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GENERALIZATION OF SOFT $\tau_1\tau_2\tau_3$- α -NORMALITIES IN SOFT TRITOPOLOGICAL SPACES

DR. G. VASUKI[1], DR. B. AMUDHAMBIGAI[2] AND K. RATHIKA[3]

[1]Associate Professor, Department of Mathematics,
Sri Sarada College for Women (Autonomous), Salem-16, India.

[2]Assistant Professor, Department of Mathematics,
Sri Sarada College for Women (Autonomous), Salem-16, India.

[3]Research Scholar, Department of Mathematics,
Sri Sarada College for Women (Autonomous), Salem-16, India.

E-mail: vasukigj@gmail.com[1], rbamudha@yahoo.co.in[2] and ksrathima@gmail.com[3].

ABSTRACT

In this paper, the concepts of soft $\tau_1\tau_2\tau_3$- α-open sets and soft $\tau_1\tau_2\tau_3$- α-continuity in soft tritopological spaces are introduced. Also various generalization of soft $\tau_1\tau_2\tau_3$- α-normal spaces and their properties are investigated in soft tritopological spaces.

Keywords: *soft $\tau_1\tau_2\tau_3$- α-open sets, soft $\tau_1\tau_2\tau_3$- α-continuous functions, soft almost$\tau_1\tau_2\tau_3$- α-normal spaces, soft almost $\tau_1\tau_2\tau_3$- α- β-normal spaces, soft $\tau_1\tau_2\tau_3$- α- k-normal spaces and soft $\tau_1\tau_2\tau_3$- α- α-normal spaces.*

1. INTRODUCTION

The concept of α-open set was introduced by O. Njastad [10] in 1995. The study of tritopological space was first initiated by Martin M.Kovar [6]. S.Palanimmal [11] study of tritopological spaces. N.F.Hameed and Moh.Yahya Abid [5] gives the definition of 123 open set in tritopological space. The concept of soft topological space is introduced in[13]. Normality plays a prominent role in general topology and several gereralized notions of normality such as almost normal [14], k-normal [12, 15], almost β-normal[4]. In [1] A.V.Arhangels'skii and Ludwig introduced the concept of α-normal and β-$normal$ spaces and Eva. Murtinova in [9] provided an example of aβ-$normal$ Tychonoff space which is not normal. In this paper, we introduce soft $\tau_1\tau_2\tau_3$- α-open sets and soft $\tau_1\tau_2\tau_3$- α-continuity in soft tritopological spaces are introduced. Also various generalization of soft $\tau_1\tau_2\tau_3$- α-normal spaces and their properties are investigated in soft tritopological spaces.

2 PRELIMINARIES

In this section, the basic concepts about soft tri topological spaces are studied.

Definition 2.1: [8] Let X be an initial universe and E be a set of parameters. Let P(X) denotes the power set of X and A be a nonempty subset of E. A pair (F, A) is called a soft set over X, where F is a mapping given by F: A → P(X). In other words, a soft set over X is a parameterized family of subsets of the universe X. For e ∈A, F(e)may be considered as the set of e-approximate elements of the soft set (F, A). Clearly, a soft set is not a set.

Definition 2.2: [7] The complement of a soft set (F, A) over Xis denoted by $(F, A)^c$andis defined by $(F, A)^c = (F^c, A)$, where F^c: A → P(X) is a mapping given by $F^c(\alpha)$ =X− F(α) for all $\alpha \in$ A.

Definition 2.3: [3] Let (F, E) be a soft set over X. The soft set (F, E) is called a soft point, denoted by (x_e, E), if for the element e ∈ E, F(e) = x and F(e') = $\emptyset_E$ for all$e' \in E - \{e\}$(briefly, denoted by x_e).

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Definition 2.4: [13] Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X, if
(1) $\emptyset$, X belong to τ .
(2) The union of any number of soft sets in τ belongs to τ.
(3) The intersection of any two soft sets in τ belongs to τ.
Then triple (X, τ,E) is called a soft topological space over X.

Definition 2.5: [2] Let (X, τ_1, E), (X, τ_2,E) and (X, τ_3,E) be the three soft topological spaces on X. Then (X, τ_1, τ_2, τ_3,E) is called a soft tritopological space. The three soft topological spaces (X, τ_1,E), (X, τ_2, E) and (X, τ_3, E) are independently satisfy the axioms of a soft topological space.

The members of $\tau_1$are called soft open sets and the complement of τ_1 open sets are called soft closed sets. And the members of $\tau_2$are called soft open sets and the complement of τ_2 open sets are called soft closed sets. Similarly, the members of $\tau_3$are called soft open sets and the complement of τ_3 open sets are called soft closed sets.

Definition 2.6: [2] Let (X, τ_1, τ_2, τ_3, E)be a soft tritopological space and (F, E) is soft set in X, then (F,E) is called a soft $\tau_1\tau_2\tau_3$-open set if (F,E) = (A, E) ∪ (B, E) ∪ (C,E), where (A, E) ∈ τ_1 , (B, E) ∈ τ_2 and (C, E) ∈ τ_3. The complement of a soft$\tau_1\tau_2\tau_3$-open set is called a soft $\tau_1\tau_2\tau_3$-closed.

Definition 2.7: [2] Let (X, τ_1, τ_2, τ_3, E) be a soft tritopological space and (F, E) is a soft set in X, then the soft $\tau_1\tau_2\tau_3$-interior of (F, E), denoted by $s.\tau_1\tau_2\tau_3$-int (F, E) is defined by $s.\tau_1\tau_2\tau_3$-int (F, E) = ∪{(B, E): (B, E) ⊆(F, E) and (B, E) is soft $\tau_1\tau_2\tau_3$- open}.

Definition 2.8: [2] Let (X, τ_1, τ_2, τ_3, E) be a soft tritopological spaceand (F, E) is a soft set in X, then the soft $\tau_1\tau_2\tau_3$-closure of (F, E), denoted by $s.\tau_1\tau_2\tau_3$-cl (F, E) is defined by$s.\tau_1\tau_2\tau_3$-cl (F, E) = ∪{(C, E): (C, E) ⊇(F, E) and (C, E) is soft $\tau_1\tau_2\tau_3$-closed}.

Definition 2.9: [2] Let (X, τ_1, τ_2, τ_3, E) be a soft tritopological space and (F, E) is a soft set in X, then (F, E) is called soft $\tau_1\tau_2\tau_3$- α-open set(or soft tri- α-open)if (F, E) ⊆s. $\tau_1\tau_2\tau_3$-int (s. $\tau_1\tau_2\tau_3$-cl (s. $\tau_1\tau_2\tau_3$-int(F, E))).The complement of a soft $\tau_1\tau_2\tau_3$- α-open set is called a soft $\tau_1\tau_2\tau_3$- α-closed set.

3. Soft $\tau_1\tau_2\tau_3$- α-normalities

In this section, we introduce the concepts of soft $\tau_1\tau_2\tau_3$- α-continuous functions, soft $\tau_1\tau_2\tau_3$- α- regular open sets, soft $\tau_1\tau_2\tau_3$- α- k-normal spaces, soft almost $\tau_1\tau_2\tau_3$- α-normal spaces, soft $\tau_1\tau_2\tau_3$- α- α-normal spaces and soft almost $\tau_1\tau_2\tau_3$- α- β-normal spaces in soft tritopological spaces are introduced and study of some their properties. Also the characterization of soft almost $\tau_1\tau_2\tau_3$- α- β-normal space established.

Definition 3.1: Let (X, τ_1, τ_2, τ_3, E) be a soft tritopological space. For any soft set (A, E) over X, the **soft $\tau_1\tau_2\tau_3$- α-interior of (A, E)**(briefly, s $\tau_1\tau_2\tau_3$- α-int) is defined as follows s $\tau_1\tau_2\tau_3$- α-int (A, E) = ∪{(F, E): (F, E) ⊆ (A, E) and (F, E) is a soft $\tau_1\tau_2\tau_3$- α-open set over X}.

Definition 3.2: Let (X, τ_1, τ_2, τ_3, E) be a soft tritopological space. For any soft set(A, E) over X, the **soft $\tau_1\tau_2\tau_3$- α-closure of (A, E)**(briefly, s $\tau_1\tau_2\tau_3$- α-cl) is defined as follows s $\tau_1\tau_2\tau_3$- α-cl (A, E) = ∩{(F, E): (F, E) ⊇ (A, E) and (F, E) is a soft $\tau_1\tau_2\tau_3$- α-closed set over X}.

Definition 3.3: Let (X, $\tau_1, \tau_2, \tau_3, E_1$) and (Y, $\sigma_1, \sigma_2, \sigma_3, E_2$) be two soft tritopological spaces. Any function f : (X, $\tau_1, \tau_2, \tau_3, E_1$) →(Y, $\sigma_1, \sigma_2, \sigma_3, E_2$) is called **soft$\tau_1\tau_2\tau_3$-continuous** if f^{-1}(U, E_2) is soft $\tau_1\tau_2\tau_3$-open set in (X, $\tau_1, \tau_2, \tau_3, E_1$) for each soft $\tau_1\tau_2\tau_3$-open set (U, E_2) in (Y, $\sigma_1, \sigma_2, \sigma_3, E_2$).

Definition 3.4: Let (X, $\tau_1, \tau_2, \tau_3, E_1$) and (Y, $\sigma_1, \sigma_2, \sigma_3, E_2$) be two soft tritopological space. Any function f: (X, $\tau_1, \tau_2, \tau_3, E_1$) →(Y, $\sigma_1, \sigma_2, \sigma_3, E_2$) is called **soft$\tau_1\tau_2\tau_3$- α-continuous** if f^{-1}(U, E_2) is soft $\tau_1\tau_2\tau_3$- α-open set in (X, $\tau_1, \tau_2, \tau_3, E_1$) for each soft $\tau_1\tau_2\tau_3$-open set (U,E_2) in (Y, $\sigma_1, \sigma_2, \sigma_3, E_2$).

Definition 3.5: Let (X, $\tau_1, \tau_2, \tau_3, E_1$) and (Y, $\sigma_1, \sigma_2, \sigma_3, E_2$) be two soft tritopological spaces. Any function f : (X, $\tau_1, \tau_2, \tau_3, E_1$) →(Y, $\sigma_1, \sigma_2, \sigma_3, E_2$) is called **soft$\tau_1\tau_2\tau_3$- α-open (soft$\tau_1\tau_2\tau_3$- α-closed)** if f (U,E_1) is soft$\sigma_1\sigma_2\sigma_3$- α-open set (soft $\tau_1\tau_2\tau_3$- α-closed set) in (Y, $\sigma_1, \sigma_2, \sigma_3, E_2$) for each soft $\tau_1\tau_2\tau_3$- α-open set (soft $\tau_1\tau_2\tau_3$- α-closed set) (U, E_1) in (X, $\tau_1, \tau_2, \tau_3, E_1$).

Definition 3.6: A soft tritopological space (X, τ_1, τ_2, τ_3,E) is said to be a soft**$\tau_1\tau_2\tau_3$- $\alpha_{1/2}$space** if every soft $\tau_1\tau_2\tau_3$- α-open set in (X, τ_1, τ_2, τ_3, E) is $\tau_1\tau_2\tau_3$-open set.

Definition 3.7: Let (X, τ_1, τ_2, τ_3, E) be a soft tritopological space. Any soft set (A, E) over X is said to be **soft $\tau_1\tau_2\tau_3$- α-regular open** if (A, E) = $s\ \tau_1\tau_2\tau_3$- α-int($s\ \tau_1\tau_2\tau_3$- α- cl(A, E)). The complement of a soft $\tau_1\tau_2\tau_3$- α-regular open set is called a soft $\tau_1\tau_2\tau_3$- α-regular closed.

Definition 3.8: A soft tritopological space(X, τ_1, τ_2, τ_3, E) is said to be **soft$\tau_1\tau_2\tau_3$- α- k-normal** if for every pair of disjoint soft regular$\tau_1\tau_2\tau_3$- α-closed sets (A, E) and (B, E) over X, there exist disjoint soft $\tau_1\tau_2\tau_3$- α-open sets (U,E) and (V, E) over X such that (A,E) ⊆ (U, E) and (B,E) ⊆ (V, E).

Definition 3.9: A soft tritopological space (X, τ_1, τ_2, τ_3, E) is said to be **soft almost $\tau_1\tau_2\tau_3$- α- normal** if for every pair of disjoint soft $\tau_1\tau_2\tau_3$- α-closed sets (A, E) and (B, E) over X, one of which is soft $\tau_1\tau_2\tau_3$- α-regular closed, there exist disjoint soft $\tau_1\tau_2\tau_3$- α-open sets (U, E) and (V, E) over X such that (A, E) ⊆ (U, E) and (B, E) ⊆ (V, E).

Definition 3.10: A soft tritopological space (X, τ_1, τ_2, τ_3,E) is said to be**soft $\tau_1\tau_2\tau_3$- α- α- normal** if for any two disjoint soft $\tau_1\tau_2\tau_3$- α-closed sets (A, E) and (B,E)over X, there exist disjoint soft $\tau_1\tau_2\tau_3$- α-open sets (U,E)and (V, E) over X such that $s\tau_1\tau_2\tau_3$- α- $cl\big((A,E) \cap (U,E)\big)$ = (A,E) and $s\ \tau_1\tau_2\tau_3$- α- $cl\big((B,E) \cap (U,E)\big) = (B,E)$.

Definition 3.11: A soft tritopological space (X, τ_1, τ_2, τ_3, E) is said to be **soft almost $\tau_1\tau_2\tau_3$- α- β- normal** if for every pair of disjoint soft $\tau_1\tau_2\tau_3$- α-closed sets (A, E) and (B, E) over X , one of which is soft $\tau_1\tau_2\tau_3$- α-regular closed, there exist disjoint soft $\tau_1\tau_2\tau_3$- α-open sets (U, E) and (V, E) over X such that $s\ \tau_1\tau_2\tau_3$- α- $cl((A,E) \cap (U,E))$ = (A,E), $s\ \tau_1\tau_2\tau_3$- α- $cl((B,E) \cap (U,E))$ =(B,E) and $s\ \tau_1\tau_2\tau_3$- α- $cl(U,E) \cap s\ \tau_1\tau_2\tau_3$- α- $cl(V,E) = \emptyset_E$.

Definition 3.12: A soft tritopological space (X, τ_1, τ_2, τ_3, E) is said to be soft**semi- $\tau_1\tau_2\tau_3$- α-normal**if for every soft $\tau_1\tau_2\tau_3$- α-closed (A,E) over X contained in soft $\tau_1\tau_2\tau_3$- α-open set (U, E) over X, there exists a soft$\tau_1\tau_2\tau_3$- α-regular open set (V, E)over X such that (A,E) ⊂ $(V,E) \subset (U,E)$.

Definition 3.13: Any soft tritopological space (X, τ_1, τ_2, τ_3, E) is called a **soft $\tau_1\tau_2\tau_3$- Hausdorff space** if for any two distinct soft points x_e , y_e ∈ (X, E)for all e ∈ E,there exist soft $\tau_1\tau_2\tau_3$-open sets (U, E) and (V, E) over X such that x_e ∈(U, E), y_e ∈(V, E) and (U, E) ∩ (V, E) = $\emptyset_E$.

Definition 3.14: A soft $\tau_1\tau_2\tau_3$-Hausdorff space (X,τ_1, τ_2, τ_3, E) is said to be **soft extremally$\tau_1\tau_2\tau_3$- α-disconnected** if the soft $\tau_1\tau_2\tau_3$- α-closure of every $\tau_1\tau_2\tau_3$- α-open set in (X,τ_1, τ_2, τ_3, E) is $s\ \tau_1\tau_2\tau_3$- α-open.

Proposition 3.1: Every soft almost$\tau_1\tau_2\tau_3$- α-normal space is softalmost $\tau_1\tau_2\tau_3$- α- β-normal.

Proof: Let (X, τ_1, τ_2, τ_3, E) be a soft almost$\tau_1\tau_2\tau_3$- α-normal space. Let (A, E)and (B,E) be two disjoint soft $\tau_1\tau_2\tau_3$- α-closed sets in (X, τ_1, τ_2, τ_3, E) one of which (say (A, E)) is soft $\tau_1\tau_2\tau_3$- α-regular closed. Since (X, τ_1, τ_2, τ_3, E) is soft almost $\tau_1\tau_2\tau_3$- α-normal, there exist disjoint soft $\tau_1\tau_2\tau_3$- α-open sets (W, E) and (V, E) over X containing (A,E) and (B, E)respectively. Since (W, E) ∩ (V, E) =$\emptyset_E$,(W,E) ∩ ($s\ \tau_1\tau_2\tau_3$- α- cl(V,E)) = $\emptyset_E$. Let(U,E) = $s\ \tau_1\tau_2\tau_3$- α- int(A, E).Then$s\tau_1\tau_2\tau_3$- α- cl((A,E) ∩ (U,E)) = (A, E), $s\ \tau_1\tau_2\tau_3$- α - $cl((B,E) \cap$ (V, E))=(B, E) and $s\tau_1\tau_2\tau_3$- α- $cl(U,E) \cap s\tau_1\tau_2\tau_3$- α- $cl(V,E) = \emptyset_E$. So,(X, τ_1, τ_2, τ_3, E) is soft almost$\tau_1\tau_2\tau_3$- α- β-normal.

Remark: The converse of Proposition 3.1 need not be true as shown in Example 3.1.s

Example 3.1: Let X ={a, b, c, d} and E ={e}. Let τ_1 ={$\emptyset_E$, X , (F_1,E), (F_2,E), (F_3,E)}, τ_2 ={$\emptyset_E$, X, (F_4,E), (F_5,E)} and τ_3 ={$\emptyset_E$, X, (F_6,E),(F_7,E)}, where (F_1,E) ={(e, {b})}, (F_2,E) ={(e, {a, d})}, (F_3,E) ={(e,{a, b, d})} , (F_4,E) ={(e,{b, d})},(F_5,E) ={(e,{ b, c, d})},(F_6,E) ={(e,{b, c})}and (F_7,E) = {(e, {a, b, c})} are soft open sets over X. Clearly τ_1, $\tau_2$andτ_3 are soft topological spaces and so (X, τ_1, τ_2, τ_3,E) is soft tritopological space. The collection of soft $\tau_1\tau_2\tau_3$- α-open sets over X is {$\emptyset_E$, X, (F_1,E),(F_2,E), (F_3,E), (F_4,E), (F_5,E),(F_6,E), (F_7,E) }. Let (A, E) = {(e, {b, c})}, then (A, E) is$s\ \tau_1\tau_2\tau_3$- α- regulary closed set. For the soft $\tau_1\tau_2\tau_3$- α-closed set (B, E) ={(e, {d})}, there exist disjoint soft $\tau_1\tau_2\tau_3$- α-open sets (U, E) = $\{(e,\{b\})\}$and (V, E) ={(e, {a, d})}. Hence, (X, τ_1, τ_2, τ_3, E) is soft almost$\tau_1\tau_2\tau_3$- α- β-normal space but soft almost$\tau_1\tau_2\tau_3$- α-normal space.

Proposition 3.2: Every soft tritopological space (X, τ_1, τ_2, τ_3, E) which is both soft extremally $\tau_1\tau_2\tau_3$- α-disconnected and soft almost $\tau_1\tau_2\tau_3$- α- β-normal is soft almost$\tau_1\tau_2\tau_3$- α-normal.

Proof: Let (X, τ_1, τ_2, τ_3, E) be an soft extremally$\tau_1\tau_2\tau_3$- α-disconnected andsoft almost$\tau_1\tau_2\tau_3$- α- β-normal space. Let (A,E) be a soft $\tau_1\tau_2\tau_3$- α-regular closed set over X which is disjoint from the soft$\tau_1\tau_2\tau_3$- α-closed set (B, E) over X. By soft almost$\tau_1\tau_2\tau_3$- α- β-normalityof (X,τ_1, τ_2, τ_3, E), there exist disjoint soft $\tau_1\tau_2\tau_3$- α-open sets (U, E) and (V, E) over X such that $\mathrm{s}\ \tau_1\tau_2\tau_3$- α- $\mathrm{cl}\big((\mathrm{A,E}) \cap (\mathrm{U,E})\big) = (\mathrm{A,E})$, $s\ \tau_1\tau_2\tau_3$- α - cl(B, E) ∩ (V,E))=(B, E) and $s\ \tau_1\tau_2\tau_3$- α- $cl(U,E) \cap s\tau_1\tau_2\tau_3$- α- $cl(V,E) = \emptyset_E$. Thus (A,E) ⊂ $s\ \tau_1\tau_2\tau_3$- α- cl(U,E) and (B,E)⊂ $s\tau_1\tau_2\tau_3$- α- $cl(V,E)$. By the soft extremally $\tau_1\tau_2\tau_3$- α-disconnectedness of (X, τ_1, τ_2, τ_3, E), $s\tau_1\tau_2\tau_3$- α- $cl(U,E)$and $s\tau_1\tau_2\tau_3$- α- $cl(V,E)$ are disjoint soft $\tau_1\tau_2\tau_3$-α-open sets containing (A, E) and (B,E) respectively. Hence, (X, τ_1, τ_2, τ_3, E) is soft almost$\tau_1\tau_2\tau_3$-α-normal.

Proposition 3.3: For any soft tritopological space (X, τ_1, τ_2, τ_3, E), the following statements are equivalent:

1. (X, τ_1, τ_2, τ_3, E) is soft almost$\tau_1\tau_2\tau_3$-α-β-normal.
2. Whenever (F, E), (G, E) are disjoint soft$\tau_1\tau_2\tau_3$-α-closed sets over X and (F, E)is soft $\tau_1\tau_2\tau_3$-α-regular closed, there is a soft$\tau_1\tau_2\tau_3$-α-open set (V, E)over X such that (G,E) $= s\,\tau_1\tau_2\tau_3$-α-$cl((V,E) \cap (G,E))$ and (F, E) $\cap$ $s\,\tau_1\tau_2\tau_3$-α-cl(V, E) $= \emptyset_E$.
3. Whenever (F, E) is soft$\tau_1\tau_2\tau_3$-α-closed over X, (U, E) is a soft$\tau_1\tau_2\tau_3$-α-regularopen set over X and (F, E) $\subseteq (U,E)$, there is asoft$\tau_1\tau_2\tau_3$-α-open set (V, E) over X such that (F, E) $= s\,\tau_1\tau_2\tau_3$-α-$cl((V,E) \cap (F,E)) \subseteq s\tau_1\tau_2\tau_3$-$\alpha$-$cl(V,E) \subseteq$ (U, E).

Proof:

(1)⇒(2): Suppose that (F, E), (G, E) are disjoint soft$\tau_1\tau_2\tau_3$-α-closed setsover X and (F,E) is soft$\tau_1\tau_2\tau_3$-α-regular closed.Since (X, τ_1, τ_2, τ_3, E) is soft almost $\tau_1\tau_2\tau_3$-α-β-normal, there exist soft$\tau_1\tau_2\tau_3$-α-open sets (U, E) and (V, E) over X such that (F, E) $= s\tau_1\tau_2\tau_3$-α-$cl((U,E) \cap (F,E) \subseteq s\tau_1\tau_2\tau_3$-$\alpha$-$cl(U,E)$,
(G,E) $= s\tau_1\tau_2\tau_3$-α-$cl((G,E) \cap (V,E)) \subseteq s\tau_1\tau_2\tau_3$-$\alpha$-$cl(V,E)$ and
$s\tau_1\tau_2\tau_3$-α-$cl((U,E) \cap s\tau_1\tau_2\tau_3$-$\alpha$-$cl(V,E)) = \emptyset_E$. Then (F,E) $\cap\, s.\tau_1\tau_2\tau_3$-$\alpha$-$cl(V,E) = \emptyset_E$.

(2)⇒ (1): Suppose that(F, E), (G, E) are disjoint soft$\tau_1\tau_2\tau_3$-α-closed sets over X and (F,E) is soft $\tau_1\tau_2\tau_3$-α-regular closed. By the assumption, there exists a soft $\tau_1\tau_2\tau_3$-α-open set (V, E) over X such that (G, E) $= s\,\tau_1\tau_2\tau_3$-α-$cl((V,E) \cap (G,E))$ and (F, E) $\cap\, s\tau_1\tau_2\tau_3$-$\alpha$-$cl(V,E) = \emptyset_E$. Let (U, E) = soft $\tau_1\tau_2\tau_3$-α-int(F,E). Then (F, E) $= s\,\tau_1\tau_2\tau_3$-α-$cl((U,E) \cap (F,E))$ and $s\tau_1\tau_2\tau_3$-α-$cl(U,E) \cap s\,\tau_1\tau_2\tau_3$-$\alpha$-$cl(V,E) =$ (F, E) $\cap$ $s\,\tau_1\tau_2\tau_3$-α-$cl(V,E) = \emptyset_E$.

(1) ⇒(3): Suppose that (F, E) is soft $\tau_1\tau_2\tau_3$-α-closed and (U, E) is soft $\tau_1\tau_2\tau_3$-α-regular open over X and (F, E) $\subseteq$ (U, E). Since (U, E) is soft $\tau_1\tau_2\tau_3$-α- regular open, $X_E -$ (U, E) is soft $\tau_1\tau_2\tau_3$-α- regular closed set over X.

Since (X, τ_1, τ_2, τ_3, E) is soft almost β-$\tau_1\tau_2\tau_3$-α-normal, there are soft $\tau_1\tau_2\tau_3$-α-open sets(O,E) and (V, E) over X such that $X_E -$ (U, E) $= s\tau_1\tau_2\tau_3$-α-$cl((O,E) \cap (X_E - $ (U, E))) $\subseteq s\tau_1\tau_2\tau_3$-$\alpha$-$cl(O,E)$,
(F, E) $= s\tau_1\tau_2\tau_3$-α-$cl((V,E) \cap (F,E)) \subseteq s\tau_1\tau_2\tau_3$-$\alpha$-$cl(V,E)$and $s\tau_1\tau_2\tau_3$-α-$cl(O,E) \cap s\tau_1\tau_2\tau_3$-$\alpha$-$cl(V,E) = \emptyset_E$.Then($X_E -$(U, E))$\cap\, s\tau_1\tau_2\tau_3$-$\alpha$-$cl(V,E) = \emptyset_E$ which means that $s\tau_1\tau_2\tau_3$-α-$cl(V,E) \subseteq$ (U,E).

(3) ⇒(2): Suppose that(F, E), (G, E) are disjoint soft$\tau_1\tau_2\tau_3$-α-closed sets over X and (F, E) is soft $\tau_1\tau_2\tau_3$-α-regular closed. Then (G, E) $\subseteq X_E -$(F, E) and ($X_E -$ (F, E)) is soft $\tau_1\tau_2\tau_3$-α- regular open set. By the hypothesis, there is a soft$\tau_2\tau_3$-α-open set (V, E) over X such that (G,E) $= s\tau_1\tau_2\tau_3$-α-$cl\big((V,E) \cap (G,E)\big) \subseteq s\tau_1\tau_2\tau_3$-$\alpha$-$cl(V,E) \subseteq X_E -$(F,E). Then (F, E) $\cap\, s\tau_1\tau_2\tau_3$-$\alpha$-$cl(V,E) = \emptyset_E$.

Proposition 3.4: A soft tritopological space (X, τ_1, τ_2, τ_3, E) is soft almost$\tau_1\tau_2\tau_3$-α-normal if and only if it is both soft almost $\tau_1\tau_2\tau_3$-α-β-normal and soft $\tau_1\tau_2\tau_3$-α- k-normal.

Proof: Let (X, τ_1, τ_2, τ_3, E) be soft almost$\tau_1\tau_2\tau_3$-α-β-normal and soft$\tau_1\tau_2\tau_3$-α- k-normal. Let (A, E) and (B, E) be two disjoint soft $\tau_1\tau_2\tau_3$-α-closed sets over Xin which (A, E) is soft $\tau_1\tau_2\tau_3$-α-regular closed. By soft almost $\tau_1\tau_2\tau_3$-α-β-normality of (X, τ_1, τ_2, τ_3, E), there exist disjoint soft-$\tau_1\tau_2\tau_3$-α-open sets (U, E) and (V,E) over X such that $s\tau_1\tau_2\tau_3$-α-$cl((A,E) \cap (U,E)) =$ (A,E), $s\tau_1\tau_2\tau_3$-α-$cl\big((B,E) \cap (V,E)\big) =$ (B,E) and $s\tau_1\tau_2\tau_3$-α-$cl(U,E) \cap s\tau_1\tau_2\tau_3$-$\alpha$-$cl(V,E) = \emptyset_E$. Thus (A,E) $\subset s\tau_1\tau_2\tau_3$-$\alpha$-$cl(U,E)$ and (B, E)$\subset s\tau_1\tau_2\tau_3$-$\alpha$-$cl(U,E)$. Here $s\tau_1\tau_2\tau_3$-α-$cl(U,E)$ and $s\tau_1\tau_2\tau_3$-α-$cl(V,E)$ are disjoint soft $\tau_1\tau_2\tau_3$-α-regular closed sets over X. So by soft $\tau_1\tau_2\tau_3$-α- k-normality of (X, τ_1, τ_2, τ_3, E), there exist disjoint soft $\tau_1\tau_2\tau_3$-α-open sets (W_1, E) and (W_2, E) over X such that $s\tau_1\tau_2\tau_3$-α-$cl(U,E) \subseteq (W_1, E)$ and $s\tau_1\tau_2\tau_3$-α-$cl(U,E) \subseteq (W_2, E)$. Hence,(X, τ_1, τ_2, τ_3, E) is soft almost $\tau_1\tau_2\tau_3$-α-normal. Necessity of the statement can be proved by using Proposition 3.1 and by using similar proof for soft $\tau_1\tau_2\tau_3$-α- k-normal space.

Proposition 3.5: Every soft tritopological space (X, τ_1, τ_2, τ_3, E) which is bothsoft $\tau_1\tau_2\tau_3$-α-seminormal and soft almost$\tau_1\tau_2\tau_3$-α-β-normal is soft $\tau_1\tau_2\tau_3$-α-α-normal.

Proof: Let (X,τ_1, τ_2, τ_3, E) be a soft semi$\tau_1\tau_2\tau_3$-α-normal andsoft almostβ-$\tau_1\tau_2\tau_3$-α-normal. Let (A, E) and (B,E) be two disjoint soft$\tau_1\tau_2\tau_3$-α- closed sets over X. Thus (A, E) $\subset$(X_E – (B, E)). By soft $\tau_1\tau_2\tau_3$-α-seminormality of (X, $\tau_1, \tau_2, \tau_3, E$), there exists a soft$\tau_1\tau_2\tau_3$-$\alpha$- regular open set (F, E) over X such that (A, E) $\subset$(F,E) $\subset$(X_E –(B, E)). Now (A, E) and $X_E -$ (F, E) are disjoint soft$\tau_1\tau_2\tau_3$-α-closed sets over X in which $X_E -$ (F, E) is a soft$\tau_1\tau_2\tau_3$-α-regular closed set containing (B, E). Since (X, τ_1, τ_2, τ_3, E) issoft almost $\tau_1\tau_2\tau_3$-α-β-normal, there exist disjoint soft almost$\tau_1\tau_2\tau_3$-α-open sets (U, E) and (V, E) over X such that

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$s\,\tau_1\tau_2\tau_3$- α- cl((A,E) ∩ (U, E))= (A,E),$s\,\tau_1\tau_2\tau_3$- α- $cl((V,E)\cap(X_E-(F,E)))=(X_E-(F,E))$ and $s\tau_1\tau_2\tau_3$- α- $cl(U,E)\cap s\tau_1\tau_2\tau_3$- α- $cl(V,E)=\emptyset_E$. So (A, E) $= s\tau_1\tau_2\tau_3$- α- $cl\big((A,E)\cap(U,E)\big)\subset s\tau_1\tau_2\tau_3$- α- $cl(U,E)$ and$\big(X_E-$ (F, E)$\big)= s\tau_1\tau_2\tau_3$- α- $cl((X_E-$ (F, E)) ∩ (V, E))⊂ $s\,\tau_1\tau_2\tau_3$- α- $cl(V,E)$.Thus(U,E) and (W, E)= $X_E-\big(s\tau_1\tau_2\tau_3$- α- $cl(U,E)\big)$ are two disjoint soft$\tau_1\tau_2\tau_3$- α-open sets over X such that $s\tau_1\tau_2\tau_3$- α- $cl\big((A,E)\cap(U,E)\big)=$ (A, E) and (B, E) ⊂(W, E).Therefore, $s\tau_1\tau_2\tau_3$- α- $cl((W,E)\cap(B,E))=$ (B,E) and (X, τ_1,τ_2,τ_3, E) is soft$\tau_1\tau_2\tau_3$- α- α-normal space.

Proposition 3.6: Suppose that (X,τ_1,τ_2,τ_3, E_1) and (Y,$\sigma_1,\sigma_2,\sigma_3$, E_2) are any two soft tritopological spaces, (X,τ_1,τ_2,τ_3,E_1) is soft almost$\tau_1\tau_2\tau_3$- α- β-normal and$f$:(X,τ_1,τ_2,τ_3,E_1) → (Y,$\sigma_1,\sigma_2,\sigma_3$,$E_2$) is onto, soft $\tau_1\tau_2\tau_3$- α-continuous, soft $\tau_1\tau_2\tau_3$- α-open,soft $\tau_1\tau_2\tau_3$- α-closed and(Y,$\sigma_1,\sigma_2,\sigma_3$,, E_2) is soft$\sigma_1\sigma_2\sigma_3$- $\alpha_{1/2}$space.Then (Y,$\sigma_1,\sigma_2,\sigma_3$,E) is soft almost$\sigma_1\sigma_2\sigma_3$- α- β- normal.

Proof: Suppose that (F, E_2), (G, E_2)are disjoint soft $\sigma_1\sigma_2\sigma_3$- α-closed sets over Y and (F,E_2)is soft $\tau_1\tau_2\tau_3$- α-regular closed. Since (Y,τ_1,τ_2,τ_3,E_2) is soft$\sigma_1\sigma_2\sigma_3$- $\alpha_{1/2}$space, (F, E_2), (G, E_2) are soft $\sigma_1\sigma_2\sigma_3$-closed sets over Y. Since f is soft$\tau_1\tau_2\tau_3$- α-continuous, f^{-1}(F, E_2) and f^{-1}(G, E_2) are disjoint soft $\tau_1\tau_2\tau_3$- α-closed sets over X. Clearly $f^{-1}(F,E_2)= s\,\sigma_1\sigma_2\sigma_3$- α- cl $(\big(f^{-1}\big($s $\sigma_1\sigma_2\sigma_3$- α- int(F, E_2)$\big)\big)$. Suppose that (W,E_1) is soft $\tau_1\tau_2\tau_3$- α-open over X such that (W, E_1) ∩ f^{-1}(F, E_2) $\neq\emptyset_E$. Then$f(W,E_1)$ is soft $\sigma_1\sigma_2\sigma_3$- α-open over Y and $f(W,E_1)\cap$ (F,E_2) $= f(W,E_1)\cap s\,\sigma_1\sigma_2\sigma_3$- α- $cl\big($s $\sigma_1\sigma_2\sigma_3$- int (F, E_2)$\big)\neq\emptyset_E$ which implies that $f(W,E_1)$ ∩($s\,\sigma_1\sigma_2\sigma_3$- α- int(F, E_2)) $\neq\emptyset_E$. Hence (W, E_1) ∩ f^{-1}(s $\sigma_1\sigma_2\sigma_3$- α- int(F,E_2))$\neq\emptyset_E$ and so f^{-1}(F,E_2) $= s\,\sigma_1\sigma_2\sigma_3$- α- $cl(f^{-1}\big(s\,\sigma_1\sigma_2\sigma_3$- α- int(F, E_2)$\big)$).Therefore, f^{-1}(F, E_2) is soft $\tau_1\tau_2\tau_3$- α-regular closed set. So there exist a$s\tau_1\tau_2\tau_3$- α-open set (U, E_1)over X such that f^{-1}(G,E_2) $= s\tau_1\tau_2\tau_3$- α- $cl(f^{-1}$(G, E_2) ∩ (U, E_1)) and $s\tau_1\tau_2\tau_3$- α- cl(U, E_1) ∩ f^{-1}(F,E_2) $=\emptyset_E$, $f(s\tau_1\tau_2\tau_3$- α- cl(U, E_1) ∩ (F, E_2)) $=\emptyset_E$. Also, note that $f(U,E_1)$ is soft $\sigma_1\sigma_2\sigma_3$- α-open and $f(s\tau_1\tau_2\tau_3$- α- cl(U, E_1)) is soft $\sigma_1\sigma_2\sigma_3$- α-closed. Since $f(s\tau_1\tau_2\tau_3$- α- cl(U, E_1) is soft $\sigma_1\sigma_2\sigma_3$- α-closed set containing $f(U,E_1)$, $s\tau_1\tau_2\tau_3$- α- $cl\big(f(U,E_1)\big)\subseteq f(s\tau_1\tau_2\tau_3$- α- cl(U, E_1)).
So $s\sigma_1\sigma_2\sigma_3$- α- $cl(f(U,E_1)\cap$ (F,E_2)) $=\emptyset_E$. It remains to show that (G, E_2) $= s\sigma_1\sigma_2\sigma_3$- α- $cl((G,E_2)\cap f(G,E_2))$. Let y_e ∈(G, E_2) for e ∈ E and (O, E_2) be soft $\tau_1\tau_2\tau_3$- α-open set over Ycontainingy_e.Then $f^{-1}(y)\subseteq[\,f^{-1}(G,E_2)\cap f^{-1}(O,E_2)]$.

Since $f^{-1}(G,E_2)= s\tau_1\tau_2\tau_3$- α- $cl(f^{-1}$(G, E_2) ∩ (U, E_1)), $f^{-1}(G,E_2)\cap(U,E_1)\cap f^{-1}(O,E_2)\neq\emptyset_E$. Hence, (G, E_2) ∩ $f(U,E_1)\cap(O,E_2)= f\big(f^{-1}(G,E_2)\big)\cap f$(U, E_1) ∩ $f\big(f^{-1}(O,E_2)\big)$⊇$f[f^{-1}(G,E_2)\cap$ (U, E_1) ∩ f^{-1}(O, E_2)]$\neq\emptyset_E$, as desired.

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