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# MINIMAL IDEAL αψ SUBMAXIMAL IN MINIMAL STRUCTURE SPACES

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***ABSTRACT***

*In this paper we introduce the notion of mIαψ locally closed set, mIαψ closed and mIαψ locally m∗ closed sets in ideal minimal spaces and investigate some of their properties. Further we study the mIαψ submaximal space and derive some of their properties.*

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## 1. INTRODUCTION

O.Njastad [13] introduced the concept of α closed sets in topological spaces. The notion of ψ closed sets are introduced by M.K.R.S.Veerakumar [17]. R.Devi *et al.*,[5] intoduced the concept if αψ closed sets in topological spaces. The notion of minimal structures and minimal spaces as a generalization of topology and topological spaces were introduced in [11, 12]. Some other results about minimal spaces can be seen in [1, 2, 15]. Ideal in topological spaces were introduced by Kuratowski [10] and Vaidyanathaswamy [18]. Jankovic and Hamlet [9] gave the notion on I-open sets in ideal topological spaces. O.B.Ozbakir and E.D.Yildirim [14] have defined the minimal local function A∗ m in ideal minimal spaces (X, M, I).In Maki [11] *et al.*, introduced the notion of minimal structure and minimal spaces as a generalization of topological spaces on a given non empty set. A subfamily M of the power set P(X) of a non empty set X is a minimal structure, if $\phi$, X ∈ M and (X, M) is called a minimal spaces. A subset A of X is said to be m open [11] if A ∈ X, then $m - int(A) = \cup\{U : U \subset A, U \in M\}$. A minimal spaces (X, M) has the property [U] and property [I] if " the arbitrary union of m open sets is m open" and "the any finite intersection of m open sets is m open" [15] respectively. Hewit[8] introduced the submaximallity in general topology. Arhangel'skii *et al.*, [1] a systematic formulation on submaximallity in topology. The concept of ideal submaximal space was investicated by Erdal Akici *et al.*, [7]. Parimala *et al.* [16] studied the concept of minimal locally closed set in minimal structure spaces. R.Chitra [4] introduced a new concept of m∗ extremely disconnected ideal minimal space. M.Parimala *et al.*, [17] studied submaximal in terms of ideal minimal spaces. In this paper, we introduce the notion of mIαψ locally m∗ closed, mIαψ submaximal space and we discussed about their properties and relationships.

## 2. PRELIMINARIES

**Definition 2.1:** [14] Let (X, M) be a minimal spaces with an ideal I on X and (.) ∗ m: P(X) → P(X) be a set operator. For a subset A ⊂ X, $A^*_m(I, M) = \{x \in X : U_m \cap A \notin I \text{ for every } U_m \in U_m(x)\}$ is called the minimal local function of A. Throughout this paper A∗ m(I, M) is denoted by simply $A^*_m$.

**Lemma 2.2:** [14] Let (X, M) be a minimal structure space with I, I ′ ideals on X and A⊂ X and B ⊂ X. Then

(a) $A^*_m \cup B^*_m \subset (A \cup B)^*_m$
(b) $A^*_m = m - cl(A^*_m) \subset m - cl(A)$
(c) $(A^*_m)^*_m \subset A^*_m$
(d) $A \subset B \Rightarrow A^*_m \subset B^*_m$
(e) $I \subset I' \Rightarrow A^*_m(I') \subset A^*_m(I)$

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**Lemma 2.3:** [14] The set operator m-cl* satisfies the following conditions:

(1) $A \subset m - cl^*(A)$
(2) $m - cl^*(\phi) = \phi$ and $mcl^*(X) = X$
(3) If $A \subset B$ then $m - cl^*(A) \subset m - cl^*(B)$
(4) $m - cl^*(A) \cup m - cl^*(B) \subset m - cl^*(A \cap B)$

**Lemma 2.4:** [14] If (X, M) has property [I], then $mcl^*(m-cl^*(A)) = m-cl^*(A)$ and $m- cl^*(A) \cup m - cl^*(B) = m-cl^*(A \cup B)$.

**Lemma 2.5:** [14] If (X, M) has property [U], and $I = \phi$ then $A^*_m = m - cl(A)$. In this case $m - cl^*(A) = m - cl(A)$

**Lemma 2.6:** [14] Let (X, M, I) be an ideal minimal spaces and $A \subset X$. If A is m* dense in itself, then $A^*_m = mcl(A^*_m) = mcl(A) = mcl^*(A)$.

**Definition 2.7:** Let (X, M) be a minimal space is called

(1) m* dense in itself [14] if $A \subset A^*_m$.
(2) m* closed [14] if $A^*_m \subset A$.
(3) mg closed [6] if $mcl(A) \subseteq U$ whenever $A \subseteq U$ and U is m open.
(4) mgs closed [6] if $mscl(A) \subseteq U$ whenever $A \subseteq U$ and U is m open.
(5) mπg closed [6] if $mcl(A) \subseteq U$ whenever $A \subseteq U$ and U is mπopen.
(6) mπgs closed [6] if $mscl(A) \subseteq U$ whenever $A \subseteq U$ and U is mπopen.

**Definition 2.8:** Let (X, M, I) be an ideal minimal spaces is called

(1) mI closed [14] if $A \subset m - int(A^*_m)$
(2) $mI_g$ closed [14] if $A^*_m \subseteq U$ whenever $A \subseteq U$ and U is m open

**Lemma 2.9:** [4] m* dense sets are pre m I open sets.

**Definition 2.10:** [4] A subsets H of Y is defined as a m∗ dense set if $mcl^*(H) = Y$.

**Lemma 2.11:** [17] Let Y be an ideal minimal space, if a subset H is a pre m I open set, then H can be expressed as $H = K \cap L$, where K is m open set and L is m* dense set.

## 3. MINIMAL IDEAL αψ SUBMAXIMAL IN MINIMAL STUCTURE SPACES

**Definition 3.1:** Let H be a subset of mIαψ locally m* closed set if there exist mIαψ open set S and m* closed set T then $H = S \cap T$.

**Definition 3.2:** A subset H of an ideal minimal space (Y, M, I) is $mI_{\alpha\psi}$ closed if $mcl^* \subseteq V$ whenever $H \subseteq V$ and V is mαψ open.

**Theorem 3.3:** Let G be a m closed set then G is $mI_{\alpha\psi}$ closed set.

**Proof:** Let us take V be a mαψ open and $G \subseteq V$. Here G is m closed set in Y, then $mcl(G) = G$, which implies $mcl(G) \subseteq V$. Therefore $mcl^*(G) \subseteq mcl(G) \subseteq V$. Since $mcl^*(G) = G \cup G^*m$ and $G \subseteq V$, therefore $G^*m \subseteq V$, where V is mαψ open. Hence G is $mI_{\alpha\psi}$ closed set.

**Theorem 3.4:** Let us consider a ideal minimal space (Y, M, I) and $A \subset Y$ is m* dense in itself then the following are equivalent.

(i) H is mIαψ locally m* closed set.
(ii) $H = S \cap mcl^*(H)$ where S is $mI_{\alpha\psi}$ open set.
(iii) $mcl^*(H) - H = H^*_m - H$ and also $mI_{\alpha\psi}$ closed set.
(iv) $H \cup (Y - mcl^*(H)) = H \cup (Y - H^*_m$ and also $mI_{\alpha\psi}$ open sets.
(v) $H \subset mint(H \cup (Y - H^*_m))$.

**Proof:**

**(i) ⇒ (ii):** If H is $mI_{\alpha\psi}$ locally m* closed set, then $H = S \cap T$ where S is $mI_{\alpha\psi}$ open and T is m* closed, that is $T^*_m \subset T$. Since $H = S \cap T$, we have $H \subset S$ and $H \subset mcl^*(H)$. Hence $H \subset S \cap mcl^*(H)$. Since T is m* closed and $H \subset T$, $mcl^*(H) \subset mcl^*(T)$, which implies $S \cap mcl^*(H) \subset S \cap mcl^*(T) \subset S \cap T = H$. Then $H = S \cap mcl^*(H)$.

**(ii) ⇒ (iii):** Let $mcl^*(H) - H = H^*_m - H = H^*_m \cap (Y - H) = H^*_m \cap (Y - (S \cap mcl^*(H))) = H^*m \cap (Y - S)$. Then $H^*m \cap (Y - S) \subset U$, which implies $(Y - S) \subset ((Y - H^*m) \cup U$. Since S is mIαψ open set $(Y - S)$ is mIαψ closed set. Therefore by Definition [3.2], we have $mcl^*(Y - S) \subset ((Y - H^*m) \cup S$. Also $H^*m \cap mcl^*(Y - S) \subset U$.

Now $H^*_m \cap (Y - S) \subset H*m$ and $H*m \cap (Y - S) \subset (Y - S)$ by Lemma [2.4], $H*m \cap (Y - S))*m \subset (H*m)*m \subset H^*_m$ and $H^*_m \cap (Y - S))^*_m \subset (Y - S)^*_m \subset mcl*(Y - S)$. Hence $(H^*_m \cap (Y - S))^*_m \subset H^*_m \cap mcl*(Y - S) \subset U$. Also $mcl*(H) - H = H*m \cap (Y - S)$. Hence we get $(mcl*(H) - H)^*_m \subset H^*_m \cap mcl*(Y - S) \subset U$. That is $(mcl*(H) - H)^*_m \subset U$ and U is mαψ open set. Therefore $mcl*(H) - H)*m \subset U$ whenever $mcl*(H) - H \subset U$, U is mαψ open set this implies that $mcl*(H) - H = H^*_m - H$ is a mIαψ closed set.

**(iii)⇒ (iv):** Since mcl∗ (H)−H is mIαψ closed. (Y − (mcl∗ (H)−H)) is mIαψ open, which implies $H \cup (mcl*(H) - H)) \Rightarrow H \cup (Y - (H - \cup H^*_m))$ is mIαψ open and hence $H \cup (Y - H^*_m)$ is mIαψ open.

**(iv) ⇒ (v):** Since $H \subset (H \cup (Y - H^*_m))$, $mint(H) \subset mint(H \cup (Y - H^*_m))$. Therefore $H \subset mint(H) \subset mint(H \cup (Y - H^*_m))$ and hence $H \subset mint(H \cup (Y - H^*_m))$.

**(v) ⇒ (i):** By (v) we have $H \cup (Y - H^*_m) \subset mint(H \cup (Y - H^*_m))$. By (v), $H \cup (Y - H^*_m)) = H \cup (Y - H^*_m)$ is mIαψ open set. Also $H \cup (Y - H^*_m))\cap mcl*(H) = (H \cap mcl*(H)) \cup (Y - mcl*(H)) \cap mcl*(H))) = (H \cap (H \cup H^*_m)) \cup \phi = H \cup (H \cap H^*_m) \cup \phi = H$. Since H is m∗ dense set. Also H is m∗ dense set, $H \subset H^*_m$. Hence $mcl*(H) = H*m$. Therefore $(mcl*(H))^*_m = (H^*_m)^*_m = H^*_m = mcl*(H)$. That is $(mcl*(H))^*_m \subset mcl*(H)$. Hence mcl∗ (H) is m∗ closed set. That is $H = (H \cup (Y - H^*_m) \cap mcl*(H)$ such that $H = (H \cup (Y - H^*_m))$ is mIαψ open and mcl∗ (H) is m∗ closed. Therefore H is mIαψ locally m∗ closed set.

**Theorem 3.5:** An ideal minimal space with property [I], then the conditions are equivalent,

(i) Each subset Y is mIαψ locally m∗ closed set.
(ii) Each m∗ dense set is mIαψ open set.

**Proof:**
**(i) ⇒ (ii):** This is proved by Theorem [3.4].

**(ii)⇒ (i):** Let $G \subset Y$. Let $U = G \cup (Y - mcl*(G))$. Then $mcl*(U) = mcl*(G \cup (Y - mcl*(G))$. Then $mcl*(U) = mcl*(G\cup(Y - mcl*(G))) = (G\cup(Y - mcl*(G)))\cup G\cup(Y - mcl*(G)))*m = (G\cup(Y - mcl*(G)))*m = (G\cup(Y - mcl*(G)))\cup G*m \cup(Y - mcl*(G))*m = mcl*(G)\cup mcl*(Y - mcl*(G)) = mcl*(G)\cup(Y - mcl*(G)) = Y$. Then we have $mcl*(U) = Y$. Then by definition of m∗ dense set. By (ii) U is mIαψ open set. Hence by Theorem [3.4] G is mIαψ locally m∗ closed set.

**Definition 3.6:** Let (Y, M, I) be an ideal minimal spaces is called mIαψ submaximal if each m∗ dense subset of Y is mIαψ open.

**Remarks 3.7:**

(i) Property [U] refers union of two mIαψ closed set is a mIαψ closed set.
(ii) Property [I] refers any two intersection of mIαψ closed set is a mIαψ closed set.

**Theorem 3.8:** An ideal minimal space satisfying the property [I] then the following statements are equivalent.

(i) An ideal minimal space is a mIαψ submaximal space.
(ii) If H is a pre m I open set then H is a mIαψ open set.

**Proof:**
**(i) ⇒ (ii):** Let (Y, M, I) be a mIαψ submaximal space and $H \subset Y$ be a pre m I open set, then $H = K \cap L$, where K is a m open set and L is a m∗ dense set. Here Y is mIαψ submaximal space and L is mIαψ open set by definition. Also K is mIαψ open set. Then by property [I], H is also mIαψ open.

**(ii)⇒ (i):** Let H be m∗ dense subset of Y, then by Lemma [2.4] H is pre m I open set. By assumption H is a mIαψ open set. Hence Y is mIαψ submaximal space.

**Theorem 3.9:** Let (Y, M, I) be an ideal minimal space if the property [I] satisfies then the following statements are equivalent.

(i) Y is mIαψ submaximal space.
(ii) If $H \subseteq Y$, then H is mIαψ locally m∗ closed set.
(iii) Any m∗ dense set and a mIαψ open subset of Y.

**Proof:**
**(i) ⇒ (ii):** Let (Y, M, I) be a mIαψ submaximal space by definition each m∗ dense set is mIαψ open set by Theorem [3.5], every m∗ dense and mIαψ open is mIαψ locally m∗ closed set.
**(ii)⇒ (iii):** Let H be m∗ dense set by (ii), H is mIαψ locally m∗ closed set. By Theorem [3.4], there exists mIαψ open set K such that $H = K\cap mcl*(H)$. Now consider H is m∗ dense set, then $mcl*(H) = Y$.
Hence $H = K \cap mcl*(H) = K \cap Y = K$ and K is mIαψ open set in Y .

**(ii)⇒ (i):** Here A is m∗ dense set by (iii) H = K ∩L, where K is mIαψ open set and L is m∗ dense set. Hence mcl∗ (L) = Y. Since L is m∗ closed set mcl∗ (L) = L = Y. Therefore H = K∩ L = K ∩ Y = K and it is mIαψ open set. Hence Y is mIαψ submaximal space.

**Theorem 3.10:** An ideal minimal space (Y, M, I), the following statements are equivalent.

(i) (Y, M, I) is a mIαψ submaximal space.
(ii) If H is not mIαψ open set, then H − (mint(mcl∗ (H))) ≠ ϕ.

**Proof**

**(i) ⇒ (ii):** Contradiction, H − (mint(mcl∗ (H))) = ϕ. Hence H ⊂ mint(mcl∗ (H))), which means H is pre m I open set. Let Y is a mIαψ submaximal space, by Theorem [3.8] H is mIαψ open which is contrary by the assumption. Hence H − (mint(mcl∗ (H))) = ϕ.

**(ii)⇒ (i):** Assume H is pre m I open set but it is not mIαψ open set, then by (ii) H − mint(mcl∗ (H)) ≠ ϕ, which implies H * mint(mcl∗ (H)). That is H is not pre m I open set which is contradiction by our assumption. Hence H is mIαψ open set and hence by Theorem [3.8] Y is mIαψ submaximal space.

**Theorem 3.11:** An ideal minimal space (Y, M, I), if the property [I] satisfies, then the following statements are equivalent.

(i) (Y, M, I) is a mIαψ submaximal space.
(ii) The mIαψ open set γ = {K −H : K is mIαψ open and mint∗ (H) = ϕ}.

**Proof:**

**(i) ⇒ (ii):** Let Y be a mIαψ submaximal space. Assume η = {K− H: K is mIαψ open and mint∗ (H) = ϕ}. To prove γ = η. Let us take an element A ∈ γ. Since A = A − ϕ and mint∗ (ϕ) = ϕ, A ∈ η. Hence γ ⊂ η. Let A ∈ η, to prove A is mIαψ open set. Since A ∈ η , A can be written as A = K− H such that K is a mIαψ open and mint ∗ (H) = ϕ. Also A = H− K = K ∩ (Y − H). Since mint∗ (H) = ϕ, Y − mint∗ (Y − H) = mcl∗ (H) = Y . Hence Y− H is m∗ dense set. Since Y is mIαψ submaximal Y− H is m∗ dense implies (Y − H) is mIαψ open set. Hence A = K ∩ (Y − H) is mIαψ open set and so η ⊂ γ. Therefore η = γ.

**(ii)⇒ (i):** Assume H is pre m I open set, by Lemma [2.4] H be the intersection of the sets S and T such that S is m open and T is m∗ dense. Hence mcl∗ (T) = Y and so mint∗ (Y −T) = ϕ. That is H = S∩T = S−(Y −T) and mint∗ (Y −T) = ϕ. By Theorem [3.3] S is mIαψ open set. Therefore H ⊂ γ and hence H is mIαψ open set. By assumption, the pre m I open set A is mIαψ open. Hence by Theorem [3.8] Y is mIαψ submaximal space.

**Theorem 3.12:** Let (Y, M, I), satisfying the property [I], then the following statements are equivalent.

(i) (Y, M, I) is a mIαψ submaximal space.
(ii) There exist a mIαψ closed set mcl∗ (H) − H for each subset H of Y.

**Proof:**

**(i) ⇒ (ii):** Assume Y be mIαψ submaximal and let H ⊂ Y. To prove mcl∗ (H) − H is mIαψ closed set. It is enough to prove Y − (mcl∗ (H)−H) is mIαψ open. That is to prove that mcl∗ (H ∪ (Y − (mcl∗ (H))) = mcl∗ (H) ∪ mcl∗ (Y − mcl∗ (H))) = Y. Hence (Y − (mcl∗ (H) − H)) is a m∗ dense set. Also since Y is mIαψ submaximal space, by definition each m∗ dense set is mIαψ open set. Hence (Y − (mcl∗ (H) − H)) is mIαψ open set. Threfore (mcl∗ (H) − H) is mIαψ closed set.

**(ii)⇒ (i):** Let (mcl∗ (H) − H) is mIαψ closed set and let H ⊂ Y is m∗ dense set in Y. Hence mcl∗ (H) = Y implies (Y − H) is mIαψ closed set. So H is mIαψ open. Hence Y is mIαψ submaximal space.

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