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ON #pg-CONTINUOUS MAPS AND #pg-COMPACT SPACES

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ABSTRACT

In this paper the concept of #pg-closed set is introduced. Besides studying some properties, the interrelations of #pg-closed set with other related sets are studied. Characterizations and properties of #pg-continuous map is discussed. Equivalently #pg-compact spaces are introduced and some interesting properties are discussed.

Keywords: **pre semi-open set, pw-closed set, #pg-closed set, #pg-continuous map, $T_{\#pg}$-space, #pg-irresolute map, #pg-compact space.*

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1. INTRODUCTION

The notion of pre-open set was introduced by Mashhour *et al.* [12]. Cameron [5], Benchalli [3], and Syed Ali Fathima and Mariasingam [19, 20] introduced and investigated regular semi-open sets, rw-closed sets, #rg-closed sets, #rg-closure, #rg-continuous maps, $T_{\#rg}$-space and #RG-compact spaces. We introduce and study the concept of #pg-closed sets, #pg-continuous maps and #pg-compact spaces.

2. PRELIMINARIES

Throughout this paper X, Y and Z denote the topological spaces (X, T), (Y, S) and (Z, R) respectively on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a topological space X, the closure of A, interior of A, semi-closure of A, semipre-closure of A, the complement of A and #rg-closure of A are denoted by cl(A), int(A), scl(A), spcl(A), X\A and #rg-cl(A) respectively. We recall the following definitions and results.

Definition 2.1: A subset A of a space X is called
1. a pre-open set [11] if A ⊆ intcl (A) and a pre-closed set if clint (A) ⊆ A.
2. a semi-open set [11] if A ⊆ clint (A) and a semi-closed set if intcl (A) ⊆ A.
3. a regular open set [17] if A = intcl (A) and a regular closed set if A = clint (A).
4. a π-open set [21] if A is a finite union of regular open sets.
5. a regular semi-open set [5] if there is a regular open U such that U ⊆ A ⊆ cl(U).
6. a semi-preopen set [1] if A ⊆ cl(int(cl(A))) and a semi-preclosed set if int(cl(int(A))) ⊆ A.

Definition 2.2: A subset A of a space X is called
1. a generalized closed set (briefly, g-closed) [10] if cl(A) ⊆ U whenever A ⊆ U and U is open in X.
2. a weakly generalized closed set (briefly, wg-closed) [13] if cl(int(A)) ⊆ U whenever A ⊆ U and U is open in X.
3. a π-generalized closed set (briefly, πg-closed) [8] if cl(A) ⊆ U whenever A ⊆ U and U is π-open in X.
4. a weakly closed set (briefly, w-closed) [15] if cl(A) ⊆ U whenever A ⊆ U and U is semi-open in X.
5. a rw-closed set [3] if cl(A) ⊆ U whenever A ⊆ U and U is regular semi-open in X.
6. a *g-closed set [20] if cl(A) ⊆ U whenever A ⊆ U and U is w-open in X.

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(7) a generalized semi-closed set (briefly, gs-closed) [2] if scl(A) ⊆ U whenever A ⊆ U and U is open in X.
(8) a generalized semi pre-closed set (briefly, gsp-closed) [7] if spcl(A) ⊆ U whenever A ⊆ U and U is open in X.
(9) a #rg-closed set [18] if cl(A) ⊆ U whenever A ⊆ U and U is rw-open in X.

Definition 2.3: [18] For a subset A of a space X, #rg-cl(A) = ⋂ {F : A ⊆ F, F is #rg closed in X} is called the #rg-closure of A.

Definition 2.4: A map f : (X, T) → (Y, S) is called
(1) continuous [4] if $f^{-1}(V)$ is closed set in X for every closed subset V of Y.
(2) π-continuous [8] if $f^{-1}(V)$ is π-closed set in X for every closed subset V of Y.
(3) πg-continuous [8] if $f^{-1}(V)$ is πg-closed set in X for every closed subset V of Y.
(4) wg-continuous [13] if $f^{-1}(V)$ is wg-closed set in X for every closed subset V of Y.
(5) gs-continuous [6] if $f^{-1}(V)$ is gs-closed set in X for every closed subset V of Y.
(6) gsp-continuous [7] if $f^{-1}(V)$ is gsp-closed set in X for every closed subset V of Y.
(7) #rg-continuous[19] if $f^{-1}(V)$ is #rg-closed set in X for every closed subset V of Y.

Definition 2.5: A space X is called $T_{\#rg}$-space [18] if every #rg-closed set in it is closed.

Definition 2.6: [19] Let (X, T) be a topological space and $T_{\#rg}$ = {V ⊆ X : #rg-cl(X\V) = X\V}.

Definition 2.7: [19] A function f : (X, T)→ (Y, S) is called #rg **-**irresolute if $f^{-1}(V)$ is #rg-closed in (X, T) for every #rg-closed subset V of (Y, S).

Definition 2.8: [16]A family S_n of # rg-open subsets of a topological space (X, T) is said to be # rg-open cover of X, if X ⊆ { S_n : n∈ I }.

Definition 2.9: [9] A topological space (X, T) is said to be compact space if every open cover of X has a finite subcover.

3. #pg-CLOSED SETS AND THEIR BASIC PROPERTIES

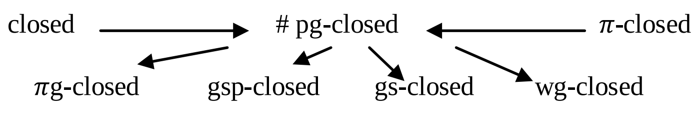
In this section, we introduce and study #pg-closed sets.

Definition 3.1: A subset A of a space X is called *pre semi-open (briefly, *ps-open) if there is a pre-open set U such that U ⊆ A ⊆ cl(U).

Definition 3.2: A subset A of a space X is called pre weakly-closed (briefly, pw-closed) if cl(A) ⊆ U whenever A ⊆ U and U is *ps-open in X.

Definition 3.3: A subset A of a space X is called #pre generalized-closed (briefly, #pg-closed) if cl(A) ⊆ U whenever A ⊆ U and U is pw-open in X.

The interrelations among the set introduced and other related sets are exhibited below:



Proposition 3.1: The union of two #pg-closed subsets of X is also a #pg-closed subset of X.

Proof: Assume that A and B are #pg-closed sets in X. Let A⋃B ⊆ U and U be pw-open in X. Then A ⊆ U and B ⊆ U and U is pw-open in X. Since A and B are #pg-closed sets in X, cl(A) ⊆ U and cl(B) ⊆ U.
Hence, cl(A⋃B) = cl(A)⋃cl(B) ⊆ U. Therefore, A⋃B is a #pg-closed set in X.

Proposition 3.2: The intersection of two #pg-closed subsets of X is also a #pg-closed subset of X.

Proposition 3.3: Let A be a #pg-closed set in X. Then cl(A)\A does not contain any non-empty pw-closed set in X.

Proof Let U be a non-empty pw-closed subset of cl(A)\A. Now A ⊆ X\U and X\U is pw-open in X. Since A is #pg-closed, cl(A) ⊆ X\U. Then U ⊆ X\cl(A). This is a contradiction, since by assumption, U ⊆ cl(A).

Proposition 3.4: Let A be a #pg-closed set in X. Then A is closed iff cl(A)\A is pw-closed.

Proposition 3.5: For every point x of a space X, X\{x} is #pg-closed (or) pw-open.

Proposition 3.6: Let A be a #pg-closed subset of (X, T) such that $A \subseteq B \subseteq cl(A)$. Then B is also a #pg-closed subset of (X, T).

Proof: Let $B \subseteq U$ and U be pw-open in (X, T). Since $A \subseteq B$, $A \subseteq U$ and U is a pw-open set in (X, T). Since A is #pg-closed, $cl(A) \subseteq U$. Then $cl(B) \subseteq cl(cl(A)) = cl(A) \subseteq U$. Hence, B is #pg-closed.

The converse of the above Theorem need not be true as seen from the following Example.

Example 3.1: Let X = {a, b, c, d} and T = {ϕ, X, {b}, {c}, {b, c}, {a, b, c}}. Let A = {a, d} and B = {a, b, d} Then A and B are #pg-closed set in (X, T). But A = {a, d} ⊆ B = {a, b, d} ⊄ cl(A) = {c, d}.

Proposition 3.7: If a subset A of a topological space X is both pw-open and #pg-closed. Then A is a closed set.

Proposition 3.8: Let A be pw-open and #pg-closed in X. Suppose that F is closed in X. Then $A \cap F$ is a #pg-closed set in X.

Proof: Let A be a pw-open and #pg-closed set in X and let F be a closed set in X. By Proposition 3.5, A is closed and so $A \cap F$ is closed. Since every closed set is #pg-closed, $A \cap F$ is a #pg-closed set in X. Hence, $A \cap F$ is a #pg-closed set in X.

Remark 3.1: If a subset A of a topological space X is
- (i) open and g-closed, then A is #pg-closed.
- (ii) w-open and *g-closed, then A is #pg-closed.
- (iii) π-open and πg-closed, then A is #pg-closed.
- (iv) semi-open and w-closed, then A is #pg-closed.
- (v) *ps-open and pw-closed, then A is #pg-closed.
- (vi) open and wg-closed, then A is #pg-closed.

Definition 3.4: A space X is called a $T_{\#pg}$-space if every #pg-closed set in it is closed.

Proposition 3.9: Every $T_{1/2}$-space is $T_{\#pg}$-space.

4. #pg-CONTINUOUS MAPS

In this section, we introduce and study #pg-continuous maps.

Definition 4.1: For a subset A of a space X, #pg-cl(A) = ⋂{F : A ⊆ F, F is #pg-closed in X} is called the #pg-closure of A.

Definition 4.2: Let (X, T) be a topological space and $T_{\#pg}$ = {V ⊆ X: #pg-cl(X\V) = X\V}.

Definition 4.3: A map f : (X, T) → (Y, S) is called #pg-continuous if $f^{-1}(V)$ is #pg-closed in X for every closed subset V in Y.

Definition 4.4: A map f : (X, T) → (Y, S) is called #pg-irresolute if $f^{-1}(V)$ is #pg-closed in (X, T) for every #pg-closed subset V of (Y, S).

Remark 4.1 Let A and B be subsets of (X, T). Then
1. #pg-cl(ϕ) = ϕ and #pg-cl(X) = X.
2. If A ⊆ B, then #pg-cl(A) ⊆ #pg-cl(B).
3. A ⊆ #pg-cl(A).
4. If A is #pg-closed, then #pg-cl(A) = A.

Proposition 4.1: Suppose $T_{\#pg}$ is a topology. If A is #pg-closed in (X, T), then A is closed in (X, $T_{\#pg}$).

Proposition 4.2: A set A ⊆ X is #pg-open iff F ⊆ int(A) whenever F ⊆ A and F is pw-closed.

Proposition 4.3: Let X be a space in which every singleton set is pw-closed. Then f: (X, T)↦ (Y, S) is #pg-continuous iff x ∈ int(f^{-1}(V)) for every open subset V of Y which contains f(x).

Proposition 4.4: Let f : (X, T) → (Y, S) be a map. Let (X, T) and (Y, S) be any two spaces such that $T_{\#pg}$ is a topology on X. Then the following statements are equivalent:

(i) For every subset A of X, f(#pg-cl(A)) ⊆ cl(f(A)).
(ii) f : (X, $T_{\#pg}$) → (Y, S) is continuous.

Proof:
(i)⇒(ii): Suppose (i) holds. Let A be a closed set in Y. By (i), f(#pg-cl(f^{-1}(A))) ⊆ cl(f(f^{-1}(A))) ⊆ cl(A) = A. So #pg-cl(f^{-1}(A)) ⊆ f^{-1}(A). Also f^{-1}(A) ⊆ #pg-cl(f^{-1}(A)). Hence, #pg-cl(f^{-1}(A)) = f^{-1}(A). This implies $(f^{-1}(A))^c \in T_{\#pg}$. Thus, f^{-1}(A) is closed in (X, $T_{\#pg}$). Hence, f is continuous.
(ii)⇒(i): Suppose (ii) holds. Let A be a subset of X. Then cl(f(A)) is closed in Y. Since f : (X, $T_{\#pg}$) → (Y, S) is continuous, f^{-1}(cl(f(A))) is closed in (X, $T_{\#pg}$). By Definition 4.2, #pg-cl(f^{-1}(cl(f(A))) = f^{-1}(cl(f(A))).
Now A ⊆ f^{-1}(f(A)) ⊆ f^{-1}(cl(f(A))), A ⊆ f^{-1}(cl(f(A))), which implies #pg-cl(A) ⊆ #pg-cl f^{-1}(cl(f(A))) = f^{-1}(cl(f(A))). Therefore, f(#pg-cl(A)) ⊆ cl(f(A)).

The interrelations among the map introduced and other related maps are exhibited below:

$$\begin{tikzcd}
\text{continuous} \arrow[r] & \text{\#pg-continuous} \arrow[dl] \arrow[d] \arrow[dr] \arrow[drr] & \pi\text{-continuous} \arrow[l] & \\
\pi g\text{-continuous} & \text{gsp-continuous} & \text{gs-continuous} & \text{wg-continuous}
\end{tikzcd}$$

Proposition 4.5: Let f : (X, T) → (Y, S) be a function. Then the following are equivalent:

(i) f is #pg-continuous.
(ii) The inverse mage of each open set in (Y, S) is #pg-open in (X, T).
(iii) The inverse mage of each closed set in (Y, S) is #pg-closed in (X, T).

Proof:
(i)⇒(ii): Suppose (i) holds. Let V be open in Y. Then Y\V is closed in Y. Since f is #pg-continuous, f^{-1}(Y\V) is #pg-closed in X. But f^{-1}(Y\V) = X\ f^{-1}(V) which is #pg-closed in X. Therefore, f^{-1}(V) is #pg-open in X. Hence, the inverse mage of each open set in (Y, S) is #pg-open in (X, T).

(ii)⇒(iii): Suppose (ii) holds. Let V be a closed set in Y. Then Y\V is open in Y. Since the inverse mage of each open set in (Y, S) is #pg-open in (X, T), f^{-1}(Y\V) is #pg-open. But f^{-1}(Y\V) = X\ f^{-1}(V) which is #pg-open. Therefore, f^{-1}(V) is #pg-closed in X. Hence, the inverse mage of each closed set in (Y, S) is #pg-closed in (X, T).

(iii)⇒(i): Suppose (iii) holds. Let V be a closed set in Y. Since, the inverse image of each closed set in (Y, S) is #pg-closed in (X, T), f^{-1}(V) is #pg-closed in X. Hence, f is #pg-continuous.

Proposition 4.6: If a map f : (X, T) → (Y, S) is #pg-continuous, then f(#pg-cl(A)) ⊆ cl(f(A)) for every subset A of X.

5. #pg-COMPACT SPACES

In this section we introduce and study the concept of #pg-compact spaces.
Definition 5.1: A family {S_n : n∈ I} of #pg-open subsets of a topological space (X, T) is said to be #pg-open cover of X, if X ⊆ ∪{S_n : n∈ I}.

Definition 5.2: A topological space (X, T) is said to be #pg-compact space if every #pg-open cover of X has a finite subcover.

Proposition 5.1: Let (X, T) be a #pg-compact space. Then a #pg-closed subset of (X, T) is a #pg-compact set.

Proof: Let (X, T) be a #pg-compact space, A ⊆ X be a #pg-closed set and {S_n : n∈ I}be #pg-open cover of A. Then A ⊆ ∪S_n. Since A^c is a # pg-open set in X, X ⊆ ∪S_n ∪A^c. Now ∪S_n ∪A^c is a #pg-open cover of X and X is a #pg-compact space. Hence, X has finite subcover, such that X ⊆ S_1 ∪ S_2 , . . . , ∪S_n ∪A^c and A ∩ A^c = ϕ. Thus, A ⊆S_1 ∪ S_2 , . . . , ∪S_n. Therefore, A is a #pg-compact set.

Proposition 5.2: If f: (X, T)→(Y, S) is a #pg -continuous map from a #pg-compact space (X, T) onto a topological space (Y, S), then (Y, S) is a compact space.

Proof: Let {A_k : k∈ I} be any open cover of (Y, S). Since f is a #pg-continuous map, {f^{-1} (A_k): k∈ I} is a #pg-open cover of X. By hypothesis, X has a finite sub cover {f^{-1} (A_{k1}), f^{-1} (A_{k2}), . . . , f^{-1}(A_{kn})}. That is, there exists k1, k2, . . . , kn, such that X ⊆ ∪{f^{-1}(A_{ki}): i=1,2,….n}. Since f is onto, Y = f(X) ⊆ ∪{f(f^{-1}(A_{ki}) : i = 1, 2,…, n}, which equals ∪{A_{ki}: i = 1, 2, …, n}. Therefore, Y is compact.

Proposition 5.3: If f : (X, T)→ (Y, S) is a #pg-irresolute map from a #pg-compact space (X, T) onto a topological space (Y, S), then (Y, S) is a compact space.

Proposition 5.4: If f : (X, T)→ (Y, S) is a #pg-irresolute map from a #pg-compact space (X, T) onto a topological space (Y, S), then (Y, S) is a #pg-compact space.

Proof: Let $\{A_k$: k∈ I} be #pg-open cover of Y. Since f is #pg-irresolute, {f⁻¹ (A_k) : k∈I} is a #pg-open cover of X. By hypothesis, X has a finite sub cover { f⁻¹(A_{k1}), f⁻¹ (A_{k2}), . . . , f⁻¹ (A_{kn})}. That is, there exist k1, k2, . . . , kn, such that X ⊆ ∪{f⁻¹(A_{ki}) : i = 1, 2, …, n}. Since f is onto, Y = f(X) ⊆ ∪{f(f⁻¹(A_{ki})) : i = 1, 2,…, n} = ∪{(A_{ki}) : i = 1, 2,…, n}. Therefore, Y is a #pg-compact space.

Proposition 5.5: Let f : (X, T) →(Y, S) be a #pg-irresolute map and G be a subset of X. If G is #pg-compact relative to X, then the image f(G) is #pg-compact relative to Y.

Proof: Let {A_k : k∈I} be a collection of #pg-open sets in Y, such that f(G) ⊆ ∪{A_k : k∈I}. Then G ⊆ ∪{f⁻¹(A_k) : k∈ I}, where f⁻¹(A_k) is a #pg-open set in X for each k∈ I. Since G is #pg-compact, there exists {A_1, A_2,..., A_n} such that G ⊆ ∪{f⁻¹(A_k) : k = 1, 2,…, n}. Then f(G) ⊆ ∪{f(f⁻¹(A_k)) : k = 1,2,…,n}. Hence, f(G) ⊆ ∪{(A_k) : k = 1, 2,..., n}. Therefore, f(G) is a #pg-compact space relative to Y.

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