FUZZY OPTIMIZATION MODELING IN THE ANALYSIS OF HUMAN STRESS

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ABSTRACT

In this paper, a survey is taken about the human stress in which most of the people gets affected. Other than allopathic medicine, different forms of scientifically appropriate and acceptable medicines, such as the Ayurveda, Homeopathy and yoga are some of the treatment given to the stress. In this paper, fuzzy optimization model is developed based on linear programming problem to minimize the overall treatment cost by distributing the various treatments to the different stress in order to minimize the human productivity loss.

Keywords: Multi Objective Linear Model, Fuzzy Linear Programming Problem, Fuzzy Number and Chronic stress.

2000 Mathematical Subject Classification: 90C70, 90C26.

1. INTRODUCTION

Major categories of modelling approaches are classical optimization techniques, linear programming, non-linear programming, geometric programming, dynamic programming, integer programming etc. In recent years there has been a dramatic increase in the application to optimize techniques to the study the health care facilities and to strengthen the health care service delivery system. To indicate the side spread scope of the subject some special typical applications in health care analysis are given below. Simplex method is widely used in programming problems as it maximizes the profit to a greater extent. Fuzzy simplex method is used due to the uncertainty in the data collected. All these data are subject to changes and hence fuzzy simplex method can give us a better optimal solution than the ordinary Linear Programming Problem.

L.A. Zadeh has introduced the concept called Fuzzy set. It can be used in wide range of domains where information are incomplete and imprecise. A fuzzy number is a quantity, whose values are imprecise and it gives single valued numbers. Rajarajeswari et al. presented a new operation on hexagonal fuzzy numbers. Ranking fuzzy number is used mainly in decision-making, data analysis, artificial intelligence and even in various fields of the operation research. In fuzzy environment, ranking fuzzy numbers are very important in decision making procedure. Allahviranloo et al. proposed a new method for solving fully fuzzy linear programming problems by using of ranking function. Rommelfanger and Delgado, Verdegay, Vila has also proposed a general model for the FLP problems. While comparing the Fuzzy linear programming problem, Fully Fuzzy linear programming parameters of the objective function are also fuzzy numbers. Campos and Vardegay considered linear programming problems with fuzzy constraints and fuzzy coefficients in both left and right hand of the constraints set.

In this paper, survey is taken about the human stress in which most of the people gets affected. Stress is the feeling of pressure, strain or tension that comes from dealing with challenging situations. It is a part of life and it happens to everyone. However too much stress, or a strong response to stress can be harmful. Among the conditions which seem in some way to be stress-related are: high blood pressure, coronary heart disease, asthma, migraine, diabetes, ulcers, insomnia, and depression. Other than allopathic medicine, different forms of scientifically appropriate and acceptable medicines, such as the yoga, Ecotherapy and Aromatherapy etc,… are some of the treatment given to the stress. In this paper, fuzzy optimization model is developed based on linear programming problem to minimize the overall treatment cost and curing time by distributing the various treatments to the different stress in order to minimize the human productivity loss. In order to minimize these problems, fuzzy trapezoidal linear programming problem has been used and finally suggestions for curing stress is given.
2. PRELIMINARIES

Definition 2.1: [8] Let R be the set of all real numbers. We assume a fuzzy number \( \widetilde{A} \) that can be expressed for all \( x \in \mathbb{R} \) in the form

\[
\mu_{\widetilde{A}}(X) = \begin{cases} 
\mu_{\lambda}\text{\( X \)} & \text{for } a \leq x \leq b \\
W & \text{for } b \leq x \leq c \\
\mu_{\lambda}\text{\( X \)} & \text{for } c \leq x \leq d \\
0 & \text{otherwise}
\end{cases}
\]

where \( 0 \leq w \leq 1 \) is a constant, \( a, b, c, d \) are real numbers, such that \( a < b \leq c < d, \mu_{\lambda}\text{\( X \)}: [a, b] \rightarrow [0, w] \) are two strictly monotonic and continuous functions from \( R \) to the closed interval \([0, w]\). Since \( \mu_{\lambda}\text{\( X \)} \) is continuous and strictly increasing, the inverse function \( \mu_{\lambda}^{-1}\text{\( X \)} \) exists. Similarly \( \mu_{\lambda}^{-1}\text{\( X \)} \) is continuous and strictly decreasing the inverse function of \( \mu_{\lambda}^{-1}\text{\( X \)} \) also exist. The inverse functions of \( \mu_{\lambda}^{-1}\text{\( X \)} \) and \( \mu_{\lambda}^{-1}\text{\( X \)} \) can be denoted by \( \mu_{\lambda}^{-1}\text{\( X \)} \) and \( \mu_{\lambda}^{-1}\text{\( X \)} \) respectively. \( \mu_{\lambda}^{-1}\text{\( X \)} \) and \( \mu_{\lambda}^{-1}\text{\( X \)} \) are continuous on \([0, w]\) that means both \( \int_0^w \mu_{\lambda}^{-1}\text{\( X \)} \) and \( \mu_{\lambda}^{-1}\text{\( X \)} \).

Definition 2.2: [6] A fuzzy set \( A \) of the real line \( \mathbb{R} \) with membership functions \( \mu_{A}: X \rightarrow [0,1] \) is called fuzzy number if,

i) \( A \) must be normal and convex fuzzy set

ii) The support of \( A \) must be bounded.

iii) \( \alpha_A \) must be closed interval for every \( \alpha \in [0,1] \).

Definition 2.3: [4] A trapezoidal fuzzy number denoted by \( \widetilde{A} \) is defined as \((a_1, a_2, a_3, a_4)\) where the membership function

\[
\mu = \begin{cases} 
0, & x \leq a_1 \\
\frac{x - a_1}{a_1 - a_2}, & a_1 \leq x \leq a_2 \\
1, & a_2 \leq x \leq a_3 \\
\frac{a_4 - x}{a_4 - a_3}, & a_3 \leq x \leq a_4 \\
0, & x \geq a_4
\end{cases}
\]

Definition 2.4: [1] If \( X \) is a collection of objects generically by \( x \), then a fuzzy set \( \widetilde{A} \) in \( X \) is a set of ordered pairs. \( \mu_{A}(X) \) is called the membership function or grade of membership of that maps to the membership space.

Definition 2.5: [7] Let \( R: F(A) \rightarrow \mathbb{R} \), where \( F(A) \) be the set of all Trapezoidal Fuzzy Numbers and \( R \) be the set of real numbers. The ranking of trapezoidal fuzzy numbers \( \widetilde{A} \) is defined and denoted as

\[
R(\widetilde{A}) = \frac{a_1 + a_2}{2} + \frac{1}{4}(a_4 - a_3)
\]

Definition 2.6: [7] A feasible solution to a linear programming problem is a set of values for the variables \( x_1, x_2, ..., x_n \) which satisfy the contraints.

Definition 2.7: [7] There can be one or more feasible solutions to a linear programming problem. A feasible solution which optimises the objective function is known as optimal solution.

3. MATHEMATICAL FORMULATION OF FUZZY LINEAR PROGRAMMING PROBLEM

In this paper based on the factors that are responsible for human stress among men and women, a minimization problem is formed. Then in new dimension, the various reasons for human stress is collected. Using Big-M Method minimization problem is calculated.

In some problems, the slack variables cannot provide the initial basic feasible solution. In these problems atleas one of the constraints is of = or \( \geq \) type. To solve such linear programming problems we used the Big M method.

3.1 A Fuzzy Version of Simplex Algorithm for Big M Method

For the solution of any Fuzzy Number Linear Programming by Simplex Algorithm, the existence of an initial basic feasible fuzzy solution is always assumed.
The steps for the computation of an optimum fuzzy solution are as follows:

i) Express the given problem in the standard form.

ii) The given data is transformed into FLPP by using arithmetic operation of trapezoidal fuzzy numbers (i.e.) the formula for ranking function is applied.

iii) Add non-negative artificial variables to the left side of each of the equations corresponding to constraints of the type = .

iv) However addition of these artificial variables causes violation of the corresponding constraints. Therefore, it is necessary to get rid of these variables so that they do not appear in the final solution. This is achieved by assigning a very large penalty (-M for maximization and M for minimization) in the objective function.

v) Solve the modified LPP by Simplex Method, until any one of the three cases arise:
   - If no artificial variable appears in the basis and the optimality conditions are satisfied, then the current solution is an optimal basic feasible solution.
   - It at least one artificial variable is present in the basis with zero value, then the current optimum basic feasible solution is a degenerate one.
   - If at least one artificial variable is present in the basis with a positive value, then the given Fuzzy Linear Programming Problem does not possess a fuzzy optimal basic feasible solution. In this case, the given problem is said to have a fuzzy pseudo-optimum basic feasible solution.

4. PROPOSED MODEL FOR TREATMENT OF HUMAN STRESS

In this section, the application is based on Big-M method for computing minimum treatment cost of human stress in order to minimize the human productivity loss. The objective is to determine how the various treatment may be distributed to the different stress so as to minimize the overall treatment cost.

In this paper, three types of stress are considered. They are Acute stress, Chronic stress and Traumatic stress. Some of the major factors to reduce stress are considered. They are Yoga and Mediation, Ecotherapy, Aromatherapy, taking healthy food, exercise, etc. Also, a survey is taken and details are collected from three different group of patients where each group is affected by three different stresses. Also in each group four patients are considered. Among various treatments given stresses, three basic treatments are considered. They are yoga, Ecotherapy and Aromatherapy. Because these three treatments are more than enough for curing human stress.

In each type of stress, time taken by each patients in each group in one day for doing yoga, Ecotherapy and Aromatherapy are considered. Also the cost for curing all the stresses for each patient in each group are given in Table 1. The overall treatment time for a patient in stress must do yoga for at least 4 hours per day, Ecotherapy for at least 9 hours per day and Aromatherapy for at least 7 hours per day.

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Time taken by four patients affected by (in hours / per day)</th>
<th>Treatment cost per day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yoga and Mediation</td>
<td>1, 4, 3, 5, 6, 5, 4, 2, 3, 2, 2, 4</td>
<td>650, 700, 775, 875</td>
</tr>
<tr>
<td>Ecotherapy</td>
<td>3, 5, 3, 7, 5, 8, 4, 6, 5, 4, 8, 6</td>
<td>950, 850, 1050, 850</td>
</tr>
<tr>
<td>Aromatherapy</td>
<td>2, 3, 2, 4, 8, 3, 4, 6, 6, 3, 2, 4</td>
<td>800, 750, 700, 1004</td>
</tr>
</tbody>
</table>

To calculate overall treatment cost for each stress, the time taken by four patients in each group is reduced to single data by using the Ranking function defined in the Definition 2.5. The calculated data is given in the Table 2.

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Time taken by four patients affected by (in hours / per day)</th>
<th>Treatment cost per day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yoga and Mediation</td>
<td>3, 5, 7, 3</td>
<td>700</td>
</tr>
<tr>
<td>Ecotherapy</td>
<td>5, 7, 4</td>
<td>950</td>
</tr>
<tr>
<td>Aromatherapy</td>
<td>3, 6, 5</td>
<td>850</td>
</tr>
</tbody>
</table>

Let

\[ x_1 = \text{Total number of days required to cure Acute stress} \]
\[ x_2 = \text{Total number of days required to cure Chronic stress} \]
\[ x_3 = \text{Total number of days required to cure Traumatic stress} \]

Since the objective is to minimize the treatment cost, the objective function and the complete LPP can be represented as follows:
Minimize: \[ Z = (550, 675, 700, 725)x_1 + (830, 880, 950, 975)x_2 + (750, 810, 850, 875)x_3 \]
Subject to
\[
\begin{align*}
3x_1 + 5x_2 + 3x_3 & \geq 4 \\
5x_1 + 7x_2 + 4x_3 & \geq 9 \\
3x_1 + 6x_2 + 5x_3 & \geq 7 \\
x_1, x_2, x_3 & \geq 0
\end{align*}
\]
We convert to maximization using \( \text{MinZ} = -\text{Max} (-Z) = -\text{MaxZ}^* \)
Maximize: \[ Z^* = (-725, -700, -675, -550)x_1 + (-975, -950, -880, -830)x_2 + (-875, -850, -810, -750)x_3 \]
Subject to
\[
\begin{align*}
3x_1 + 5x_2 + 3x_3 & \geq 4 \\
5x_1 + 7x_2 + 4x_3 & \geq 9 \\
3x_1 + 6x_2 + 5x_3 & \geq 7 \\
x_1, x_2, x_3 & \geq 0
\end{align*}
\]
The problem is converted to canonical form by adding surplus and artificial variables as appropriate
1. As the constraint 1 is of type ‘\( \geq \)’ we should subtract surplus variable \( S_1 \) and add artificial variable \( A_1 \).
2. As the constraint 2 is of type ‘\( \geq \)’ we should subtract surplus variable \( S_2 \) and add artificial variable \( A_2 \).
3. As the constraint 3 is of type ‘\( \geq \)’ we should subtract surplus variable \( S_3 \) and add artificial variable \( A_3 \).

After introducing surplus variables and artificial variables:
Minimize: \[ Z = (550, 675, 700, 725)x_1 + (830, 880, 950, 975)x_2 + (750, 810, 850, 875)x_3 + 0S_1 + 0S_2 + 0S_3 \]
Subject to
\[
\begin{align*}
3x_1 + 5x_2 + 3x_3 - S_1 & = 4 \\
5x_1 + 7x_2 + 4x_3 - S_2 & = 9 \\
3x_1 + 6x_2 + 5x_3 & = 7 \\
x_1, x_2, x_3, S_1, S_2, S_3 & \geq 0
\end{align*}
\]
To find the Initial Basic Feasible Solution, the artificial variables \( A_1, A_2, A_3 \) are introduced and the present form of the given FLPP is
Maximize: \[ Z^* = (-725, -700, -675, -550)x_1 + (-975, -950, -880, -830)x_2 + (-875, -850, -810, -750)x_3 + 0S_1 + 0S_2 + 0S_3 - MA_1 - MA_2 \]
Subject to
\[
\begin{align*}
3x_1 + 5x_2 + 3x_3 - S_1 & = 4 \\
5x_1 + 7x_2 + 4x_3 & = 9 \\
3x_1 + 6x_2 + 5x_3 & = 7 \\
x_1, x_2, x_3, S_1, S_2, A_1, A_2, A_3 & \geq 0
\end{align*}
\]
After converting to fuzzy linear programming problem, the problem is solved to get the optimal solution in minimizing the treatment cost for the stress. The required optimal solution is \( x_1 = 0, x_2 = 1.985 \) and \( x_3 = 0 \).

Hence
\[
\begin{align*}
\text{Max } Z^* &= (-975, 950, -800, 830) (1.985)  \\
&= (-1253.75, -1221.605, -1131.592, -1067.297)  \\
\text{Min } Z &= -\text{Max}Z^* = (1067.297, 1131.592, 1221.605, 1253.75)  \\
&= \text{Rs. } 1107.48
\end{align*}
\]
Thus from the above calculations, it is clear that among the three types of stress, patients affected by Chronic stress must undergo treatment for 2 days and the treatment cost for first patient is Rs.1068, second patient is Rs.1132, third patient is Rs.1222 and for fourth patient is Rs.1254. After applying the Ranking function, the overall treatment cost is Rs.1108.

Suggestions:
- Meditation: Focus on your breath. Breathe in and out slowly and deeply and visualize a serene environment such as a deserted beach or grassy hill.
- Workplace: We found that employees who have a stressful workday end to bring their negative feelings from the workplace to the dinner table, as manifested in eating more than usual and opting for more junk foods instead of healthy foods.
- Exercise: Regular physical exercise is good for physical and mental health. It provides an outlet for frustrations and releases mood-enhancing endorphins. Yoga can be particularly effective at reducing anxiety and stress.
• Play music: Soft, calming music can lower blood pressure and relax our mind and body. Lend a hand to a relative or neighbour, or volunteer in community. Helping others will take our mind off of our own anxiety and fears.
• Establish a regular, relaxing bedtime routine. Avoid stimulants like coffee, chocolate and nicotine before going to sleep and never watch TV, never use the computer or never pay bills before going to bed. Read a book, listen to soft music or meditate instead.
• Sleep disorders are characterized by abnormal sleep patterns that interfere with physical, mental and emotional functioning. So deep sleep in the night time is very important to reduce the stress.

5. CONCLUSION

This paper has developed fuzzy Linear Programming Model in order to distribute the various treatments to the different human stress so as to minimize the overall treatment cost of patients. By minimizing the overall treatment cost, human stress may be minimized. This work will be an innovative application of fuzzy linear programming technique in health care. Consuming healthy foods and practicing exercise regularly may help to decrease the level of stress. Even there are certain ways to put down stress, they are listening to music, walking, meditation, etc., may reduce the stress level. Meditation in one of the best way to overcome from stress and even reading books and gardening can also helps to calm down the stress level. Planning and doing the things accordingly may lower the stress level.

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