GENERALIZATION
OF SOFT $\tau_1\tau_2\tau_3\cdot\alpha$ -NORMALITIES IN SOFT TRITOPOLITICAL SPACES

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ABSTRACT

In this paper, the concepts of soft $\tau_1\tau_2\tau_3\cdot\alpha$-open sets and soft $\tau_1\tau_2\tau_3\cdot\alpha$-continuity in soft tritopological spaces are introduced. Also various generalization of soft $\tau_1\tau_2\tau_3\cdot\alpha$-normal spaces and their properties are investigated in soft tritopological spaces.

Keywords: soft $\tau_1\tau_2\tau_3\cdot\alpha$-open sets, soft $\tau_1\tau_2\tau_3\cdot\alpha$-continuous functions, soft almost $\tau_1\tau_2\tau_3\cdot\alpha$-normal spaces, soft almost $\tau_1\tau_2\tau_3\cdot\alpha$-$\beta$-normal spaces, soft $\tau_1\tau_2\tau_3\cdot\alpha$-$\kappa$-normal spaces and soft $\tau_1\tau_2\tau_3\cdot\alpha$-$\alpha$-normal spaces.

1. INTRODUCTION

The concept of $\alpha$-open set was introduced by O. Njastad [10] in 1995. The study of tritopological space was first initiated by Martin M.Kovar [6]. S.Palanimmal [11] study of tritopological spaces. N.F.Hameed and Moh.Yahya Abid [5] gives the definition of $\tau_1\tau_2\tau_3$ open set in tritopological space. The concept of soft topological space is introduced in [13]. Normality plays a prominent role in general topology and several generalized notions of normality such as almost normal [14], k-normal [12, 15], almost $\beta$-normal[4]. In [1] A.V.Arhangel’skii and Ludwig introduced the concept of $\alpha$-normal and $\beta$-normal spaces and Eva. Murtinova in [9] provided an example of a $\beta$-normal Tychonoff space which is not normal. In this paper, we introduce soft $\tau_1\tau_2\tau_3\cdot\alpha$-open sets and soft $\tau_1\tau_2\tau_3\cdot\alpha$-continuity in soft tritopological spaces are introduced. Also various generalization of soft $\tau_1\tau_2\tau_3\cdot\alpha$-normal spaces and their properties are investigated in soft tritopological spaces.

2 PRELIMINARIES

In this section, the basic concepts about soft tri topological spaces are studied.

Definition 2.1: [8] Let $X$ be an initial universe and $E$ be a set of parameters. Let $P(X)$ denotes the power set of $X$ and $A$ be a nonempty subset of $E$. A pair $(F, A)$ is called a soft set over $X$, where $F$ is a mapping given by $F: A \rightarrow P(X)$. In other words, a soft set over $X$ is a parameterized family of subsets of the universe $X$. For $e \in A$, $F(e)$ may be considered as the set of $e$-approximate elements of the soft set $(F, A)$. Clearly, a soft set is not a set.

Definition 2.2: [7] The complement of a soft set $(F, A)$ over $X$ is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$, where $F^c: A \rightarrow P(X)$ is a mapping given by $F^c(\alpha) = \emptyset$ for all $\alpha \in A$.

Definition 2.3: [3] Let $(F, E)$ be a soft set over $X$. The soft set $(F, E)$ is called a soft point, denoted by $(x_e, E)$, if for the element $e \in E$, $F(e) = x$ and $F(e') = \emptyset$ for all $e' \in E - \{e\}$ (briefly, denoted by $x_e$).
Definition 2.4: [13] Let $\tau$ be the collection of soft sets over $X$, then $\tau$ is said to be a soft topology on $X$, if

1. $\emptyset \in \tau$.
2. The union of any number of soft sets in $\tau$ belongs to $\tau$.
3. The intersection of any two soft sets in $\tau$ belongs to $\tau$.

Then triple $(X, \tau, E)$ is called a soft topological space over $X$.

Definition 2.5: [2] Let $(X_1, \tau_1, E_1), (X_2, \tau_2, E_2)$ and $(X_3, \tau_3, E_3)$ be the three soft topological spaces on $X$. Then $(X_1, \tau_1, \tau_2, \tau_3, E_3)$ is called a soft tritopological space. The three soft topological spaces $(X_1, \tau_1, E_1), (X_2, \tau_2, E_2)$ and $(X_3, \tau_3, E_3)$ are independently satisfy the axioms of a soft topological space.

The members of $\tau_1$ are called soft open sets and the complement of $\tau_1$ open sets are called soft closed sets. And the members of $\tau_2$ are called soft open sets and the complement of $\tau_2$ open sets are called soft closed sets. Similarly, the members of $\tau_3$ are called soft open sets and the complement of $\tau_3$ open sets are called soft closed sets.

Definition 2.6: [2] Let $(X_1, \tau_1, \tau_2, \tau_3, E_3)$ be a soft tritopological space and $(F, E)$ is soft set in $X$, then $(F, E)$ is called a soft $\tau_1\tau_2\tau_3$-open set if $(F, E) \subseteq (A, E) \cup (B, E)$, where $(A, E) \in \tau_1$ and $(B, E) \in \tau_2$ and $(C, E) \in \tau_3$. The complement of a soft $\tau_1\tau_2\tau_3$-open set is called $\tau_1\tau_2\tau_3$-closed.

Definition 2.7: [2] Let $(X_1, \tau_1, \tau_2, \tau_3, E_3)$ be a soft tritopological space and $(F, E)$ is a soft set in $X$, then the soft $\tau_1\tau_2\tau_3$-interior of $(F, E)$, denoted by $\tau_1\tau_2\tau_3$-int $(F, E)$ is defined by $\tau_1\tau_2\tau_3$-int $(F, E) = \{ (B, E) : (B, E) \subseteq (F, E)$ and $(B, E)$ is soft $\tau_1\tau_2\tau_3$-open set).

Definition 2.8: [2] Let $(X_1, \tau_1, \tau_2, \tau_3, E_3)$ be a soft tritopological space and $(F, E)$ is a soft set in $X$, then the soft $\tau_1\tau_2\tau_3$-closure of $(F, E)$, denoted by $\tau_1\tau_2\tau_3$-cl $(F, E)$ is defined by $\tau_1\tau_2\tau_3$-cl $(F, E) = \{ (C, E) : (C, E) \supseteq (F, E)$ and $(C, E)$ is soft $\tau_1\tau_2\tau_3$-closed set).

Definition 2.9: [2] Let $(X_1, \tau_1, \tau_2, \tau_3, E_3)$ be a soft tritopological space and $(F, E)$ is a soft set in $X$, then $(F, E)$ is called soft $\tau_1\tau_2\tau_3$-open set (or soft tri-$\alpha$-open) if $(F, E) \subseteq \tau_1\tau_2\tau_3$-int $(s\tau_1\tau_2\tau_3$-cl $(s\tau_1\tau_2\tau_3$-int $(F, E))$. The complement of a soft $\tau_1\tau_2\tau_3$-open set is called a soft $\tau_1\tau_2\tau_3$-closed set.

3. Soft $\tau_1\tau_2\tau_3$-$\alpha$-normalities

In this section, we introduce the concepts of soft $\tau_1\tau_2\tau_3$-$\alpha$-continuous functions, soft $\tau_1\tau_2\tau_3$-$\alpha$-regular open sets, soft $\tau_1\tau_2\tau_3$-$\alpha$-$k$-normal spaces, soft almost $\tau_1\tau_2\tau_3$-$\alpha$-$\alpha$-normal spaces, soft $\tau_1\tau_2\tau_3$-$\alpha$-$\alpha$-normal spaces and soft almost $\tau_1\tau_2\tau_3$-$\alpha$-$\beta$-normal spaces in soft tritopological spaces are introduced and study of some their properties. Also the characterization of soft almost $\tau_1\tau_2\tau_3$-$\alpha$-$\alpha$-$\beta$-normal space established.

Definition 3.1: Let $(X_1, \tau_1, \tau_2, \tau_3, E_3)$ be a soft tritopological space. For any soft set $(A, E)$ over $X$, the soft $\tau_1\tau_2\tau_3$-$\alpha$-interior of $(A, E)$ (briefly, $\tau_1\tau_2\tau_3$-$\alpha$-int) is defined as follows $\tau_1\tau_2\tau_3$-$\alpha$-int $(A, E) = \cup \{ (F, E) : (F, E) \subseteq (A, E) \}$. And $(F, E)$ is a soft $\tau_1\tau_2\tau_3$-$\alpha$-open set over $X$.

Definition 3.2: Let $(X_1, \tau_1, \tau_2, \tau_3, E_3)$ be a soft tritopological space. For any soft set $(A, E)$ over $X$, the soft $\tau_1\tau_2\tau_3$-$\alpha$-closure of $(A, E)$ (briefly, $\tau_1\tau_2\tau_3$-$\alpha$-cl) is defined as follows $\tau_1\tau_2\tau_3$-$\alpha$-cl $(A, E) = \cap \{ (F, E) : (F, E) \supseteq (A, E) \}$. And $(F, E)$ is a soft $\tau_1\tau_2\tau_3$-$\alpha$-closed set over $X$.

Definition 3.3: Let $(X_1, \tau_1, \tau_2, \tau_3, E_3)$ and $(Y, \sigma_1, \sigma_2, \sigma_3, E_2)$ be two soft tritopological spaces. Any function $f : (X_1, \tau_1, \tau_2, \tau_3, E_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3, E_2)$ is called soft $\tau_1\tau_2\tau_3$-$\alpha$-continuous if $f^{-1}(U, E_2)$ is soft $\tau_1\tau_2\tau_3$-open set in $(X_1, \tau_1, \tau_2, \tau_3, E_3)$ for each soft $\tau_1\tau_2\tau_3$-open set $(U, E_2)$ in $(Y, \sigma_1, \sigma_2, \sigma_3, E_2)$.

Definition 3.4: Let $(X_1, \tau_1, \tau_2, \tau_3, E_3)$ and $(Y, \sigma_1, \sigma_2, \sigma_3, E_2)$ be two soft tritopological spaces. Any function $f : (X_1, \tau_1, \tau_2, \tau_3, E_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3, E_2)$ is called soft $\tau_1\tau_2\tau_3$-$\alpha$-continuous if $f^{-1}(U, E_2)$ is soft $\tau_1\tau_2\tau_3$-$\alpha$-open set in $(X_1, \tau_1, \tau_2, \tau_3, E_3)$ and each soft $\tau_1\tau_2\tau_3$-open set $(U, E_2)$ in $(Y, \sigma_1, \sigma_2, \sigma_3, E_2)$.

Definition 3.5: Let $(X_1, \tau_1, \tau_2, \tau_3, E_3)$ and $(Y, \sigma_1, \sigma_2, \sigma_3, E_2)$ be two soft tritopological spaces. Any function $f : (X_1, \tau_1, \tau_2, \tau_3, E_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3, E_2)$ is called soft $\tau_1\tau_2\tau_3$-$\alpha$-open (soft $\tau_1\tau_2\tau_3$-$\alpha$-closed) if $f(U, E_1)$ is soft $\tau_1\tau_2\tau_3$-$\alpha$-open (soft $\tau_1\tau_2\tau_3$-$\alpha$-closed) set in $(Y, \sigma_1, \sigma_2, \sigma_3, E_2)$ for each soft $\tau_1\tau_2\tau_3$-$\alpha$-open set (soft $\tau_1\tau_2\tau_3$-$\alpha$-closed set) $(U, E_1)$ in $(X_1, \tau_1, \tau_2, \tau_3, E_3)$.

Definition 3.6: Let $(X_1, \tau_1, \tau_2, \tau_3, E_3)$ is said to be a soft $\tau_1\tau_2\tau_3$-$\alpha_1/2$-space if every soft $\tau_1\tau_2\tau_3$-$\alpha$-open set in $(X_1, \tau_1, \tau_2, \tau_3, E_3)$ is $\tau_1\tau_2\tau_3$-open set.
Definition 3.7: Let \((X, \tau_1, \tau_2, \tau_3, E)\) be a soft tritopological space. Any soft set \((A, E)\) over \(X\) is said to be soft \(\tau_1\tau_2\tau_3\alpha\)-regular open if \((A, E) = s \tau_1\tau_2\tau_3\alpha\)-int\((s \tau_1\tau_2\tau_3\alpha\)-cl\((A, E)\)) \(\in\) \(X\). The complement of a soft \(\tau_1\tau_2\tau_3\alpha\)-regular open set is called a soft \(\tau_1\tau_2\tau_3\alpha\)-regular closed.

Definition 3.8: A soft tritopological space \((X, \tau_1, \tau_2, \tau_3, E)\) is said to be soft almost \(\tau_1\tau_2\tau_3\alpha\)-k-normal if for every pair of disjoint soft regular \(\tau_1\tau_2\tau_3\alpha\)-closed sets \((A, E)\) and \((B, E)\) over \(X\), there exists disjoint soft \(\tau_1\tau_2\tau_3\alpha\)-open sets \((U, E)\) and \((V, E)\) over \(X\) such that \((A, E) \subseteq (U, E)\) and \((B, E) \subseteq (V, E)\).

Definition 3.9: A soft tritopological space \((X, \tau_1, \tau_2, \tau_3, E)\) is said to be soft almost \(\tau_1\tau_2\tau_3\alpha\)-normal if for every pair of disjoint soft \(\tau_1\tau_2\tau_3\alpha\)-closed sets \((A, E)\) and \((B, E)\) over \(X\), one of which is soft \(\tau_1\tau_2\tau_3\alpha\)-regular closed, there exist disjoint soft \(\tau_1\tau_2\tau_3\alpha\)-open sets \((U, E)\) and \((V, E)\) over \(X\) such that \((A, E) \subseteq (U, E)\) and \((B, E) \subseteq (V, E)\).

Definition 3.10: A soft tritopological space \((X, \tau_1, \tau_2, \tau_3, E)\) is said to be soft semi- \(\tau_1\tau_2\tau_3\alpha\)-normal if for any two disjoint soft \(\tau_1\tau_2\tau_3\alpha\)-closed sets \((A, E)\) and \((B, E)\) over \(X\), there exist disjoint soft \(\tau_1\tau_2\tau_3\alpha\)-open sets \((U, E)\) and \((V, E)\) over \(X\) such that \(s\tau_1\tau_2\tau_3\alpha\)-int\((A, E)\) \(\cap\) \((U, E)) = (A, E)\) and \(s\tau_1\tau_2\tau_3\alpha\)-cl\((B, E)\) \(\cap\) \((U, E)) = (B, E)\).

Definition 3.11: A soft tritopological space \((X, \tau_1, \tau_2, \tau_3, E)\) is said to be soft almost \(\tau_1\tau_2\tau_3\alpha\)-normal if for every pair of disjoint soft \(\tau_1\tau_2\tau_3\alpha\)-closed sets \((A, E)\) and \((B, E)\) over \(X\), one of which is soft \(\tau_1\tau_2\tau_3\alpha\)-regular closed, there exist disjoint soft \(\tau_1\tau_2\tau_3\alpha\)-open sets \((U, E)\) and \((V, E)\) over \(X\) such that \(s\tau_1\tau_2\tau_3\alpha\)-int\((A, E)\) \(\cap\) \((U, E)) = (A, E)\) and \(s\tau_1\tau_2\tau_3\alpha\)-cl\((B, E)\) \(\cap\) \((U, E)) = (B, E)\).

Definition 3.12: A soft tritopological space \((X, \tau_1, \tau_2, \tau_3, E)\) is said to be soft extremally \(\tau_1\tau_2\tau_3\alpha\)-disconnected if the soft \(\tau_1\tau_2\tau_3\alpha\)-closure of every \(\tau_1\tau_2\tau_3\alpha\)-open set in \((X, \tau_1, \tau_2, \tau_3, E)\) is soft almost \(\tau_1\tau_2\tau_3\alpha\)-open.

Proposition 3.1: Every soft almost \(\tau_1\tau_2\tau_3\alpha\)-normal space is soft almost \(\tau_1\tau_2\tau_3\alpha\)-normal.

Definition 3.13: Any soft tritopological space \((X, \tau_1, \tau_2, \tau_3, E)\) is called a soft \(\tau_1\tau_2\tau_3\alpha\)-Hausdorff space if for any two distinct soft points \(x_\alpha, y_\beta \in (X, E)\) for all \(\alpha, \beta \in E\), there exist soft \(\tau_1\tau_2\tau_3\alpha\)-open sets \((U, E)\) and \((V, E)\) over \(X\) such that \(x_\alpha \in (U, E)\) and \(y_\beta \in (V, E)\).

Definition 3.14: A soft \(\tau_1\tau_2\tau_3\alpha\)-Hausdorff space \((X, \tau_1, \tau_2, \tau_3, E)\) is said to be soft extremally \(\tau_1\tau_2\tau_3\alpha\)-disconnected if the soft \(\tau_1\tau_2\tau_3\alpha\)-closure of every \(\tau_1\tau_2\tau_3\alpha\)-open set in \((X, \tau_1, \tau_2, \tau_3, E)\) is soft almost \(\tau_1\tau_2\tau_3\alpha\)-open.

Proposition 3.2: Every soft tritopological space \((X, \tau_1, \tau_2, \tau_3, E)\) which is both soft extremally \(\tau_1\tau_2\tau_3\alpha\)-disconnected and soft almost \(\tau_1\tau_2\tau_3\alpha\)-normal is soft almost \(\tau_1\tau_2\tau_3\alpha\)-normal.
disjoint soft $\tau_1\tau_2\tau_3\alpha$-open sets containing (A, E) and (B, E) respectively. Hence, $(X, \tau_1, \tau_2, \tau_3)$ is soft almost $\tau_1\tau_2\tau_3\alpha$-normal.

**Proposition 3.3:** For any soft tritopological space $(X, \tau_1, \tau_2, \tau_3, E)$, the following statements are equivalent:

1. $(X, \tau_1, \tau_2, \tau_3, E)$ is soft almost $\tau_1\tau_2\tau_3\alpha$-normal.
2. Whenever $(F, E), (G, E)$ is soft disjoint $\tau_1\tau_2\tau_3\alpha$-closed sets over $X$ and $(F, E)$ is soft $\tau_1\tau_2\tau_3\alpha$-regular closed, there is a soft $\tau_1\tau_2\tau_3\alpha$-open set $(V, E)$ over $X$ such that $(G, E) = s\tau_1\tau_2\tau_3\alpha cl((V, E) \cap (G, E))$ and $(F, E) \cap s\tau_1\tau_2\tau_3\alpha cl(V, E) = \emptyset$.
3. Whenever $(F, E)$ is soft $\tau_1\tau_2\tau_3\alpha$-closed over $X$, $(U, E)$ is a soft $\tau_1\tau_2\tau_3\alpha$-regular open set over $X$ and $(F, E) \subseteq (U, E)$, there is a soft $\tau_1\tau_2\tau_3\alpha$-open set $(V, E)$ over $X$ such that $(F, E) = s\tau_1\tau_2\tau_3\alpha cl((V, E) \cap (F, E)) \subseteq s\tau_1\tau_2\tau_3\alpha cl(V, E) \subseteq s\tau_1\tau_2\tau_3\alpha cl((V, E) \cap (F, E)) = \emptyset$.

**Proof:**

(1)$\Rightarrow$(2): Suppose that $(F, E), (G, E)$ is disjoint soft $\tau_1\tau_2\tau_3\alpha$-closed sets over $X$ and $(F, E)$ is soft almost $\tau_1\tau_2\tau_3\alpha$-closed. Since $(X, \tau_1, \tau_2, \tau_3, E)$ is soft almost $\tau_1\tau_2\tau_3\alpha$-normal, there exist soft $\tau_1\tau_2\tau_3\alpha$-open sets $(U, E)$ and $(V, E)$ over $X$ such that $(F, E) = s\tau_1\tau_2\tau_3\alpha cl(U, E) \cap s\tau_1\tau_2\tau_3\alpha cl(V, E)$ and $(G, E) = s\tau_1\tau_2\tau_3\alpha cl((G, E) \cap (V, E)) \subseteq s\tau_1\tau_2\tau_3\alpha cl(V, E)$. Then $(F, E) \cap (G, E) = \emptyset$.

(2)$\Rightarrow$(1): Suppose that $(F, E), (G, E)$ is disjoint soft $\tau_1\tau_2\tau_3\alpha$-closed sets over $X$ and $(F, E)$ is soft $\tau_1\tau_2\tau_3\alpha$-regular closed. By the assumption, there exists a soft $\tau_1\tau_2\tau_3\alpha$-open set $(V, E)$ over $X$ such that $(G, E) \cap s\tau_1\tau_2\tau_3\alpha cl(V, E)$ and $(F, E) \cap s\tau_1\tau_2\tau_3\alpha cl(V, E) = \emptyset$. Then $(F, E) \cap (G, E) = \emptyset$.

(3)$\Rightarrow$(2): Suppose that $(F, E), (G, E)$ is disjoint soft $\tau_1\tau_2\tau_3\alpha$-closed sets over $X$ and $(F, E)$ is soft $\tau_1\tau_2\tau_3\alpha$-regular closed. Then $(G, E) = X_E - (F, E)$ and $(X_E - (F, E))$ is soft $\tau_1\tau_2\tau_3\alpha$-regular open set over $X$. By the hypothesis, there is a soft $\tau_1\tau_2\tau_3\alpha$-open set $(V, E)$ over $X$ such that $(G, E) = s\tau_1\tau_2\tau_3\alpha cl((V, E) \cap (G, E)) \subseteq s\tau_1\tau_2\tau_3\alpha cl(V, E) = \emptyset$.

**Proposition 3.4:** A soft tritopological space $(X, \tau_1, \tau_2, \tau_3, E)$ is soft almost $\tau_1\tau_2\tau_3\alpha$-normal if and only if it is both soft almost $\tau_1\tau_2\tau_3\alpha$-normal and soft $\tau_1\tau_2\tau_3\alpha$-k-normal.

**Proof:** Let $(X, \tau_1, \tau_2, \tau_3, E)$ be soft almost $\tau_1\tau_2\tau_3\alpha$-normal and soft $\tau_1\tau_2\tau_3\alpha$-k-normal. Let $(A, E)$ and $(B, E)$ be two disjoint soft $\tau_1\tau_2\tau_3\alpha$-closed sets over $X$ which $(A, E)$ is soft $\tau_1\tau_2\tau_3\alpha$-regular closed. By soft almost $\tau_1\tau_2\tau_3\alpha$-normality of $(X, \tau_1, \tau_2, \tau_3, E)$, there exist disjoint soft $\tau_1\tau_2\tau_3\alpha$-open sets $(U, E)$ and $(V, E)$ over $X$ such that $s\tau_1\tau_2\tau_3\alpha cl((A, E) \cap (U, E)) = (A, E)$, $s\tau_1\tau_2\tau_3\alpha cl((B, E) \cap (V, E)) = (B, E)$ and $s\tau_1\tau_2\tau_3\alpha cl((U, E) \cap (V, E)) = \emptyset$. Thus $(A, E) \subseteq s\tau_1\tau_2\tau_3\alpha cl(U, E)$ and $(B, E) \subseteq s\tau_1\tau_2\tau_3\alpha cl(V, E)$. Here $s\tau_1\tau_2\tau_3\alpha cl(U, E)$ and $s\tau_1\tau_2\tau_3\alpha cl(V, E)$ are disjoint soft $\tau_1\tau_2\tau_3\alpha$-regular closed sets over $X$. So by soft $\tau_1\tau_2\tau_3\alpha$-k-normality of $(X, \tau_1, \tau_2, \tau_3, E)$, there exist disjoint soft $\tau_1\tau_2\tau_3\alpha$-open sets $(W_1, E)$ and $(W_2, E)$ over $X$ such that $s\tau_1\tau_2\tau_3\alpha cl((U, E) \cap (W_1, E)) = (W_1, E)$ and $s\tau_1\tau_2\tau_3\alpha cl((U, E) \cap (W_2, E)) = (W_2, E)$. Hence, $(X, \tau_1, \tau_2, \tau_3, E)$ is soft almost $\tau_1\tau_2\tau_3\alpha$-normal.

**Proposition 3.5:** Every soft tritopological space $(X, \tau_1, \tau_2, \tau_3, E)$ which is both soft $\tau_1\tau_2\tau_3\alpha$-seminormal and soft almost $\tau_1\tau_2\tau_3\alpha$-k-normal is soft $\tau_1\tau_2\tau_3\alpha$-k-normal.

**Proof:** Let $(X, \tau_1, \tau_2, \tau_3, E)$ be a soft seminormal $\tau_1\tau_2\tau_3\alpha$-normal and soft almost $\tau_1\tau_2\tau_3\alpha$-k-normal. Let $(A, E)$ and $(B, E)$ be two disjoint soft $\tau_1\tau_2\tau_3\alpha$-closed sets over $X$. Thus $(A, E) \subseteq (X_E - (B, E))$. By soft almost $\tau_1\tau_2\tau_3\alpha$-k-normality of $(X, \tau_1, \tau_2, \tau_3, E)$, there exists a soft $\tau_1\tau_2\tau_3\alpha$-regular open set $(F, E)$ over $X$ such that $(A, E) \subseteq (F, E)$ and $(B, E) \subseteq (X_E - (F, E))$. Now $(A, E)$ is disjoint soft $\tau_1\tau_2\tau_3\alpha$-open sets over $X$ in which $(X_E - (F, E))$ is soft $\tau_1\tau_2\tau_3\alpha$-regular closed set containing $(B, E)$. Since $(X, \tau_1, \tau_2, \tau_3, E)$ is soft almost $\tau_1\tau_2\tau_3\alpha$-k-normal, there exist disjoint soft almost $\tau_1\tau_2\tau_3\alpha$-open sets $(U, E)$ and $(V, E)$ over $X$.
Proposition 3.6: Suppose that \((X, \tau_1, \tau_2, \tau_3, E_1)\) is a soft tritopological space, and \((Y, \sigma_1, \sigma_2, \sigma_3, E_2)\) are any two soft tritopological spaces, \((X, \tau_1, \tau_2, \tau_3, E_1)\) is soft almost \(\tau_1 \tau_2 \tau_3\)-\(\alpha\)-normal and \(f(X, \tau_1, \tau_2, \tau_3, E_1)\) is soft \(\tau_1 \tau_2 \tau_3\)-\(\alpha\)-continuous, and \((Y, \sigma_1, \sigma_2, \sigma_3, E_2)\) is soft \(\sigma_1 \sigma_2 \sigma_3\)-\(\alpha\)-continuous. Then \((Y, \sigma_1, \sigma_2, \sigma_3, E_2)\) is soft \(\sigma_1 \sigma_2 \sigma_3\)-\(\alpha\)-normal space.

Proof: Suppose that \((F, E_2)\) and \((G, E_2)\) are disjoint soft \(\sigma_1 \sigma_2 \sigma_3\)-\(\alpha\)-closed sets in \(Y\) and \((F, E_2)\) is soft \(\tau_1 \tau_2 \tau_3\)-\(\alpha\)-regular closed. Since \((Y, \tau_1, \tau_2, \tau_3, E_1)\) is soft \(\sigma_1 \sigma_2 \sigma_3\)-\(\alpha\)-space, \((F, E_2)\) and \((G, E_2)\) are soft \(\sigma_1 \sigma_2 \sigma_3\)-\(\alpha\)-closed sets in \(Y\). Since \(f\) is soft \(\tau_1 \tau_2 \tau_3\)-\(\alpha\)-continuous, \(f^{-1}(F, E_2)\) and \(f^{-1}(G, E_2)\) are disjoint soft \(\tau_1 \tau_2 \tau_3\)-\(\alpha\)-closed sets over \(X\). Clearly \(f^{-1}(F, E_2) = s \sigma_1 \sigma_2 \sigma_3 \cdot \alpha \cdot cl((s \sigma_1 \sigma_2 \sigma_3 \cdot int(F, E_2)))\). Suppose that \((W, E_1)\) is soft \(\tau_1 \tau_2 \tau_3\)-\(\alpha\)-open over \(X\), such that \((W, E_1) \cap f^{-1}(F, E_2) \neq \emptyset\). Then \((W, E_1)\) is soft \(\sigma_1 \sigma_2 \sigma_3\)-\(\alpha\)-open over \(Y\) and \(f(W, E_1) \cap (F, E_2) = f(\tau_1 \tau_2 \tau_3 \cdot \alpha \cdot cl((s \sigma_1 \sigma_2 \sigma_3 \cdot int(F, E_2))) \cap \emptyset\). If \(f(W, E_1) \cap (F, E_2) \neq \emptyset\), then \((W, E_1)\) is soft \(\sigma_2 \sigma_3\)-\(\alpha\)-open in \(Y\) and \(f(W, E_1) \cap (F, E_2) \neq \emptyset\). It follows that \((W, E_1)\) is soft \(\sigma_2 \sigma_3\)-\(\alpha\)-open in \(Y\). Therefore, \(f^{-1}(G, E_2)\) is soft \(\tau_1 \tau_2 \tau_3\)-\(\alpha\)-regular closed set. So there exist soft \(\tau_1 \tau_2 \tau_3\)-\(\alpha\)-open set \((U, E_1)\) over \(X\) such that \(f^{-1}(G, E_2) = \tau_1 \tau_2 \tau_3 \cdot \alpha \cdot cl\left(f^{-1}(G, E_2) \cap (U, E_1)\right)\) and \(\tau_1 \tau_2 \tau_3 \cdot \alpha \cdot cl\left(U, E_1\right) \cap \left(f^{-1}(F, E_2) \cap (U, E_1)\right) = \emptyset\). Also, note that \(f(U, E_1)\) is soft \(\sigma_1 \sigma_2 \sigma_3\)-\(\alpha\)-open and \(\tau_1 \tau_2 \tau_3 \cdot \alpha \cdot cl\left(U, E_1\right)\) is soft \(\sigma_1 \sigma_2 \sigma_3\)-\(\alpha\)-closed. Since \(f\) is soft \(\tau_1 \tau_2 \tau_3\)-\(\alpha\)-continuous, \(f(U, E_1)\) is soft \(\tau_1 \tau_2 \tau_3\)-\(\alpha\)-closed and \(f(\tau_1 \tau_2 \tau_3 \cdot \alpha \cdot cl\left(U, E_1\right)) \subseteq \emptyset\). It follows that \((G, E_2) = \sigma_1 \sigma_2 \sigma_3 \cdot \alpha \cdot cl\left((G, E_2) \cap \emptyset\right)\). Let \(y \in (E_2)\) for \(e \in E\) and \((O, E_2)\) be soft \(\tau_1 \tau_2 \tau_3\)-\(\alpha\)-open set over \(Y\). Then \(f^{-1}(Y) \subseteq f^{-1}(G, E_2) \cap f^{-1}(O, E_2) \neq \emptyset\), as desired.

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