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A STUDY ON (M/M/1): (N/FCFS) QUEUEING MODEL UNDER MONTE CARLO SIMULATION IN A MULTI SPECIALITY HOSPITAL

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ABSTRACT

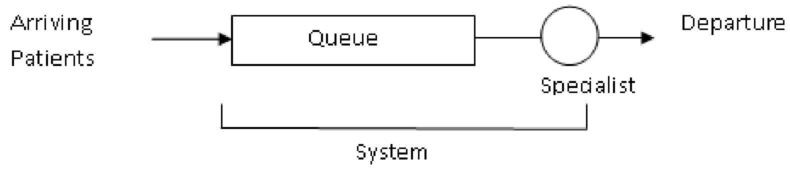
In this study, we analyse the queueing model under single server, the system capacity is limited to N. The arrival and service data are collected in a multi speciality hospital in two cases i.e., inpatients and emergency patients. Chi-square test is used for test the goodness of fit in arrival and service distribution. Monte Carlo Simulation method is used to analyse the queue length. We compare the Simulation result with analytical.

Keywords: *Arrival, Service, System Capacity, (M/M/1):(N/FCFS) queueing model, Monte Carlo Simulation, Queue length, Chi-square test.*

1. INTRODUCTION

Queues or waiting lines are very common in everyday life. Customers arrive at service counters and are attended by one or more of the servers and the customers leave the system after the service. The service starts from the first person or a thing in the sequence. Analysing and providing the service, servers related to the queue is defined as Queueing theory. Now it can be explained briefly about the topics viz: i) Queuing Theory, ii) Simulation Method, iii) Analytical Method and iv) Chi – Square Test. Queueing theory has originated in the research by Agner Krarup Erlang (1878-1929) a Danish Engineer cum mathematician while creating models to describe the telephone exchange and that was the first paper published which is now called as queueing theory [3]. Characteristics of queueing system are (i) Arrival pattern (ii) Service pattern (iii) Queue discipline (iv) System capacity (v) Number of service channels and (vi) Number of service phases.

M/M/1 QUEUING MODEL



The simulation was introduced by Mr. John von Neumann and Mr.Stanishlaw ulam. It is a method of solving decision-making problems by designing, constructing and manipulating a model of the real system. It is defined to be the action of performing experiments on a model of a given system. Mr. J.V. Neumann and Mr. Stanishlaw gave the first important application in the behaviour of Neutrons in a Nuclear Shielding Problem with remarkable success. Banks, J., (2001) [2] has described that in a Monte Carlo simulation, a random value is selected for each of the task, based on the range of estimates. The model is calculated based on this random value. The result of the model is recorded, and the process is repeated. A typical Monte Carlo simulation calculates the model hundreds or thousands of times, each time using different randomly-selected values. When the simulation is completed, there is a large number of results from the model, each is based on random input values. These results are used to describe the likelihood, or probability, of reaching various results in the model.

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Chi-square test is used to test the suitability of a distribution and the independence of the attributes. It is used to test the significance of the difference between the observed frequencies in a sample and expected frequencies obtained from the theoretical distribution. Karl Pearson developed a test for testing the significance of discrepancy between experimental values and the theoretical values obtained under some hypothesis. This is χ^2 test fitness. The test statistic is given by $\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$ where O_i is the observed frequency in the i^{th} class interval and E_i is the expected frequency in that class interval. The expected frequency for each class interval is computed as E_i is the np_i where p_i is the theoretical, hypothesized probability associated with the i^{th} class interval.

The aims of this paper are i) to check the goodness-of-fit of arrival and service distribution, and ii) to determine the queue length both in Simulation and Analytical method.

Ishan, P. Lade, Sandeep, A. Chowriwar and Pranay, B. Sawaitul (2013) have described the use of queueing systems to decrease the waiting time of patients in the queueing system using simulation method [5]. Shanmugasundaram, S. and Punitha, S., (2014) has analyzed the application of simulation in queueing model in tollgate [8]. Syed Shujauddin Sameer (2014) has helped in understanding the behaviour of a queueing system using simulation and also obtains certain very useful parameters. In this paper, simulation provides a good strategy to analyze the client – server systems and to solve the complicated problems. One of the important applications of the Simulation is the analysis of the waiting line problem and it is classified into Deterministic Model, Probabilistic Model, Static Model and Dynamic Model [12]. Shanmugasundaram, S. and Banumathi, P. (2016) have aimed to reduce the queue length, system length, queue time and system time in Southern railway using simulation in queueing analysis. Simulation is a mimic of reality that exists or which is contemplated. Simulation is most effectively used as a queueing analysis [7]. Soemon Takakuwa and Athula Wijewickrama (2008) have described the application of the simulation for patients coming to the hospital, the pertinent parameters like waiting time, service time, waiting time and service time ratio [11]. Shanmugasundaram, S. and Umarani, P., (2016) have discussed the applications of simulation in queueing model at a medical center and how to calculate the queue time, system time, queue length and system length in the Simulation table [10]. They (2016) also have analyzed the queueing system in a simulation model of a medical center in order to develop an efficient procedure for reducing the waiting time of the patients in the queue [9]. Ishan, P. Lade, V. P. Sakhare, M. S. Shelke and P. B. Sawaitul (2015) have analyzed the applications of queueing model using Simulation and to reduce the average waiting time of patients for chemotherapy section in the radiation therapy and oncology department [6]. Gateri Judy Muthoni, Stephen Kimani, Joseph wafula (2014) have compared the existing prediction models and come up with Monte Carlo Simulation model to predict the number of patients in the queue [4]. Alireza Saremi, Payman Jula, Tarek ElMekkawy, G. GaryWang (2012) have addressed the appointment scheduling of outpatient surgeries in a multistage operating room department with stochastic service times serving multiple patient types and have described to minimize the patients wait time and patients completion time by using Simulation-based optimization method [1].

2. DESCRIPTION OF THE MODEL

Initially, the arrival data of inpatients (Admitted patients) and emergency patients in a multi speciality hospital has been collected for a week, during the month of September 2017. Every department in a hospital has been assigned with a specialist for Inpatients and emergency patients. The actual queue length of the patients at the hospital is calculated through Monte Carlo Simulation and Analytical methods with (M/M/1) : (N/FCFS). The results of both the models were identical and they are illustrated in Table 11. As the objective of this paper is to minimize or nullify the queue length of the patients, Chi – Square test was used to check whether the arrival and the service of the patients are uniformly distributed. Generally, it is very tiresome for the patients to wait at a hospital as they may be both physically and psychologically weak. Further, patients have to wait for a long time in the queue for their turn which is undesirable. The problem of patients' waiting in the queue is solved in this paper. The number of patients accommodated in each department (N) is 25.

Table-1: Tag Number Table for Arrival Chosen

S. No.	Type of Patients	Probability	Cumulative Probability	Tag numbers
1	In Patients	0.4	0.4	0 – 39
2	Emergency Patients	0.6	1	40 – 100

Table-2: Tag Number Table for Day Choosen

S. No.	DAY	Probability	Cumulative probability	Tag number
1	MONDAY	0.13	0.13	0 – 12
2	TUESDAY	0.13	0.26	13 – 25
3	WEDNESDAY	0.13	0.39	26 – 38
4	THURSDAY	0.13	0.52	39 – 51
5	FRIDAY	0.17	0.69	52 – 68
6	SATURDAY	0.17	0.86	69 – 85
7	SUNDAY	0.14	1	86 – 100

Table-3: Arrival Distributions for in Patients

S.No.	DAY	No of Patients	Probability
1	MONDAY	31	0.16
2	TUESDAY	28	0.14
3	WEDNESDAY	26	0.13
4	THURSDAY	32	0.16
5	FRIDAY	27	0.14
6	SATURDAY	29	0.15
7	SUNDAY	25	0.12
8	TOTAL	198	-

Table-4: Chi-Square Test for in Patients Arrival
DISTRIBUTION

S.NO.	DAY	No of Patients f	E_i	χ^2
1	MONDAY	31	28.28	0.26
2	TUESDAY	28	28.28	0.002
3	WEDNESDAY	26	28.28	0.18
4	THURSDAY	32	28.28	0.49
5	FRIDAY	27	28.28	0.06
6	SATURDAY	29	28.28	0.02
7	SUNDAY	25	28.28	0.38
8	TOTAL	198	-	1.392

Null Hypothesis H_0: The arrival follows uniform distribution.

The level of significance: $\alpha = 0.01$. Degrees of freedom = 7. Calculated value of χ^2 = 1.392 Tabulated value of χ^2 for 6 degrees of freedom at 1% level of significance is 16.812. Since $\chi^2 < \chi 2_{0.01}$, It is accepted as H_0 and concluded that the arrival follows the Uniform distribution and it is suitable for the given time of interval.

Table-5: Tag Numbr Table for in Patients Arrival Distribution

S. No.	DAY	No of Patients	Probability	Cumulative probability	Tag number
1	MONDAY	31	0.16	0.16	0 – 15
2	TUESDAY	28	0.14	0.30	16 – 29
3	WEDNESDAY	26	0.13	0.43	30 – 42
4	THURSDAY	32	0.16	0.59	43 – 58
5	FRIDAY	27	0.14	0.73	59 – 72
6	SATURDAY	29	0.15	0.88	73 – 87
7	SUNDAY	25	0.12	1	88 – 100
8	TOTAL	198	-	-	-

Table-6: Arrival Distributions For Emergency Patients

S.No.	DAY	No of Patients	Probability
1	MONDAY	33	0.16
2	TUESDAY	25	0.12
3	WEDNESDAY	26	0.12
4	THURSDAY	31	0.15
5	FRIDAY	37	0.18
6	SATURDAY	29	0.14
7	SUNDAY	28	0.13
8	TOTAL	209	-

Table-7: Chi-Square Test For Emergency Patients Arrival Distribution

S.NO.	DAY	No of Patients f	E_i	χ^2
1	MONDAY	33	29.85	0.33
2	TUESDAY	25	29.85	0.78
3	WEDNESDAY	26	29.85	0.50
4	THURSDAY	31	29.85	0.04
5	FRIDAY	37	29.85	1.71
6	SATURDAY	29	29.85	0,02
7	SUNDAY	28	29.85	0.11
Total	TOTAL	209	-	3.49

Null Hypothesis H_0: The arrival follows uniform distribution.

The level of significance: $\alpha = 0.01$. Degrees of freedom = 7. Calculated value of χ^2 = 3.49. Tabulated value of χ^2 for 6 degrees of freedom at 1% level of significance is 16.812. Since $\chi^2 < \chi 2_{0.01}$, It is accepted as H_0 and concluded that the arrival follows the Uniform distribution and it is suitable for the given time of interval.

Table-8: Tag Numbr Table For Emergency Patients Arrival Distribution

S. No.	DAY	No of Patients	Probability	Cumulative probability	Tag number
1	MONDAY	33	0.16	0.16	0 – 15
2	TUESDAY	25	0.12	0.28	16 – 27
3	WEDNESDAY	26	0.12	0.40	28 – 39
4	THURSDAY	31	0.15	0.55	40 – 54
5	FRIDAY	37	0.18	0.73	55 – 72
6	SATURDAY	29	0.14	0.87	73 – 86
7	SUNDAY	28	0.13	1	87 – 100
8	TOTAL	209	-	-	-

Table-9: Service Distributions for In and Emergency Patients

S. No.	DAY	No of Patients	Probability
1	MONDAY	18	0.13
2	TUESDAY	21	0.15
3	WEDNESDAY	19	0.14
4	THURSDAY	23	0.17
5	FRIDAY	20	0.14
6	SATURDAY	21	0.15
7	SUNDAY	17	0.12
8	TOTAL	139	-

Table-10: Chi-Square Test For Service Distributions

S.NO.	DAY	No of Patients f	E_i	χ^2
1	MONDAY	18	19.85	0.17
2	TUESDAY	21	19.85	0.06
3	WEDNESDAY	19	19.85	0.03
4	THURSDAY	23	19.85	0.49
5	FRIDAY	20	19.85	0.001
6	SATURDAY	21	19.85	0.06
7	SUNDAY	17	19.85	0.41
Total	TOTAL	139	-	1.221

Null Hypothesis H_0: The service follows uniform distribution.

The level of significance: $\alpha = 0.01$. Degrees of freedom = 7. Calculated value of χ^2 = 1.221 Tabulated value of χ^2 for 6 degrees of freedom at 1% level of significance is 16.812. Since $\chi^2 < \chi 2_{0.01}$, It is accepted as H_0 and concluded that the service follows the Uniform distribution and it is suitable for the given time of interval.

Table-10: Tag Numbr Table For Service Distribution

S.No.	DAY	No of Patients	Probability	Cumulative probability	Tag number
1	MONDAY	18	0.13	0.13	0 – 12
2	TUESDAY	21	0.15	0.28	13 – 27
3	WEDNESDAY	19	0.14	0.42	28 – 41
4	THURSDAY	23	0.17	0.59	42 – 58
5	FRIDAY	20	0.14	0.73	59 – 72
6	SATURDAY	21	0.15	0.88	73 – 87
7	SUNDAY	17	0.12	1	88 – 100
8	TOTAL	139	-	-	-

Table-11: Simulation Table

S. No.	R.No (DAY)	DAY	R.No For Patients	Type of Patients	R.No for Arrival	Arrival	R.No For Service	Service	System capacity (25)	Queue
1	06	MON	92	EME	43	31	54	23	23	8
2	69	SAT	77	EME	78	29	32	19	19	10
3	85	SAT	81	EME	00	33	86	21	21	12
4	48	THU	13	IN	44	32	40	19	19	13
5	20	TUE	76	EME	79	29	08	18	18	11
6	29	WED	12	IN	72	27	24	21	21	6
7	60	FRI	02	IN	87	29	74	21	21	8
8	07	MON	47	EME	29	26	18	21	21	5
9	26	WED	24	IN	26	28	05	18	18	10
10	30	WED	20	IN	67	27	52	23	23	4
11	18	TUE	44	EME	80	29	54	23	23	6
12	51	THU	82	EME	02	33	32	19	19	14
13	29	WED	14	IN	84	29	86	21	21	8
14	37	WED	59	EME	68	37	40	19	19	18
15	89	SUN	11	IN	11	31	08	18	18	13
16	70	SAT	25	IN	05	31	24	21	21	10
17	04	MON	32	IN	91	25	74	21	21	4
18	96	SUN	39	IN	04	31	18	21	21	10
19	57	FRI	69	EME	75	29	05	18	18	11
20	16	TUE	72	EME	21	25	52	23	23	2
21	96	SUN	85	EME	60	37	99	17	17	20
22	77	SAT	43	EME	73	29	48	23	23	6
23	70	SAT	98	EME	80	29	05	18	18	11
24	21	TUE	17	IN	96	25	31	19	19	6

25	62	FRI	47	EME	32	26	81	21	21	5
26	79	SAT	59	EME	35	26	38	19	19	7
27	27	WED	16	IN	25	28	63	20	20	8
28	01	MON	86	EME	92	28	13	21	21	7
29	08	MON	64	EME	17	25	35	19	19	6
30	36	WED	75	EME	15	33	04	18	18	15
31	24	TUE	85	EME	70	37	71	20	20	17
32	84	SAT	82	EME	22	25	40	19	19	6
33	74	SAT	06	IN	08	31	75	20	20	11
34	36	WED	81	EME	33	26	45	23	23	3
35	71	SAT	71	EME	24	25	37	19	19	6
36	61	FRI	52	EME	43	31	47	23	23	8
37	16	TUE	04	IN	66	27	03	18	18	9
38	02	MON	80	EME	38	26	62	20	20	6
39	83	SAT	12	IN	25	28	91	17	17	11
40	83	SAT	69	EME	77	29	71	20	20	9
41	92	SUN	02	IN	44	32	07	18	18	14
42	77	SAT	26	IN	15	31	95	17	17	14
43	81	SAT	56	EME	90	28	03	18	18	10
44	13	TUE	18	IN	77	29	70	20	20	9
45	76	SAT	16	IN	01	31	87	21	21	10
46	12	MON	36	IN	84	29	32	19	19	10
47	02	MON	51	EME	67	37	82	21	21	16
48	47	THU	11	IN	49	32	37	19	19	13
49	27	TUE	62	EME	58	37	86	21	21	16
50	20	TUE	18	IN	89	25	92	17	17	8

1. R.No – RANDOM NUMBER

3. SIMULATION CALCULATION

Queue length of emergency patients – 5
Queue length of in patients – 4

4. ANALYTICAL CALCULATION (M/M/1) : (N/FCFS)

Average arrival rate (λ) – 0.03 and Average service rate (μ) – 0.05
Queue length of emergency patients – 2
Queue length of in patients – 4

5. NUMERICAL STUDY

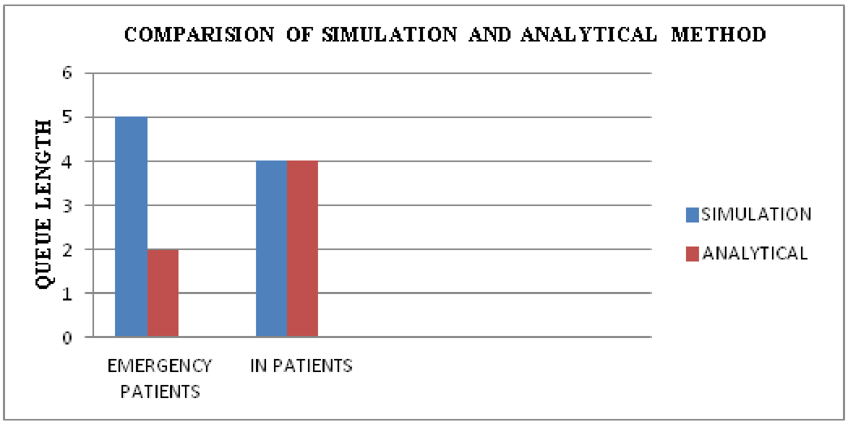

Figure-1

Figure 1: Bar Chart comparing the **Queue length** of emergency and in patients for both Simulation and Analytical Methods (M/M/1): (N/FCFS) model). In the chart, the first bar indicates the result through simulation method and the second bar indicates the result through Analytical method.

6. CONCLUSION

In both the cases inpatients and emergency patients, the Simulation result coincides with the Analytical result. Also the bar chart indicates the coincidence.

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