if and only if it is pairwise fuzzy e -connected between every pair of its non-empty fuzzy subsets.

Proof: Necessity: Let λ and μ be any pair of non-empty fuzzy subsets of X. Suppose (X, τ_1, τ_2) is not pairwise fuzzy e -connected between λ and μ. Then there is an (i, j)-fuzzy e -clopen set δ in X such that $\lambda \leq \delta$ and $\neg(\delta q \mu)$. Since λ and μ are non-empty, it follows that δ is a non-empty proper (i, j) -fuzzy e -clopen subset of X. Hence (X, τ_1, τ_2) is not pairwise fuzzy e -connected.

Sufficiency: Suppose (X, τ_1, τ_2) is not pairwise fuzzy e -connected. Then there exists a non-empty proper (i, j)-fuzzy e -clopen subset δ of X. Consequently, (X, τ_1, τ_2) is not pairwise fuzzy e -connected between δ and $1 - \delta$, a contradiction.

Remark 3.3: If fuzzy bitopological space (X, τ_1, τ_2) is pairwise fuzzy e - connected between a pair of its subsets then it need not necessarily hold that (X, τ_1, τ_2) is pairwise fuzzy between every pair of its subsets and so it is not necessarily pairwise fuzzy e -connected as is shown by the next example.

Example 3.4: In Example 3.3, the fuzzy sets μ_9, η_8 and η_9 be defined as follows:
μ_9(a) = 0.4, μ_9(b) = 0.3, μ_9(c) = 0.1; η_8(a) = 0.4, η_8(b) = 0.4, η_8(c) = 0.3; η_8 (a) = 0.6, η_9(b) = 0.6, η_9c) = 0.5. Then (X, τ_1, τ_2) is pairwise fuzzy e (resp. δs and δp)-connected between μ_9, and η_8, but it is not pairwise fuzzy e -connected between μ_9, and η_9. Also (X, τ_1, τ_2) is not pairwise fuzzy e (resp, δs and δp)-connected.

Theorem 3.8: Let $(Y, (\tau_1)Y, (\tau_2)Y)$ be a subspace of a fuzzy bitopological space (X, τ_1, τ_2) and let λ, μ be fuzzy sets of Y. If $(Y, (\tau_1)Y, (\tau_2)Y)$ is pairwise fuzzy e -connected between λ and μ then (X, τ_1, τ_2) is also pairwise fuzzy e -connected between λ and μ.

Proof: Evident.

Theorem 3.9: Let $(Y, (\tau_1)Y, (\tau_2)Y)$ be a subspace of a fuzzy bitopological space (X, τ_1, τ_2) and let λ, μ be fuzzy sets of Y. If (X, τ_1, τ_2) is pairwise fuzzy e -connected between λ and μ and χ_Y is bifuzzy clopen in (X, τ_1, τ_2) then $(Y, (\tau_1)Y, (\tau_2)Y)$ is pairwise fuzzy e -connected between λ and μ.

Proof: Suppose $(Y, (\tau_1)Y, (\tau_2)Y)$ is not pairwise fuzzy e -connected between λ and μ then there exists an (i, j) -fuzzy e -clopen set δ in X such that $\lambda \leq \delta$ and$\neg(\lambda q \delta)$. Since χ_Y is bifuzzy open and bifuzzy closed in (X, τ_1, τ_2), δ is (i, j)-fuzzy e -clopen in (X, τ_1, τ_2). Therefore (X, τ_1, τ_2) is not pairwise fuzzy e -connected between λ and μ. Which is a contradiction.

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