A VIEW ON SOFT $\alpha B$-CONTINUITY ON SOFT TOPOLOGICAL SPACES

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ABSTRACT
In this paper, the concept of soft $\alpha B$-open set is introduced. In this connection, the interrelations among the sets are established. Also the interrelation among soft $\alpha B$-continuity is discussed and counter examples are provided wherever necessary.

Keywords: Soft $\alpha B$-open sets, Soft $\alpha B$-continuous functions.

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1. INTRODUCTION
In 1999, Molodtsov [1] introduced the concept of soft sets with adequate parametrization for dealing with uncertainties. Later Muhammad Shabir and Munazza Naz [2] introduced the concept of soft topological spaces which are defined over an initial universe with a fixed set of parameters. Naime Tozlu and Saziye yükse1 [3, 4] introduced the concepts of soft A-sets, soft B-sets and soft C-sets. In this paper, the concept of soft $\alpha B$-open set is introduced. In this connection, the interrelations among the sets are established. Also the interrelations among soft $\alpha B$-continuity are discussed and counter examples are provided wherever necessary.

2. PRELIMINARIES
Let $U$ be an initial universe set and $E$ be a set of parameters. Let $P(U)$ denote the power set of $U$, and let $A \subseteq E$.

Definition 2.1: [1] A pair $(F, A)$ is called a soft set over $U$, where $F$ is a mapping given by $F: A \rightarrow P(U)$. In other words, a soft set over $U$ is a parameterized family of subsets of the universe $U$. For a particular $e \in A$, $F(e)$ may be considered the set of $e$-approximate elements of the soft set $(F, A)$.

Definition 2.2: [5] For two soft sets $(F, A)$ and $(G, B)$ over a common universe $U$,

(i) $(F, A)$ is a soft subset of $(G, B)$, denoted by $(F, A) \subseteq (G, B)$, if $A \subseteq B$ and $\forall e \in A, F(e) \subseteq G(e)$.

(ii) $(F, A)$ is said to be a soft superset of $(G, B)$, if $(G, B)$ is a soft subset of $(F, A)$, denoted by $(F, A) \supseteq (G, B)$.

Definition 2.3: [5] Two soft sets $(F, A)$ and $(G, B)$ over a common universe $U$ are said to be soft equal if $(F, A)$ is a soft subset of $(G, B)$ and $(G, B)$ is a soft subset of $(F, A)$.

Definition 2.4: [5] A soft set $(F, A)$ over $U$ is said to be a null soft set, denoted by $\Phi$, if $\forall e \in A, F(e) = \emptyset$.

Definition 2.5: [5] A soft set $(F, A)$ over $U$ is said to be an absolute soft set, denoted by $U$, if $\forall e \in A, F(e) = U$.

Definition 2.6: [5] The union of two soft sets $(F, A)$ and $(G, B)$ over the common universe $U$ is the soft set $(H, C)$, where $C = A \cup B$ and for all $\forall e \in C$,
This relationship is written as $(F,A) \cup (G,B) = (H,C)$.

**Definition 2.7:** [5] The intersection of two soft sets $(F,A)$ and $(G,B)$ over the common universe $U$ is the soft set $(H,C)$, where $C = A \cap B$ and $\forall e \in C$, $H(e) = F(e) \cap G(e)$. This relationship is written as $(F,A) \cap (G,B) = (H,C)$.

**Definition 2.8:** [5] The complement of a soft set $(F,A)$ denoted by $(F,A)'$ is defined by $(F,A)' = (F',A)$, where $F': A \to \mathcal{P}(U)$ is a mapping given by $F'(e) = U - F(e), \forall e \in A$. $F'$ is called the soft complement function of $F$. Clearly, $(F')'$ is the same as $F$ and $((F,A)')' = (F,A)$.

**Example 2.1:** [4] Let $X = \{x_1, x_2, x_3, x_4\}$, $E = \{e_1, e_2\}$ be a mapping from $E$ to $P(X)$ defined by,

$(F_1, E) = \{(e_1, \{x_1\}), (e_2, \{x_1\})\},$

$(F_2, E) = \{(e_1, \{x_2\}), (e_2, \{x_2\})\},$

$(F_3, E) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_2\})\},$

$(F_4, E) = \{(e_1, \{x_1, x_2, x_3\}), (e_2, \{x_1, x_2, x_3\})\},$

$(F_5, E) = \{(e_1, \{x_1, x_2, x_4\}), (e_2, \{x_1, x_2, x_3\})\},$

$(F_6, E) = \{(e_1, \{x_1, x_2\}), (e_2, \emptyset)\},$

$(F_7, E) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_2\})\},$

$(F_8, E) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_2, x_3\})\},$

$(F_9, E) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_2, x_3\})\},$

$(F_{10}, E) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_2, x_3\})\},$

$(F_{11}, E) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_3\})\}$

Then $\tau = \{\emptyset, X, (F_1, E), (F_2, E), ..., (F_{11}, E)\}$ is a soft topological space over $X$.

**Definition 2.10:** [5] Let $(X, \tau, E)$ be a soft topological space over $X$ and $(F,E)$ be a soft set over $X$. Then,

(i) the soft closure of $(F,E)$ is the soft set $\text{scl}(F,E) = \cap \{(G,E)/(G,E)$ is soft closed and $(F,E) \supseteq (G,E)\}$.

(ii) the soft interior of $(F,E)$ is the soft set $\text{sint}(F,E) = \cap \{(H,E)/(H,E)$ is soft open and $(H,E) \supseteq (F,E)\}$.

**Definition 2.12:** [3] Let $(X, \tau, E)$ be a soft topological space. A soft set $(F,E)$ is called soft t-open set in $X$, if $\text{sint}(\text{scl}(F,E)) = \text{sint}(F,E)$.

(iii) the soft interior of $(F,E)$ is the soft set $\text{sint}(F,E) = \cap \{(H,E)/(H,E)$ is soft open and $(H,E) \supseteq (F,E)\}$.

**Definition 2.13:** [3] Let $(X, \tau, E)$ be a soft topological space. A soft set $(F,E)$ is called soft B-open(closed) set in $X$, if $(F,E) = (G,E) \cap (H,E)$, where $(G,E)$ is a soft open(closed) set and $(H,E)$ is a soft t-open (closed) set.

**Definition 2.14:** [3] Let $(X, \tau, E)$ be a soft topological space over $X$ and $(F,E)$ be a soft set over $X$. Then,

(i) the soft B-closure of $(F,E)$ is the soft set $\text{sBcl}(F,E) = \cap \{(G,E)/(G,E)$ is soft B-closed and $(F,E) \supseteq (G,E)\}$.

(ii) the soft B-interior of $(F,E)$ is the soft set $\text{sBint}(F,E) = \cap \{(H,E)/(H,E) \supseteq (F,E)\}$.

**Definition 2.15:** [4] Let $(X, \tau, E)$ be a soft topological space over $X$ and $(F,E)$ be a soft set over $X$. Then $(F,E)$ is said to be $\alpha^*$-set if $\text{sint}(\text{scl}(F,E)) = \text{sint}(F,E)$.

**Remark 2.1:** [4] Let $(X, \tau, E)$ be a soft topological space. The notion of soft $\alpha$-open sets is different from the soft $\alpha^*$-sets.
Example 2.2: [4] Let \( X = \{x_1, x_2, x_3, x_4\}, E = \{e_1, e_2\} \). Let us take the soft topology \( \tau \) on \( X \) as in example 2.1 and let \((G,E) = \{(e_1, \{x_1\} ), (e_2, \{x_2\} )\}\) be a soft set over \( X \). Here \((G,E)\) is a soft \( \alpha \)-set. Also, \((G,E)\) is a soft C-open set since every soft \( \alpha \)-set is a soft C-open set. Also, \((G,E)\) is not a soft open set.

Example 2.3: [4] Let \( X = \{x_1, x_2, x_3\}, E = \{e_1, e_2\} \) and \( F \) is a mapping from \( E \) to \( P(X) \) defined by, \((F,E) = \{(e_1, \{x_1\} ), (e_2, \{x_2\} )\}\) be a soft set over \( X \). Then \( \tau = \{\Phi, X, (F,E)\} \) is soft topology over \( X \). Let \((G,E) = \{(e_1, \{x_1, x_2\} ), (e_2, \{x_2\} )\}\) be a soft set over \( X \). Here \((G,E)\) is a soft \( \alpha \)-open set but not a soft \( \alpha \)-open set.

Definition 2.17: [3] Let \((X, \tau_1, E)\) and \((X, \tau_2, E)\) be any two soft topological spaces. A function \( f \) from a soft topological space \((X, \tau_1, E)\) to another soft topological space \((X, \tau_2, E)\) is said to be soft \( \alpha \)-continuous function iff the inverse image of every soft \( \alpha \)-open(closed) set in \((X, \tau_1, E)\) to another soft topological space \((X, \tau_2, E)\) is said to be soft \( \alpha \)-continuous function iff the inverse image of every soft open(closed) set in \((X, \tau_1, E)\) to another soft topological space \((X, \tau_2, E)\) is said to be soft \( \alpha \)-continuous function.

Definition 2.18: [3] Let \((X, \tau_1, E)\) and \((X, \tau_2, E)\) be any two soft topological spaces. A function \( f \) from a soft topological space \((X, \tau_1, E)\) to another soft topological space \((X, \tau_2, E)\) is said to be soft \( \alpha \)-continuous function iff the inverse image of every soft \( \alpha \)-open(closed) set in \((X, \tau_1, E)\) to another soft topological space \((X, \tau_2, E)\) is said to be soft \( \alpha \)-continuous function.

Definition 2.19: [4] Let \((X, \tau_1, E)\) and \((X, \tau_2, E)\) be any two soft topological spaces. A function \( f \) from a soft topological space \((X, \tau_1, E)\) to another soft topological space \((X, \tau_2, E)\) is said to be soft \( \alpha \)-continuous function iff the inverse image of every soft \( \alpha \)-open(closed) set in \((X, \tau_1, E)\) to another soft topological space \((X, \tau_2, E)\) is said to be soft \( \alpha \)-continuous function.

Remark 2.2: [4] Let \((X, \tau, E)\) be a soft topological space over \( X \) and \((F, E)\) be a soft set over \( X \). Then every soft \( \alpha \)-open set is a soft \( \alpha \)-open set.

3. SOFT \( \alpha \)-OPEN SET

Definition 3.1: Let \((X, \tau, E)\) be a soft topological space. A soft set \((F,E)\) is called soft \( \alpha \)-open(closed) set in \( X \), if \((F,A) = (G,E) \cap (H,E)\), where \((G,E)\) is a soft \( \alpha \)-open(closed) set and \((H,E)\) is a soft \( \alpha \)-open(closed) set.

Definition 3.2: Let \((X, \tau, E)\) be a soft topological space over \( X \) and \((F, E)\) be a soft set over \( X \). Then,
- \((i)\) the soft \( \alpha \)-closure of \((F,E)\) is the soft set \( s\alpha Bcl(F,E) = \cap \{(G,E)/(G,E)\ is \ soft \ \alpha \text{-}\text{closed} \} \).
- \((ii)\) the soft \( \alpha \)-interior of \((F,E)\) is the soft set \( s\alpha Bint(F,E) = \cap \{(H,E)/(H,E)\ is \ soft \ \alpha \text{-}\text{open} \} \).

Proposition 3.1: \((i)\) Finite intersection of soft \( \alpha \)-open sets is a soft \( \alpha \)-open set. \((ii)\) Finite union of soft \( \alpha \)-closed sets is a soft \( \alpha \)-closed set.

Proof:
- \((i)\) Let \((F_i, A_i)\) be a soft \( \alpha \)-open set. Then \((F_i, A_i) = (G_i, B_i) \cap (H_i, C_i)\), where \((G_i, B_i)\) is a soft \( \alpha \)-open set and \((H_i, C_i)\) is a soft \( \alpha \)-open set. Now,
\[
\bigcap_{i=1}^{n}(F_i, A_i) = \bigcap_{i=1}^{n} \{(G_i, B_i) \cap (H_i, C_i)\}
\]

Hence finite intersection of soft \( \alpha \)-open sets is a soft \( \alpha \)-open set.

- \((ii)\) Let \((F_i, A_i)\) be a soft \( \alpha \)-closed set. Then \((F_i, A_i) = (G_i, B_i) \cup (H_i, C_i)\), where \((G_i, B_i)\) is a soft \( \alpha \)-closed set and \((H_i, C_i)\) is a soft \( \alpha \)-closed set. Now,
\[
\bigcup_{i=1}^{n}(F_i, A_i) = \bigcup_{i=1}^{n} \{(G_i, B_i) \cup (H_i, C_i)\}
\]

Hence finite union of soft \( \alpha \)-closed sets is a soft \( \alpha \)-closed set.

Proposition 3.2: Let \((X, \tau, E)\) be a soft topological space over \( X \). For any two soft sets \((F, A)\) and \((G, B)\) the following statements are valid.
- \((i)\) \( s\alpha Bcl(F, \Phi) = (F, \Phi)\).
- \((ii)\) \( s\alpha Bcl(F, X) = (F, X)\).
- \((iii)\) \( s\alpha Bcl(F, A) \supseteq (F, A)\).
- \((iv)\) If \((F, A) \supseteq (G, B)\), then \( s\alpha Bcl(F, A) \subseteq s\alpha Bcl(G, B)\).
- \((v)\) \( s\alpha Bcl(s\alpha Bcl(F, A)) = s\alpha Bcl(F, A)\).
- \((vi)\) \( s\alpha Bcl(F, A) \cap (G, B) \subseteq (s\alpha Bcl(F, A)) \cap (s\alpha Bcl(G, B))\).
- \((vii)\) \( s\alpha Bcl((F, A) \cup (G, B)) = (s\alpha Bcl(F, A)) \cup (s\alpha Bcl(G, B))\).
Proposition 3.3: Let \((X, \tau, E)\) be a soft topological space over \(X\). For any two soft sets \((F,A)\) and \((G,B)\) the following statements are valid.

(i) \(s\alpha B\text{Int}(F,A) \subseteq (F,A)\).
(ii) If \((F,A) \subseteq (G,B)\), then \(s\alpha B\text{Int}(F,A) \subseteq s\alpha B\text{Int}(G,B)\).
(iii) \((s\alpha B\text{Cl}(F,A)) = s\alpha B\text{Cl}(F,A)\).
(iv) \(s\alpha B\text{Int}(F,A) = (s\alpha B\text{Int}(F,A))'\).
(v) \(s\alpha B\text{Int}(F,A) \cap (G,B) = (s\alpha B\text{Int}(F,A)) \cap (s\alpha B\text{Int}(G,B))\).
(vi) \(s\alpha B\text{Int}(F,A) \cup (G,B) = (s\alpha B\text{Int}(F,A)) \cup (s\alpha B\text{Int}(G,B))\).

Proposition 3.4: Every soft \(B\)-open set is a soft \(\alpha B\)-open set.

Proof: Let \((F,A)\) be a soft \(B\)-open set. Then, \((F,A) = (G,B) \cap (H,C)\), where \((G,B)\) is a soft open set and \((H,C)\) is a soft t-open set. Since every soft open set is a soft \(\alpha\)-open set, \((G,B)\) is a soft \(\alpha B\)-open set. Hence \((F,A)\) is a soft \(\alpha B\)-open set.

Remark 3.1: The converse of the proposition 3.4 need not be true.

Example 3.1: Let \(X\) be the set of diabetes patients in a hospital and \(E\) be the set of parameters. Let \(X = \{p_1,p_2,p_3\}\) and \(E = \{e_1,e_2,e_3,e_4,e_5,e_6\}\) where \(e_1,e_2,e_3,e_4,e_5,e_6\) be the set of parameters which stands for 'Polyurea', 'Fatigue', 'Polydipsea', 'Polyphagia', 'Weightloss', 'Slow healing of wounds' respectively. Let \(A = \{e_1,e_2\}\). Let \(F\) be a mapping \(A\) to \(P(X)\) defined by, \((F,A) = \{(e_1,\{p_1\}),(e_2,\{p_2\})\}\) is a soft set over \(X\). Then, \(\tau = \{\emptyset, X\}\) is a soft topological space over \(X\). Let \((G,A) = \{(e_1,\{p_1,p_2\}),(e_2,\{p_2\})\}\) and \((H,A) = \{(e_1,\{p_1\}),(e_2,\{\emptyset\})\}\) are soft sets over \(X\). Here \((G,A)\) is a soft \(\alpha\)-open set but not a soft open set and \((F,A)'\) is a soft t-open set. Then \((H,A)\) is a soft \(\alpha B\)-open set but not a soft \(B\)-open set.

Proposition 3.5: Every soft \(\alpha B\)-open set is a soft \(\alpha C\)-open set.

Proof: Let \((F,A)\) be a soft \(\alpha B\)-open set. Then, \((F,A) = (G,B) \cap (H,C)\), where \((G,B)\) is a soft \(\alpha\)-open set and \((H,C)\) is a soft t-open set. Since every soft t-open set is a soft \(\alpha C\)-open set, \((H,C)\) is a soft \(\alpha C\)-open set. Hence \((F,A)\) is a soft \(\alpha B\)-open set.

Remark 3.2: The converse of the proposition 3.5 need not be true.

Example 3.2: Let \(X\) be the set of diabetes patients in a hospital and \(E\) be the set of parameters. Let \(X = \{p_1,p_2,p_3\}\) and \(E = \{e_1,e_2,e_3,e_4,e_5,e_6\}\) where \(e_1,e_2,e_3,e_4,e_5,e_6\) be the set of parameters which stands for 'Polyurea', 'Fatigue', 'Polydipsea', 'Polyphagia', 'Weightloss', 'Slow healing of wounds' respectively. Let \(A = \{e_1,e_2\}\). Let \(F,G,H,I,J\) be a mapping \(A\) to \(P(X)\) defined by,

\[
\begin{align*}
(F,A) & = \{(e_1,\{p_1\}),(e_2,\{p_1,p_3\})\}, \\
(G,A) & = \{(e_1,\{p_1\}),(e_2,\{p_1\})\}, \\
(H,A) & = \{(e_1,\{p_2\}),(e_2,\{p_2\})\}, \\
(I,A) & = \{(e_1,\{p_1,p_2\}),(e_2,\{p_2\})\} \text{ and} \\
(J,A) & = \{(e_1,\{p_1,p_2\}),(e_2,\{X\})\}
\end{align*}
\]

are soft sets over \(X\). Then, \(\tau = \{\emptyset, X, (F,A),(G,A),(H,A),(I,A),(J,A)\}\) is a soft topological space over \(X\). Let \((K,A) = \{(e_1,\{p_1\}),(e_2,\{p_2\})\}\) be a soft set over \(X\). Here \((K,A)\) is a soft \(\alpha\)-set and soft \(\alpha\)-open set but not a soft t-open set. Therefore \((K,A)\) is a soft \(\alpha C\)-open set but not a soft \(\alpha B\)-open set.

Remark 3.3: The interrelations among the sets introduced are given clearly in the following diagram.

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soft B-open set ▼
soft aB-open set ▼
soft aC-open set
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4. SOFT aB-CONTINUOUS FUNCTION

Definition 4.1: Let \((X, \tau_1, E)\) and \((X, \tau_2, E)\) be any two soft topological spaces. A function \(f\) from a soft topological space \((X, \tau_1, E)\) to another soft topological space \((X, \tau_2, E)\) is said to be soft \(aB\)-continuous function iff the inverse image of every soft open(closed) set in \((X, \tau_2, E)\) is a soft \(aB\)-open(closed) set in \((X, \tau_1, E)\).

Definition 4.2: Let \((X, \tau_1, E)\) and \((X, \tau_2, E)\) be any two soft topological spaces. A function \(f\) from a soft topological space \((X, \tau_1, E)\) to another soft topological space \((X, \tau_2, E)\) is said to be soft \(\alpha\)-continuous function iff the inverse image of every soft open(closed) set in \((X, \tau_2, E)\) is a soft \(\alpha\)-open(closed) set in \((X, \tau_1, E)\).
Proposition: 4.1: Every soft B-continuous is a soft αB-continuous function.

Proof: Let \((X, \tau_1, E)\) and \((X, \tau_2, E)\) be any two soft topological spaces. Let \(f: (X, \tau_1, E) \to (X, \tau_2, E)\) be a soft B-continuous function. Let \((F, A)\) be a soft open set in \((X, \tau_2, E)\). Since \(f\) is soft B-continuous function, the inverse image \((F, A)\) is soft B-open set in \((X, \tau_1, E)\). Since every soft B-open set is a soft αB-open set, \(f^{-1}(F, A)\) is soft αB-open set in \((X, \tau_1, E)\). Hence \(f\) is a soft αB-continuous function.

Remark: 4.1: The converse of the proposition 4.1 need not be true.

Example: 4.1: Let \(\tau_1 = \{\Phi, X, (F, A), (G, A), (H, A), (I, A), (J, A)\}\) and \(\tau_2 = \{\Phi, \tilde{X}, (F, A)\}\). Let \((X, \tau_1, E)\) and \((X, \tau_2, E)\) be any two topological spaces over \(X\) and \(Y\). Let \(f: (X, \tau_1, E) \to (X, \tau_2, E)\) be a mapping. Then \(f\) is a soft αB-continuous function in \((X, \tau_1, E)\) since in example 3.1, \((H, A)\) is a soft αB-open set but not a soft B-open set in \((X, \tau_1, E)\). Therefore \(f\) is a soft αB-continuous function but not a soft B-continuous function.

Proposition: 4.2: Every soft αB-continuous is a soft αC-continuous function.

Proof: Let \((X, \tau_1, E)\) and \((X, \tau_2, E)\) be two soft topological spaces. Let \(f: (X, \tau_1, E) \to (X, \tau_2, E)\) be a soft αB-continuous function. Let \((F, A)\) be a soft open set in \((X, \tau_2, E)\). Since \(f\) is soft αB-continuous function, the inverse image \((F, A)\) is soft αB-open set in \((X, \tau_1, E)\). Since every soft αB-open set is a soft αC-open set, \(f^{-1}(F, A)\) is soft αC-open set in \((X, \tau_1, E)\). Hence \(f\) is a soft αC-continuous function.

Remark: 4.2: The converse of the proposition 4.2 need not be true.

Example: 4.2: Let \(\tau_1 = \{\Phi, \tilde{X}, (F_1, E), (F_2, E), \ldots, (F_n, E)\}\) defined as in example 2.2 and \(\tau_2 = \{\Phi, \tilde{X}, (G, E)\}\). Let \((X, \tau_1, E)\) and \((X, \tau_2, E)\) be two topological spaces over \(X\) and \(Y\). Let \(f: (X, \tau_1, E) \to (X, \tau_2, E)\) be a mapping. Then \(f\) is a soft αC-continuous function in \((X, \tau_1, E)\) since in example 3.2, \((H, A)\) is a soft αC-open set but not a soft αB-open set in \((X, \tau_1, E)\). Therefore \(f\) is a soft αC-continuous function but not a soft αB-continuous function.

Remark: 4.3: By Remark 2.1 and examples 2.2 and 2.3, any soft α-open set need not be soft αC-open set and any soft αC-open set need not be soft α-continuous set. Hence, soft α-continuous function and soft αC-continuous function are independent as shown by the following Example 4.3 and Example 4.4.

Example: 4.3: Let \(\tau_1 = \{\Phi, \tilde{X}, (F_1, E), (F_2, E), \ldots, (F_n, E)\}\) where \((G, E)\) is a defined as in example 2.2. Let \((X, \tau_1, E)\) and \((X, \tau_2, E)\) be two topological spaces over \(X\) and \(Y\). Let \(f: (X, \tau_1, E) \to (X, \tau_2, E)\) be a mapping. Then \(f\) is a soft αC-continuous function in \((X, \tau_1, E)\) since in example 2.2, \((G, E)\) is a soft αC-open set but not a soft α-open set in \((X, \tau_1, E)\). Therefore \(f\) is a soft αC-continuous function but not a soft α-continuous function.

Example: 4.4: Let \(\tau_1 = \{\Phi, \tilde{X}, (F_1, E)\}\) and \(\tau_2 = \{\Phi, \tilde{X}, (G, E)\}\) where \((F, E)\) and \((G, E)\) is a defined as in example 2.3. Let \((X, \tau_1, E)\) and \((X, \tau_2, E)\) be two topological spaces over \(X\) and \(Y\). Let \(f: (X, \tau_1, E) \to (X, \tau_2, E)\) be a mapping. Then \(f\) is a soft α-continuous function in \((X, \tau_1, E)\) since in example 2.3, \((G, E)\) is a soft α-open set but not a soft αC-open set in \((X, \tau_1, E)\). Therefore \(f\) is a soft α-continuous function but not a soft αC-continuous function.

Remark: 4.4: The interrelations among the functions introduced are given clearly in the following diagram.

![Diagram showing the interrelations among soft B-continuous, soft αB-continuous, and soft αC-continuous functions]

Theorem: 4.1: Let \((X, \tau_1, E)\) and \((X, \tau_2, E)\) be any two soft topological spaces. If \(f\) is any mapping from \((X, \tau_1, E)\) to \((X, \tau_2, E)\) then the conditions below are equivalent.

(i) The function \(f\) is soft αB-continuous function.

(ii) The inverse of every soft α-closed set is a soft αB-closed set.

Proof: The proof follows from the Definition 4.1.

Theorem: 4.2: Let \((X, \tau_1, E)\) and \((X, \tau_2, E)\) be any two soft topological spaces. If \(f\) is any mapping from \((X, \tau_1, E)\) to \((X, \tau_2, E)\) then the following conditions are equivalent.

(i) \(f\) is a soft αB-continuous function.

(ii) For every soft set \((F, A)\) of \((X, \tau_1, E)\), \(f(\text{sBInt}(F, A)) \supseteq \text{sint}(f(F, A))\).

(iii) For every soft set \((F, A)\) of \((X, \tau_2, E)\), \(f^{-1}(\text{sInt}(F, A)) \subseteq \text{sBInt}(f^{-1}(F, A))\).
Proof:

(i) \(\Rightarrow\) (ii): Let \((F,A)\) be a soft set in \((X, \tau_1, E)\). Then \(f(F,A)\) is a soft set in \((X, \tau_2, E)\). Since \(f\) is a soft \(aB\)-continuous function, \(\text{sint}(f(F,A))\) is a soft \(aB\)-open set in \((X, \tau_2, E)\). By hypothesis, \(f^{-1}(\text{sint}(f(F,A)))\) is a soft \(aB\)-open set in \((X, \tau_1, E)\). Now,
\[
(F,A) \supset f^{-1}(\text{sint}(f(F,A)))
\]
\[
\Rightarrow \text{sabInt}(F,A) \supset \text{sabInt}(f^{-1}(\text{sint}(f(F,A))))
\]
\[
\supset f^{-1}(\text{sint}(F,A))
\]
\[
\Rightarrow f(\text{sabInt}(F,A)) \supset \text{sint}(f(F,A)).
\]

Hence \(f(\text{sabInt}(F,A)) \supset \text{sint}(f(F,A))\).

(ii) \(\Rightarrow\) (iii): Let \((F,A)\) be a soft open set in \((X, \tau_2, E)\). Now, \(f^{-1}(F,A)\) be a soft open set in \((X, \tau_1, E)\).

By (ii), \(f(\text{sabInt}(f^{-1}(F,A))) \subset \text{sint}(f(f^{-1}(F,A)))\)
\[
\Rightarrow \text{sabInt}(f^{-1}(F,A)) \subset f^{-1}(\text{sint}(F,A)).
\]

Hence \(\text{sabInt}(f^{-1}(F,A)) \subset f^{-1}(\text{sint}(F,A))\).

(iii) \(\Rightarrow\) (i): Let \((F,A)\) be a soft open set in \((X, \tau_2, E)\). By hypothesis,
\[
\text{sabInt}(f^{-1}(F,A)) \supset f^{-1}(\text{sint}(F,A))
\]
\[
= f^{-1}(\text{sint}(F,A))
\]
\[
\Rightarrow \text{sabInt}(f^{-1}(F,A)) = f^{-1}(\text{sint}(F,A)).
\]

Thus, \(f^{-1}(F,A)\) is a soft \(aB\)-open set in \((X, \tau_1, E)\). Therefore \(f\) is a soft \(aB\)-continuous function.

Theorem: 4.3: Let \((X, \tau_1, E)\) and \((X, \tau_2, E)\) be any two soft topological spaces. If \(f\) is any mapping from \((X, \tau_1, E)\) to \((X, \tau_2, E)\) then the following conditions are equivalent.

(i) \(f\) is a soft \(aB\)-continuous function.

(ii) For every soft set \((F,A)\) of \((X,\tau_1,E)\), \(f(\text{sabcl}(F,A)) \subset \text{scl}(f(F,A))\).

(iii) For every soft set \((F,A)\) of \((X,\tau_2,E)\), \(\text{sabclf}(f^{-1}(F,A)) \subset f^{-1}(\text{scl}(F,A))\).

Proof:

(i) \(\Rightarrow\) (ii): Let \((F,A)\) be a soft set in \((X, \tau_1, E)\). Then \(f(F,A)\) is a soft set in \((X, \tau_2, E)\). Now, \(\text{scl}(f(F,A))\) is a soft closed set in \((X, \tau_2, E)\). By hypothesis, \(f^{-1}(\text{scl}(f(F,A)))\) is a soft \(aB\)-closed set in \((X, \tau_1, E)\). Hence,
\[
(F,A) \supset f^{-1}(\text{scl}(f(F,A)))
\]
\[
\Rightarrow \text{sabcl}(F,A) \supset \text{sabcl}(f^{-1}(\text{scl}(f(F,A))))
\]
\[
= f^{-1}(\text{scl}(f(F,A)))
\]
\[
\Rightarrow f(\text{sabcl}(F,A)) = \text{scl}(f(F,A)).
\]

Hence \(f(\text{sabcl}(F,A)) = \text{scl}(f(F,A))\).

(ii) \(\Rightarrow\) (iii): Let \((F,A)\) be a soft closed set in \((X, \tau_2, E)\). Now \(f^{-1}(F,A)\) be a soft closed set in \((X, \tau_1, E)\).

By (ii), \(f(\text{sabcl}(f^{-1}(F,A))) \subset \text{scl}(f(f^{-1}(F,A)))\)
\[
\Rightarrow \text{sabcl}(f^{-1}(F,A)) \subset f^{-1}(\text{scl}(F,A)).
\]

Hence \(\text{sabcl}(f^{-1}(F,A)) \subset f^{-1}(\text{scl}(F,A))\).

(iii) \(\Rightarrow\) (i): Let \((F,A)\) be a soft closed set in \((X, \tau_2, E)\). By hypothesis,
\[
\text{sabcl}(f^{-1}(F,A)) \supset f^{-1}(\text{scl}(F,A))
\]
\[
= f^{-1}(\text{scl}(F,A))
\]
\[
\Rightarrow \text{sabcl}(f^{-1}(F,A)) = f^{-1}(\text{scl}(F,A)).
\]

Therefore, \(\text{sabcl}(f^{-1}(F,A)) = f^{-1}(\text{scl}(F,A))\)
\[
= f^{-1}(\text{scl}(F,A))
\]

Thus \(f^{-1}(F,A)\) is a soft \(aB\)-closed set in \((X, \tau_1, E)\). Therefore \(f\) is a soft \(aB\) continuous function.

Theorem: 4.4: Let \((X, \tau_1, E)\), \((X, \tau_2, E)\) and \((X, \tau_3, E)\) be any three soft topological spaces. A function \(f: (X, \tau_1, E) \to (X, \tau_2, E)\) is a soft \(aB\)-continuous function and \(g: (X, \tau_2, E) \to (X, \tau_3, E)\) is a soft continuous function. Then \(g \circ f: (X, \tau_1, E) \to (X, \tau_3, E)\) is a soft \(aB\)-continuous function.

Proof: Let \((F,A)\) be a soft open set in \((X, \tau_2, E)\). Since \(g\) is a soft continuous function, \(g^{-1}(F,A)\) is a soft open set in \((X, \tau_2, E)\). Also since \(f\) is a soft \(aB\)-continuous function, \(f^{-1}(g^{-1}(F,A))\) is a soft \(aB\)-open set in \((X, \tau_1, E)\). Hence \(g \circ f\) is a soft \(aB\)-continuous function.
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