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$(1, \alpha) -$ DERIVATIONS IN $\Gamma -$ NEAR RING

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ABSTRACT

In this paper, we introduce the notion of$(1, \alpha)$- derivation of $\Gamma -$ near ring and give some generalizations of [1]. The purpose of this paper is to prove the following two assertions: (i) Let Mbe a semiprime$\Gamma -$ near ring, U be a subset of M such that $0 \in U, U\Gamma M \subseteq U$ and d be a $(1, \alpha)$-derivation of M. If d acts as homomorphism on U or as antihomomorphism on U under certain conditions on α, then $d(U) = \{0\}$. (ii) Let M be a prime $\Gamma -$ near ring, U be a nonzero semigroup ideal of M, and d be a$(1, \alpha)$derivation on M. If $d(x + y - x - y) = 0$ for all $x, y \in U$, then $(M, +)$ is abelian.

Key words: *prime $\Gamma -$ near ring, semiprime $\Gamma -$ near ring, semigroup ideal,$(1, \alpha) -$ derivation.*

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1. INTRODUCTION

The notion of a Γ −ring a concept more general than a ring was defined by Nobusawa [6]. As a generalization of near rings, $\Gamma -$ near ring introduced by Satyanarayana [7]. The derivationsin $\Gamma -$ near rings has been introduced by Bell and Mason [3].They obtained some basic properties of derivations in, $\Gamma -$ near ring. In [2] Argac defined a two sided $\alpha -$ derivation of a $\Gamma -$ near ring. In a similar way, we introduce the notion of a of $(1, \alpha)$ $-$ derivation of Γ −nearring and give some Generalizations of [1, 2].

2. PRELIMINARIES

Throughout this paper, M stands for a zero symmetric right$\Gamma -$ near ring. In this section, we collect all basic concepts and results in $\Gamma -$ near rings mostly from Mustafa Kazaz and Akin Alkan [5] which are required for our study.

Definition 2.1[5]: A **$\Gamma -$ near ring**M is a triple $(M, +, \Gamma)$, where

(i) $(M, +)$is a group (not necessarily abelian),
(ii) $\Gamma$is a non empty set of binary operations on M such that $(M, +, \gamma)$ is a near ring for each $\gamma \in \Gamma$.
(iii) $(x\beta y)\gamma z = x\beta(y\gamma z)$for all $x, y, z \in M$ and $\beta, \gamma \in \Gamma$

Definition 2.2[5]: A $\Gamma -$ near ring M is said to be **zero-symmetric**$\Gamma -$ near ring if $0\gamma x = 0$ for all $x \in M$and $\gamma \in \Gamma$.

Definition 2.3[5]: A $\Gamma -$ near ring M is said to be **prime $\Gamma -$ near ring** if $x\Gamma M\Gamma y = \{0\}$ for $x, y \in M$ implies$x = 0$ or $y = 0$ and **semiprime$\Gamma -$ near ring** if $x\Gamma M\Gamma x = \{0\}$ for $x \in M$implies$= 0$.

Definition 2.4[4]: A **derivation** on M is defined to be an additive endomorphism d of M satisfying the product rule

$d(x\gamma y) = d(x)\gamma y + x\gamma d(y)$for all $x, y \in M$ and $\gamma \in \Gamma$, or equivalently
$d(x\gamma y) = x\gamma d(y) + d(x)\gamma y$for all $x, y \in M$ and $\gamma \in \Gamma$

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Definition 2.5[5]: If M and M' are two $\Gamma -$ near rings, then a mapping $f: M \rightarrow M'$ such that $f(x+y) = f(x) + f(y)$ and $f(x\gamma y) = f(x)\gamma f(y)$ (resp. $f(x\gamma y) = f(y)\gamma f(x)$) for all $x, y \in M$ and $\gamma \in \Gamma$, is called a $\Gamma -$ near rings **homomorphism** (resp.an **anti- homomorphism**) on M.

Definition 2.6[5]: Let S be a nonempty subset of M and let d be a derivation on M.
If $d(x\gamma y) = d(x)\gamma d(y)$(resp. $d(x\gamma y) = d(y)\gamma d(x)$) for all $x, y \in S$ and $\gamma \in \Gamma$, then d is said to act as a **homomorphism** (resp.an **anti-homomorphis**m) on S.

Definition 2.7[5]: An additive endomorphism $d: M \rightarrow M$ of a $\Gamma -$ near ring M is called a (α, β)derivation on M if there exist two functions $\alpha, \beta: M \rightarrow M$ such that the following product rule holds: $d(x\gamma y) = d(x)\gamma\alpha(y) + \beta(x)\gamma d(y)$ forall $x, y \in M$ and $\gamma \in \Gamma$.One can easily show that if d is (α, β)derivation on M Such that

$\alpha(x+y) = \alpha(x) + \alpha(y)$ and $\beta(x+y) = \beta(x) + \beta(y)$, then $d(x\gamma y) = \beta(x)\gamma d(y) + d(x)\gamma\alpha(y)$.

Definition 2.8[5]: An additive mapping $d: M \rightarrow M$ is called a two-sided $\alpha -$ derivation if d is $(\alpha, 1)$derivation as well as $(1, \alpha)$-derivation. we should note that if $\alpha = 1$,then a two sided $\alpha -$ derivation is just a derivation .

Definition 2.9 [4]: A nonempty subset U of M is called a **semigroup right ideal** (**resp**. **semigroup left ideal**) if $U\Gamma M \subset M(M\Gamma U)$ is called **semigroup** ideal if it is both right and left semigroup ideal.

3. $(1, \alpha)$- derivation in prime and semiprime $\Gamma -$ near rings

We need the following Lemmas to prove the main theorem of this section.

Lemma 3.1: Let M be a prime $\Gamma -$ near ring and let U be a nonzero semigroup ideal of M. If $a + b = b + a$ for all $a, b \in U$,then $(M, +)$ is abelian.

Proof: By the hypothesis, we have $x\gamma a + y\gamma a - x\gamma a - y\gamma a = 0$ for all $a \in U$, $x, y \in M$ and $\gamma \in \Gamma$.Then we get $(x + y - x - y)\gamma a = 0$ for all $a \in U$, $x, y \in M$ and $\gamma \in \Gamma$. It means that $(x + y - x - y)\Gamma M\Gamma U \subseteq (x + y - x - y)\Gamma U = 0$.
Since U is a non zerosemigroup ideal then $x + y - x - y = 0$ for all $x, y \in M$ by the primeness of M. Thus $(M, +)$ is Abelian.

Lemma 3.2: Let M be right $\Gamma -$ near ring, d a$(1, \alpha)$- derivation of M and U a multiplicative semigroup of M which contains 0. If d acts as an anti-homomorphism on U and $\alpha(x) = x$, then $x\gamma 0 = 0$ for all $x \in U$.

Proof: Since $0\gamma x = 0$ for all $x \in U$ and $\gamma \in \Gamma, d$ acts as an anti-homomorphism on U, it is clear that $d(0\gamma x) = d(x)\gamma d(0) = d(x)\gamma 0$ for all $x \in U$. Taking $x\gamma 0$ instead of x one, can obtain $d(x)\gamma 0 + \alpha(x)\gamma d(0) = \alpha(x)\gamma d(0) = 0$ for all $x \in U$. Thus we have $x\gamma 0 = 0$ for all $x \in U$.

Lemma 3.3: Let M be a $\Gamma -$ near ring and Ube a multiplicative subsemigroup of M. If d is a $(1, \alpha) -$ derivation of M such that $\alpha(x\gamma y) = \alpha(x)\gamma\alpha(y)$ for all $x, y \in$ Uand $\alpha(\mathrm{U}) = \mathrm{U}$, then $n\mu(d(x)\gamma y + \alpha(x)\gamma d(y)) = n\mu d(x)\gamma y + n\mu\alpha(x)\gamma d(y)$ for all $n, x, y \in \mathrm{U}, \gamma, \mu \in \Gamma$.

Proof: Since $d(x\gamma y) = d(x)\gamma y + \alpha(x)\gamma d(y)$.From the associative law $d\big(n\mu(x\gamma y)\big) = d(n)\mu(x\gamma y) + \alpha(n)\mu d(x\gamma y)$

$= d(n)\mu(x\gamma y) + \alpha(n)\mu\big(d(x)\gamma y + \alpha(x)\gamma d(y)\big)$ since (U) = U , we have

$$d\big(n\mu(x\gamma y)\big) = d(n)\mu(x\gamma y) + n\mu\big(d(x)\gamma y + \alpha(x)\gamma d(y)\big) \tag{1}$$

On other hand for all $n, x, y \in U$ and $\gamma, \mu \in \Gamma$.

$$\begin{aligned} d\big((n\mu x)\gamma y\big) &= d(n\mu x)\gamma y + \alpha(n\mu x)\gamma d(y) \\ &= d(n)\mu(x\gamma y) + \alpha(n)\mu\big(d(x)\gamma y + \alpha(x)\gamma d(y)\big) \\ &= d(n)\mu(x\gamma y) + n\mu(d(x)\gamma y + \alpha(x)\gamma d(y)) \text{since} \alpha(\mathrm{U}) = \mathrm{U} \end{aligned} \tag{2}$$

Since Uis a semigroup ideal, comparing (1) and (2)

$$n\mu(d(x)\gamma y + \alpha(x)\gamma d(y) = n\mu d(x)\gamma y + n\mu\alpha(x)\gamma d(y)$$

Lemma 3.4: Let M be a prime $\Gamma -$ near ring and U be a non zero semigroup ideal of M. Let d be a non-zero $(1, \alpha)$ derivation on M such that $\alpha(x\gamma y) = \alpha(x)\gamma\alpha(y)$for all $x, y \in U$,$\gamma \in \Gamma$. If $x \in M$ and $d(U)\gamma x = \{0\}$ then $x = 0$.

Proof: Assume that $d(U)\gamma x = \{0\}$. Since U is a non-zero semigroupideal of M, $d(u\mu y)\gamma x = 0$ for all $x, y \in M, u \in U$ and $\gamma, \mu \in \Gamma$. Hence$0 = [d(u)\mu y + \alpha(u)\mu d(y)]\gamma x = \alpha(u)\mu d(y)\gamma x$ for all $u \in U, y \in M. \Rightarrow \alpha(u)\Gamma d(y)\Gamma x = 0$ for all $\gamma, \mu \in \Gamma$. Since M is prime, $\alpha(u) = 0$or $x = 0$. If $\alpha(u) = 0$ for all $u \in U$, then $\alpha = 0$. This is not possible. There for e $x = 0$.

Lemma 3.5: Let M be a prime $\Gamma -$ near ring and U be a non zero semigroup ideal of M and a non-zero $(1,\alpha) -$ derivation on M.If $d(x+y-x-y)=0$ for all $x,y \in U$then $\alpha(x+y-x-y)\gamma d(z)=0$ for all $x,y \in U$ and $z \in M$.

Proof: Assume that $d(x+y-x-y)=0$ for all $x,y \in U$.Let us take $y\gamma z$ and $x\gamma z$ instead of y and x respectively (where $Z \in M$ and $\gamma \in \Gamma$) We obtain

$$d(x\gamma z+y\gamma z-x\gamma z-y\gamma z)=d\big((x+y-x-y)\gamma z\big)=0$$

$$\Rightarrow d(x+y-x-y)\gamma z+\alpha(x+y-x-y)\gamma d(z)= \alpha(x+y-x-y)\gamma d(z)=0 \text{ for all } x,y \in U, z \in M,\ \gamma \in \Gamma.$$

Therefore $\alpha(x+y-x-y)\Gamma d(z)=0$ $f$or all $\gamma \in \Gamma$.

Lemma 3.6: Let M be a $\Gamma -$ near ring U be a multiplicative subsemigroup of M. Let d be $(1,\alpha)$- derivation of M such That $\alpha(x\gamma y)=\alpha(x)\gamma\alpha(y)$ for all $x,y \in U$ and $\alpha(U)=U$.

(i) If d acts as a homomorphism on U then,

$$d(y)\mu x\gamma d(y)=\alpha(y)\mu x\gamma d(y)=d(y)\mu x\gamma y \text{ for all } x,y \in U \text{ and } \mu,\gamma \in \Gamma.$$

(ii) If d acts as a anti- homomorphism on U then,

$$d(y)\gamma x\gamma d(y)=x\gamma\alpha(y)\gamma d(y)=d(y)\gamma y\gamma x \text{for all } x,y \in U\text{and } \mu,\gamma \in \Gamma.$$

Proof:

(i) Let d acts as a homomorphism on U. Then

$$d(x\gamma y)=d(x)\gamma y+ \alpha(x)\gamma d(y)=d(y\mu x\gamma y)\text{for all } x,y \in U,\gamma \in \Gamma \quad (3)$$

Substituting $y\mu x$ for x in equation (3)

$$d(y\mu x\gamma y)=d(y\mu x)\gamma y+\alpha(y\mu x)\gamma d(y)=d(y)\mu d(x\gamma y)\text{for all } x,y \in U\ \mu,\gamma \in \Gamma \quad (4)$$

By the Lemma3.3

$$d(y)\mu d(x\gamma y)=d(y)\mu d(x)\gamma y+d(y)\mu\, \alpha(x)\gamma d(y)\text{for all } x,y \in U,\mu,\gamma \in \Gamma.$$

Using this relation in equation (4), we get

$$\alpha(y)\mu\alpha(x)\gamma d(y) = d(y)\mu\, \alpha(x)\gamma d(y)$$

Since $\alpha(U)=U$,we have

$$\alpha(y)\mu x\gamma d(y)= d(y)\mu x\gamma d(y)\text{for all } x,y \in U,\mu,\gamma \in \Gamma\ .$$

Similarly taking $y\mu x$ instead of $y$in equation (3) we obtain

$$d(x)\gamma d(y\mu x)= d(x)\gamma y\mu x+\alpha(x)\gamma d(y\mu x)=d(x\gamma y)\mu d(x)\text{for all } x,y \in U,\mu,\gamma \in \Gamma\ . \quad (5)$$

On the other hand $d(x\gamma y)\mu d(x)=\big(d(x)\gamma y+\alpha(x)\gamma d(y)\big)\mu d(x)=d(x)\gamma y\mu x+\alpha(x)\gamma d(y)\mu d(x)$. using this relation in (5) we get $d(y)\mu x\gamma d(y)=\alpha(y)\mu x\gamma d(y)=d(y)\mu x\gamma y$ for all $x,y \in U,\mu,\gamma \in \Gamma$

(ii) since$d$ acts as a anti- homomorphism on U,we have

$$d(x\gamma y)=d(x)\gamma y+ \alpha(x)\gamma d(y)$$

$$\Rightarrow d(y)\gamma\, d(x)= d(x)\gamma y+ \alpha(x)\gamma d(y) \text{ for all } x,y \in U,\gamma \in \Gamma \quad (6)$$

Taking $x\gamma y$ for y in equation (6) and by the hypothesis we get

$$d(x\gamma y)\gamma\, d(x)= d(x)\gamma x\gamma y+ \alpha(x)\gamma d(x\gamma y)$$

$$\Rightarrow d(x)\gamma y\gamma\, d(x)= d(x)\gamma x\gamma y \quad (7)$$

Similarly taking$x\gamma y$ instead of x in equation (6) and by hypothesis

$$d(y)\gamma\, d(x\gamma y)= d(x\gamma y)\gamma y+ \alpha(x\gamma y)\gamma d(y)$$

$$\Rightarrow d(y)\gamma\, \alpha(x)\gamma d(y)=\alpha(x\gamma y)\gamma d(y)$$

Since $\alpha(U)=U$, we have

$$d(y)\gamma\, x\gamma d(y)=x\gamma\alpha(y)\gamma d(y)$$

Theorem 3.7: Let M be a semiprime $\Gamma -$ near ring and U be a subset of M such that $0 \in U$and $U\Gamma M \subseteq U$. Let d be a $(1,\alpha) -$ derivation on M such that $\alpha(U)=U$ and $\alpha(x\gamma y)=\alpha(x)\gamma\alpha(y)\ \forall x,y \in U$

(i) If $d$acts as an homomorphism on U then $d(U)=\{0\}$

(ii) If $d$acts as an anti- homomorphism on U and $\alpha(0)=0$then $d(U)=\{0\}$

Proof: Suppose d acts as an homomorphism on U. By the Lemma 3.6, we have

$$d(y)\mu x\gamma d(y)=d(y)\mu x\gamma y \quad \text{for all} \quad x,y \in U \text{ and } \mu,\gamma \in \Gamma \quad (8)$$

Multiply (8) by $d(z)$ where $z \in U$ and using the hypothesis that d act as an homomorphism on U together with Lemma 3.6 we get

$$d(y)\mu x\gamma(d(y)\mu z + \alpha(y)\mu d(z)) = d(y)\mu x\gamma y\mu d(z)$$
$$\Rightarrow\ d(y)\mu x\gamma d(y)\mu z + d(y)\mu x\gamma\alpha(y)\mu d(z) = d(y)\mu x\gamma y\mu d(z) \text{ by the Lemma 3.3}$$

Since $\alpha(U) = U$, we get

$$d(y)\mu x\gamma d(y)\mu z + d(y)\mu x\gamma y\mu d(z) = d(y)\mu x\gamma y\mu d(z)$$
$$\Rightarrow\ d(y)\mu x\gamma d(y)\mu z = 0 \quad \text{for all } x, y \in U \text{ and } \mu, \gamma \in \Gamma \tag{9}$$

Taking $z\eta m$ instead of x where $m \in M$

We get $d(y)\mu z\eta m\gamma d(y)\mu z = 0$ for all $x, y, z \in U, m \in M$ and $\mu, \eta, \gamma \in \Gamma$. ,

In particular

$$\Rightarrow\ d(y)\mu z\Gamma M\Gamma d(y)\mu z = 0$$

By the semiprimeness of M we conclude that

$$d(y)\mu z = \{0\} \tag{10}$$

Substitute $y\eta n$for y in equation (10)

$$d(y\eta n)\mu z = 0 \tag{11}$$

Left multiply (11) by $d(z)$where $z \in U$, we get

$$d(z)\beta d(y\eta n)\mu z = 0$$
$$\Rightarrow\ d(z)\beta d(y)\eta n\mu z + d(z)\beta\alpha(y)\eta d(n)\mu z = 0 \text{ by the Lemma 3.3}$$

Since the second summand is zero by (11) we get

$$d(z)\beta d(y)\eta n\mu z = 0$$

By the hypothesis and by using (10)

$$\alpha(z)\beta d(y)\eta n\mu z = 0$$

Since $\alpha(U) = U$, we have

$$z\beta d(y)\eta n\mu z = 0 \text{for all } x, y \in U, n \in M$$

Substitute $z = z\beta d(y)$ in above equation and since M is semiprime

$$z\beta d(y) = 0 \text{for all } y, z \in U$$

Since $\alpha(U) = U,$ (12)

$$\alpha(z)\ \beta d(y) = 0$$

Combining (10) and (12) we get

$$d(y\beta z) = 0 \text{ for all } y, z \in U$$

Replace y by $z\gamma m$

$$d(z\gamma m\beta z) = 0 \text{ for all } m \in M \text{ and } \gamma, \beta \in \Gamma, z \in U$$

By the hypothesis and by (12)

$$d(z)\gamma m\beta d(z) = 0$$
$$\Rightarrow d(z)\Gamma M\Gamma d(z) = 0$$

Hence

$$d(z) = \{0\} \text{for all } z \in U \tag{13)(ii}$$

Now assume that d acts as an anti- homomorphism on U. By the Lemma 3.6, we have

$$x\ \gamma\alpha(y)\gamma d(y) = d(y)\gamma x\gamma d(y) \tag{14}$$
$$d(y)\gamma y\gamma x = d(y)\gamma x\gamma d(y) \tag{15}$$

Replace x by $x\gamma d(y)$ in (14), we get

$$x\gamma d(y)\gamma\alpha(y)\gamma d(y) = d(y)\gamma x\gamma d(y)\gamma y + d(y)\gamma x\gamma\alpha(y)\gamma d(y))\text{by the Lemma3.3}$$

Since $\alpha(U) = U$, we have

$$x\gamma d(y)\gamma y\gamma d(y) = d(y)\gamma x\gamma d(y)\gamma\alpha(y) + d(y)\gamma x\gamma y\gamma d(y)) \tag{16}$$

Substitute $x\gamma y$ for x equation (14)

$$x\gamma y\gamma\alpha(y)\gamma d(y) = d(y)\gamma x\gamma y\gamma d(y) \quad (17)$$

multiply equation (14) by $\alpha(y)$

$$x\gamma\alpha(y)\gamma d(y)\gamma\alpha(y) = d(y)\gamma x\gamma d(y)\gamma\alpha(y) \quad (18)$$

Replace x by$y$ in equation (14)

$$y\gamma\alpha(y)\gamma d(y) = d(y)\gamma y\gamma d(y)$$

Multiply by$x$ above relation by

$$x\gamma y\gamma\alpha(y)\gamma d(y) = x\gamma d(y)\gamma y\gamma d(y) \quad (19)$$

Using (17) (18) and (19) in (16)

$$x\gamma y\gamma\alpha(y)\gamma d(y) = x\gamma\alpha(y)\gamma d(y)\gamma\alpha(y) + x\gamma y\gamma\alpha(y)\gamma d(y)$$
$$x\gamma\alpha(y)\gamma d(y)\gamma\alpha(y) = 0$$

In particular,

$$xn\gamma\alpha(y)\gamma d(y)\gamma\alpha(y) = 0 \text{ where } n \in M$$

Hence $\alpha(y)\gamma d(y)\gamma\alpha(y)\gamma M\gamma\, \alpha(y)\gamma d(y)\gamma\alpha(y) = \{0\}$

By the semiprimeness of

$$\alpha(y)\gamma d(y)\gamma\alpha(y) = 0 \text{for all } x, y \in U \quad (20)$$

According to (18) we get

$$d(y)\gamma x\, \gamma d(y)\gamma\alpha(y) = 0 \text{for all } x, y \in U \quad (21)$$

Replacing x by $\alpha(y)\gamma x\gamma n$

$$d(y)\gamma\alpha(y)\gamma x\gamma n\gamma d(y)\gamma\alpha(y)\gamma x = 0$$
$$\Rightarrow d(y)\gamma\alpha(y)\gamma x = 0 \text{for all } x, y \in U, n \in M$$

Since $\alpha(U) = U$, we get

$$d(y)\gamma y\gamma x = 0 \text{ for all } x, y \in U \quad (22)$$

Using (22) in (15) we obtain

$$d(y)\gamma x\gamma d(y) = 0 \text{for all } x, y \in U$$

And so we have

$$d(y)\gamma x\gamma n\gamma d(y)\gamma x = 0 \text{for all } x, y \in U, n \in M$$

Hence$d(y)\gamma x = 0$ for all $x, y \in U$ (23)

Therefore

$$x\gamma d(z)\gamma d(y\gamma n)\gamma x = 0 \text{for all } x, y, z \in Un \in M$$

By the hypothesis and using Lemma 3.3

$$x\gamma d(z)\gamma d(y)\gamma n\gamma x + 0 = 0$$

Since $\alpha(U) = U$ the second summand is zero by (13)

Hence $x\gamma d(z)\gamma d(y)\gamma M\gamma x = 0$

By the semiprimeness of M we get

$$0 = x\gamma d(z)\gamma d(y)$$
$$= x\gamma d(z)\gamma y + x\gamma\alpha(z)\gamma d(y)$$
$$= x\gamma\alpha(z)\gamma d(y) = 0$$

By the semiprimeness of M,

$$\alpha(z)\gamma d(y) = 0 \text{ for all } z, y \in U \quad (24)$$

Combining (23) and (24) we have

$$0 = d(x\gamma y) \text{ for all } x, y \in U$$

Replace y by $x\gamma n$. By hypothesis and Lemma 3.3

$$0 = d(x)\gamma n\gamma d(x) + \alpha(x)\gamma d(n)\gamma d(x)$$

Since the second summand is zero we get

$$d(x)\gamma n\gamma d(x) = 0$$

Therefore $d(x) = 0$ for all $x \in U$.

i.e.,$d(U) = 0$

Corollary 3.8: Let M be a semi prime $\Gamma-$near ring and d a $(1,\alpha)-$ derivation of M Such that α is onto and $\alpha(x\gamma y) = \alpha(x)\gamma\alpha(y)$ for all $x, y \in M$

(i) If $d$acts as a homomorphism on M, then $d = 0$

(ii) If $d$acts as a anti- homomorphism on M such that $\alpha(0) = 0$ then $d = 0$

Proof: Take $U = M$ in above theorem we get

(i) $d(M) = 0 \Rightarrow d = 0$

(ii) $\alpha(0) = 0$then$d(M) = 0 \Rightarrow d = 0$

Corollary 3.9: Let M be a prime $\Gamma-$ near ringand U a non zero subset of M such that $0 \in U$ and $U\Gamma M \subseteq U$. Let d be a $(1,\alpha)-$ derivation of M Such that $\alpha(U) = U$ and $\alpha(x\gamma y) = \alpha(x)\gamma\alpha(y)$ for all $x, y \in U$

(i) If $d$acts as a homomorphism on U, then $d = 0$

(ii) If $d$acts as an anti- homomorphism on U and $\alpha(0) = 0$ then $d = 0$

Proof: By the theorem 3.7, we have$d(x) = 0$ for all $x \in U$Then $d(x\gamma n) = d(x)\gamma n + \alpha(x)\gamma d(n) = \alpha(x)\gamma d(n) = 0$forall$x \in U$,$n \in M$. Replace x by $x\gamma m, \alpha(x\gamma m)\gamma d(n) = \alpha(x)\gamma\alpha(m)\gamma d(n) = 0$for all $x \in U$, $n, m \in M$ and $\gamma \in \Gamma$. Hence $x\Gamma M\,\Gamma d(n) = 0$. By the primeness of M we have $x = 0$ or $d(n) = 0$ for all $x \in U$. Since U is non zero we have$d(n) = 0$ for all $n \in M$.

Theorem 3.10: Let M be a prime $\Gamma-$ near ring U a non zero semigroup ideal of M and d a non zero$(1,\alpha)$- derivation of M such that $\alpha(x\gamma y) = \alpha(x)\gamma\alpha(y)$ for all $x, y \in U$ and $\alpha(U) = U$. If $d(x + y - x - y) = 0$ for all $x, y \in U$ then $(M, +)$ is abelian.

Proof: Suppose that $d(x + y - x - y) = 0$ for all $x, y \in U$.By the Lemma 3.5, we have $\alpha(x + y - x - y)\gamma d(z) = 0$ for all $x, y \in U$, $z \in M$ and $\gamma \in \Gamma$..Since $d \neq 0$ it follows that $\alpha(x + y - x - y) = x + y - x - y = 0$ for all $x, y \in U$. Hence$(M, +)$ is abelian by Lsemma 3.1.

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