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SOLVING UNBALANCED ASSIGNMENT PROBLEM FOR USING REVISED ONES ASSIGNMENT METHOD

C. GOWRISHANKAR, R. DHARSHINEE AND T. P. PRIYA

Department of Mathematics,
Bharathiar University Arts and Science College, Gudalur-643 212, Tamil Nadu, India.

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ABSTRACT

The assignment problem is a special case of the transportation problem in which the objective is to assign the number of resources and the number of activities are equal at a minimum cost or maximum profit (ie, balanced assignment problem).If an assignment problem has not equal number of resources and activities, then it is called an unbalanced assignment problem. As we can solve only a balanced assignment problem, so we have to convert it into a balanced assignment problem by introducing dummy resource or activity with costs one.

In this paper we attempt to introduce the algorithms and solution steps for "Revised Ones Assignment Method" for solving wide range of problems. Finally we compare with the optimal solutions for Revised Ones Assignment Method (ROA-method) to Hungarian Assignment Method (HA-method).

Keywords: *Assignment problem, Hungarian Assignment Method (HA-method), Revised Ones Assignment Method (ROA-method), Balanced assignment problem, Unbalanced assignment problem, Minimization, Maximization, Optimization.*

INTRODUCTION

Assignment problem is a most important problem in mathematics and is also discuss in real physical world. The assignment problem is completely degenerated form of a transportation problem. This is particularly important in the theory of decision making. This problem finds many applications in allocation and scheduling, for example in assigning salesmen to different regions, vehicles and drivers to different routes, products to factories, jobs to machines, contracts to bidders and research problems to teams, etc. In a normal case of assignment problem where the objective is to assign the available resources to the activity going on as so as to get the minimum cost or maximum total benefits of allocation.

In this paper we change an unbalanced assignment problem to balanced assignment problem and using "Revised Ones Assignment Method" to achieve exact optimal solution, which is same as that of Hungarian Assignment Method. In similar way, we can use to solve the maximization problem same as the minimization problem after the small changes to get optimal solution.

MATHEMATICAL FORMULATION OF ASSIGNMENT PROBLEM

Considering the assignment problem of n resources(workers) to n activities(jobs) so as to minimize the overall cost or time in such a way that each resource can associate with one and only one job.

The cost (or effectiveness) matrix (c_{ij}) is given as below:

		Activity				
		A_1	A_2	…….	A_n	Available
	R_1	C_{11}	C_{12}	…….	C_{1n}	1
Resource	R_2	C_{21}	C_{22}	…….	C_{2n}	1
	⋮	⋮	⋮	⋮	⋮	⋮
	R_n	C_{n1}	C_{n2}	……..	C_{nn}	1
	Required	1	1	…….	1	

Corresponding Author: C. Gowrishankar, Department of Mathematics,
Bharathiar University Arts and Science College, Gudalur-643 212, Tamil Nadu, India.

This cost matrix is same as that of a transportation problem except that availability at each of the resources and the requirement at each of the destinations is unity.

Let X_{ij} denote the assignment of i^{th} resource to j^{th} activity, such that

$$X_{ij} = \begin{cases} 1\ ; & if\ resource\ i\ is\ assigned\ to\ activity\ j \\ 0\ ; & otherwise \end{cases}$$

Then, the mathematical formulation of the assignment problem is

Minimize z = $\sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}X_{ij}$

Subject to the constraints

$\sum_{i=1}^{n} X_{ij}$ = 1 and $\sum_{j=1}^{n} X_{ij}$ =1 such that X_{ij} = 0 or 1
for all i = 1,2, ... ,n and j = 1,2,.......,n.

I. Hungarian Assignment (HA) Method

The Hungarian method was first introduced by Mr.$K\ddot{o}ning$ of Hungary or the reduced matrix method or the Flood's technique is used for solving assignment problems. It involves a reduction of the original matrix and finding of a set of n independent zeros, one in each row and column, which results in an optimal solution.

HA-Algorithm

Step-1: To check the given assignment problem is a balanced or unbalanced assignment problem.

- If the given problem is an balanced (ie, the number of rows is equal to the number of columns) assignment problem then we go to step (2).
- If the given problem is an unbalanced (ie, the number of rows is not equal to the number of columns) assignment problem then we have to convert it into a balanced assignment problem by introducing a dummy row or column with costs zero (ie, 0). Then we go to next step.

Step-2: To check the given problem is minimization case or maximization case.

- If the given problem is an minimization case then we go to step (3).
- If the given problem is a maximization case then we can be converted in to the equivalent minimization problem by subtracting all the matrix elements from the highest element. Then we go to next step.

Step-3: Find the minimum element of each row in the assignment matrix and write it on the right hand side of the row matrix.

Step-4: Subtract the row minimum from each row.

Step-5: Find the minimum element of each column in the assignment matrix and write it below the column matrix.

Step-6: Subtract the column minimum from each column from the reduced matrix.

Step-7: Assign one "0" to each row and column.

i. Mark (√) all unassigned rows.
ii. If a row is marked (√) and has a "0", then mark the corresponding column (if the column is not yet marked).
iii. If a column is marked and has an assignment, then mark the corresponding row (if the row is not yet marked).
iv. Repeat steps (ii) and (iii) till no more marking is possible.
v. Draw straight lines through all unmarked rows and marked columns.
vi. If the number of lines drawn is equal to the number of rows or columns, then the current solution is optimal solution. Otherwise go to next step.

Step-8: Find out the smallest number which does not have any line passing through it. Select the smallest number to subtract from all the numbers that do not have any lines passing through them and add to all those numbers that have two lines passing through them. Keep the rest of them the same.

Step-9: If we cannot get the optimal assignment in each row and column. Then repeat steps (7) and (8) successively till an optimum solution is obtained.

II. Revised Ones Assignment (ROA) Method

This section presents a new method to solve the assignment problem which is different from the preceding method. We call it "Revised Ones Assignment Method" because of making assignment in terms of ones. The new method is based on creating some ones in the assignment matrix and then tries to find a complete assignment in terms of ones. By a complete assignment we mean an assignment plan containing exactly n assigned independent ones, one in each row and one in each column.

ROA-Algorithm
Step-1: To check the given assignment problem is a balanced or unbalanced assignment problem.

- If the given problem is an balanced (ie, the number of rows is equal to the number of columns) assignment problem then we go to step (2).
- If the given problem is an unbalanced (ie, the number of rows is not equal to the number of columns) assignment problem then we have to convert it into a balanced assignment problem by introducing a dummy row or column with costs one (ie, 1). Then we go to next step.

Step-2: In a minimization (maximization) case, find the minimum (maximum) element of each row in the assignment matrix and write it on the right hand side of the row matrix.

Step-3: Divide the row minimum (maximum) from each row.

Step-4: Find the minimum (maximum) element of each column in the assignment matrix and write it below the column matrix.

Step-5: Divide the column minimum (maximum) from each column from the reduced matrix.

Step-6: Assign one "1" to each row and column.

i. Mark (√) all unassigned rows.
ii. If a row is marked (√) and has a "1", then mark the corresponding column (if the column is not yet marked).
iii. If a column is marked and has an assignment, then mark the corresponding row (if the row is not yet marked).
iv. Repeat steps (ii) and (iii) till no more marking is possible.
v. Draw straight lines through all unmarked rows and marked columns.
vi. If the number of lines drawn is equal to the number of rows or columns, then the current solution is optimal solution. Otherwise go to next step.

Step-7: Find out the minimum (maximum) number which does not have any line passing through it. Select the minimum (maximum) number to divide from all the numbers that do not have any lines passing through them and multiply to all those numbers that have two lines passing through them. Keep the rest of them the same.

Step-8: If we cannot get the optimal assignment in each row and column. Then repeat steps (6) and (7) successively till an optimum solution is obtained.

Step-9: Finally in our optimum solution for unbalanced assignment problem to put a dummy row (column) cost 1 as 0.

Numerical Comparison Of HA and ROA – Methods

Problem 1:
Solve the assignment problem using
(i) Hungarian Assignment (HA) Method.
(ii) Revised Ones Assignment Method (ROA) Method.

Consider the problem of assigning a company has six machines on which to do five jobs. Each job can be assigned to one and only one machine. The cost of each job on each machine is given in the following table:

		Machines					
		M1	M2	M3	M4	M5	M6
Jobs	A	12	10	15	22	18	8
	B	10	18	25	15	16	12
	C	11	10	3	8	5	9
	D	6	14	10	13	13	12
	E	8	12	11	7	13	10

Determine the optimum assignment schedule and minimum assignment cost.

Solution:
(i) Hungarian Assignment (HA) Method.

Consider the problem of assigning five jobs to six machines. The assignment cost are given below.

		Machines					
		M1	M2	M3	M4	M5	M6
Jobs	A	12	10	15	22	18	8
	B	10	18	25	15	16	12
	C	11	10	3	8	5	9
	D	6	14	10	13	13	12
	E	8	12	11	7	13	10

Step-1: The given problem is an unbalanced (ie, the number of rows is not equal to the number of columns) assignment problem then we have to convert it into a balanced assignment problem by introducing a dummy row with costs zero (ie, 0).

	M1	M2	M3	M4	M5	M6
A	12	10	15	22	18	8
B	10	18	25	15	16	12
C	11	10	3	8	5	9
D	6	14	10	13	13	12
E	8	12	11	7	13	10
F	0	0	0	0	0	0

Step-2: The given problem is a minimization case then we go to step.

	M1	M2	M3	M4	M5	M6	Min
A	12	10	15	22	18	8	8
B	10	18	25	15	16	12	10
C	11	10	3	8	5	9	3
D	6	14	10	13	13	12	6
E	8	12	11	7	13	10	7
F	0	0	0	0	0	0	0

Step-3: Find the minimum element of each row in the assignment matrix and write it on the right hand side of the row matrix.

Step-4: Subtract the row minimum from each row.

	M1	M2	M3	M4	M5	M6
A	4	2	7	14	10	0
B	0	8	15	5	6	2
C	8	7	0	5	2	6
D	0	8	4	7	7	6
E	1	5	4	0	6	3
F	0	0	0	0	0	0

Step-5: Find the minimum element of each column in the assignment matrix and write it below the column matrix.

	M1	M2	M3	M4	M5	M6
A	4	2	7	14	10	0
B	0	8	15	5	6	2
C	8	7	0	5	2	6
D	0	8	4	7	7	6
E	1	5	4	0	6	3
F	0	0	0	0	0	0
Min	0	0	0	0	0	0

Step-6: Subtract the column minimum from each column from the reduced matrix.

	M1	M2	M3	M4	M5	M6
A	4	2	7	14	10	0
B	0	8	15	5	6	2
C	8	7	0	5	2	6
D	0	8	4	7	7	6
E	1	5	4	0	6	3
F	0	0	0	0	0	0

Step-7: Make initial assignment
.

	M1	M2	M3	M4	M5	M6	
A	4	2	7	14	10	$\boxed{0}$	
B	$\boxed{0}$	8	15	5	6	2	√
C	8	7	$\boxed{0}$	5	2	6	
D	0	8	4	7	7	6	√
E	1	5	4	$\boxed{0}$	6	3	
F	0	$\boxed{0}$	0	0	0	0	
	√						

Here, 4th row and 5th column do not have any assignment. Thus the solution is not optimum and we go to next.
Step-8: Find out the smallest number (ie,2) which does not have any line passing through it. Select the smallest number to subtract from all the numbers that do not have any lines passing through them and add to all those numbers that have two lines passing through them. Keep the rest of them the same.

	M1	M2	M3	M4	M5	M6	
A	6	2	7	14	10	$\boxed{0}$	
B	$\boxed{0}$	6	13	3	4	0	√
C	10	7	$\boxed{0}$	5	2	6	
D	0	6	2	5	5	4	√
E	3	5	4	$\boxed{0}$	6	3	
F	2	$\boxed{0}$	0	0	0	0	
	√						

Here, 4th row and 5th column do not have any assignment. Thus the solution is not optimum and we go to next.
Step-9: Find out the smallest number (ie,2) which does not have any line passing through it. Select the smallest number to subtract from all the numbers that do not have any lines passing through them and add to all those numbers that have two lines passing through them. Keep the rest of them the same.

	M1	M2	M3	M4	M5	M6
A	6	$\boxed{0}$	5	12	8	0
B	0	4	11	1	2	$\boxed{0}$
C	12	7	$\boxed{0}$	5	2	8
D	$\boxed{0}$	4	0	3	3	4
E	5	5	4	$\boxed{0}$	6	5
F	4	0	0	0	$\boxed{0}$	2

Here we see that all zeros are either assigned or crossed out. That is, the total assigned zero's is 6 which is equal to the number of rows or columns.

The optimum assignment is: $A \to M2,\ B \to M6,\ C \to M3,\ D \to M1, E \to M4, F \to M5.$
∴ The total minimum assignment cost = $10 + 12 + 3 + 6 + 7 + 0 = Rs. 38$

(ii) Revised One's Assignment Method (ROA) Method.
Consider the problem of assigning five jobs to six machines. The assignment cost are given below.

		Machines					
		M1	M2	M3	M4	M5	M6
Jobs	A	12	10	15	22	18	8
	B	10	18	25	15	16	12
	C	11	10	3	8	5	9
	D	6	14	10	13	13	12
	E	8	12	11	7	13	10

Step-1: The given problem is an unbalanced (ie,the number of rows is not equal to the number of columns) assignment problem then we have to convert it into a balanced assignment problem by introducing a dummy row with costs one(ie,1).

	M1	M2	M3	M4	M5	M6
A	12	10	15	22	18	8
B	10	18	25	15	16	12
C	11	10	3	8	5	9
D	6	14	10	13	13	12
E	8	12	11	7	13	10
F	1	1	1	1	1	1

Step-2: Find the minimum element of each row in the assignment matrix and write it on the right hand side of the row matrix.

	M1	M2	M3	M4	M5	M6	Min
A	12	10	15	22	18	8	8
B	10	18	25	15	16	12	10
C	11	10	3	8	5	9	3
D	6	14	10	13	13	12	6
E	8	12	11	7	13	10	7
F	0	0	0	0	0	0	1

Step-3: Divide the row minimum from each row.

	M1	M2	M3	M4	M5	M6
A	1.5	1.25	1.88	2.75	2.25	1
B	1	1.8	2.5	1.5	1.6	1.2
C	3.67	3.33	1	2.67	1.67	3
D	1	2.33	1.67	2.17	2.17	2
E	1.14	1.71	1.57	1	1.86	1.43
F	1	1	1	1	1	1

Step-4: Find the minimum element of each column in the assignment matrix and write it below the column matrix.

	M1	M2	M3	M4	M5	M6
A	1.5	1.25	1.88	2.75	2.25	1
B	1	1.8	2.5	1.5	1.6	1.2
C	3.67	3.33	1	2.67	1.67	3
D	1	2.33	1.67	2.17	2.17	2
E	1.14	1.71	1.57	1	1.86	1.43
F	1	1	1	1	1	1
Min	1	1	1	1	1	1

Step-5: Divide the column minimum from each column from the reduced matrix.

	M1	M2	M3	M4	M5	M6
A	1.5	1.25	1.88	2.75	2.25	1
B	1	1.8	2.5	1.5	1.6	1.2
C	3.67	3.33	1	2.67	1.67	3
D	1	2.33	1.67	2.17	2.17	2
E	1.14	1.71	1.57	1	1.86	1.43
F	1	1	1	1	1	1

Step-6: Make initial assignment.

	M1	M2	M3	M4	M5	M6	
A	1.5	1.25	1.88	2.75	2.25	[1]	
B	[1]	1.8	2.5	1.5	1.6	1.2	√
C	3.67	3.33	[1]	2.67	1.67	3	
D	1	2.33	1.67	2.17	2.17	2	√
E	1.14	1.71	1.57	[1]	1.86	1.43	
F	1	[1]	1	1	1	1	
	√						

Here, 4th row and 5th column do not have any assignment. Thus the solution is not optimum and we go to next.

Step-7: Find out the smallest number (ie, 1.2) which does not have any line passing through it. Select the smallest number to divide from all the numbers that do not have any lines passing through them and multiply to all those numbers that have two lines passing through them. Keep the rest of them the same.

	M1	M2	M3	M4	M5	M6	
A	1.8	1.25	1.88	2.75	2.25	[1]	
B	[1]	1.5	2.08	1.25	1.33	1	√
C	4.40	3.33	[1]	2.67	1.67	3	
D	1	1.94	1.39	1.81	1.81	1.67	√
E	1.37	1.71	1.57	[1]	1.86	1.43	
F	1.2	[1]	1	1	1	1	
	√						

Here, 4th row and 5th column do not have any assignment. Thus the solution is not optimum and we go to next.

Step-8: Find out the smallest number (ie, 1.25) which does not have any line passing through it. Select the smallest number to divide from all the numbers that do not have any lines passing through them and multiply to all those numbers that have two lines passing through them. Keep the rest of them the same.

	M1	M2	M3	M4	M5	M6
A	1.8	$\boxed{1}$	1.50	2.2	1.8	$\cancel{1}$
B	$\cancel{1}$	1.2	1.66	$\cancel{1}$	1.06	$\boxed{1}$
C	5.5	3.33	$\boxed{1}$	2.67	1.67	3.75
D	$\boxed{1}$	1.55	1.11	1.45	1.45	1.67
E	1.71	1.71	1.57	$\boxed{1}$	1.86	1.79
F	1.5	$\cancel{1}$	$\cancel{1}$	$\cancel{1}$	$\boxed{1}$	1.25

Here we see that all ones are either assigned or crossed out. That is, the total assigned zero's is 6 which is equal to the number of rows or columns.

The optimum assignment is : $A \to M2,\ B \to M6,\ C \to M3,\ D \to M1, E \to M4, F \to M5.$

$\therefore$ The total minimum assignment cost = $10 + 12 + 3 + 6 + 7 + 1 = Rs. 39$

Step-9: Finally in our optimum solution the dummy row cost$(F, M5)$ take it 1 as 0.
$\therefore$ The total minimum assignment cost = $10 + 12 + 3 + 6 + 7 + 0 = Rs. 38$

Comparison of Optimal Values of two Methods

Example	HA- method	ROA-method	Optimum Value
1	38	38	38

Problem 2:
Solve the assignment problem using
- (i) Hungarian Assignment (HA) Method.
- (ii) Revised Ones Assignment Method (ROA) Method.

Consider the problem of assigning a company has five machines that are used for four jobs. Each job can be assigned to one and only one machine. The cost of each job on each machine is given in the following table:

		Machines				
		M1	M2	M3	M4	M5
Jobs	A	6.20	7.80	5.00	10.10	8.20
	B	7.10	8.40	6.10	7.30	5.90
	C	8.70	9.20	11.10	7.10	8.10
	D	4.80	6.40	8.70	7.70	8.00

Determine the optimum assignment schedule and maximum assignment profit.

Solution:
(i) Hungarian Assignment (HA) Method.
Consider the problem of assigning four jobs to five machines. The assignment cost are given below.

		Machines				
		M1	M2	M3	M4	M5
Jobs	A	6.20	7.80	5.00	10.10	8.20
	B	7.10	8.40	6.10	7.30	5.90
	C	8.70	9.20	11.10	7.10	8.10
	D	4.80	6.40	8.70	7.70	8.00

Step-1: The given problem is an unbalanced (ie, the number of rows is not equal to the number of columns) assignment problem then we have to convert it into a balanced assignment problem by introducing a dummy row with costs zero (ie, 0).

	M1	M2	M3	M4	M5
A	6.20	7.80	5.00	10.10	8.20
B	7.10	8.40	6.10	7.30	5.90
C	8.70	9.20	11.10	7.10	8.10
D	4.80	6.40	8.70	7.70	8.00
E	0	0	0	0	0

Step-2: The given problem is a maximization case then we can be converted in to the equivalent minimization problem by subtracting all the matrix elements from the highest element which is 11.10. Then we go to next step.

	M1	M2	M3	M4	M5
A	4.9	3.3	6.1	1.0	2.9
B	4.0	2.7	5.0	3.8	5.2
C	2.4	1.9	0	4.0	3.0
D	6.3	4.7	2.4	3.4	3.1
E	11.10	11.10	11.10	11.10	11.10

Step-3: Find the minimum element of each row in the assignment matrix and write it on the right hand side of the row matrix.

	M1	M2	M3	M4	M5	Min
A	4.9	3.3	6.1	1.0	2.9	1.0
B	4.0	2.7	5.0	3.8	5.2	2.7
C	2.4	1.9	0	4.0	3.0	0
D	6.3	4.7	2.4	3.4	3.1	2.4
E	11.10	11.10	11.10	11.10	11.10	11.10

Step-4: Subtract the row minimum from each row.

	M1	M2	M3	M4	M5
A	3.9	2.3	5.1	0	1.9
B	1.3	0	2.3	1.1	2.5
C	2.4	1.9	0	4.0	3.0
D	3.9	2.3	0	1.0	0.7
E	0	0	0	0	0

Step-5: Find the minimum element of each column in the assignment matrix and write it below the column matrix.

	M1	M2	M3	M4	M5
A	3.9	2.3	5.1	0	1.9
B	1.3	0	2.3	1.1	2.5
C	2.4	1.9	0	4.0	3.0
D	3.9	2.3	0	1.0	0.7
E	0	0	0	0	0
Min	0	0	0	0	0

Step-6: Subtract the column minimum from each column from the reduced matrix.

	M1	M2	M3	M4	M5
A	3.9	2.3	5.1	0	1.9
B	1.3	0	2.3	1.1	2.5
C	2.4	1.9	0	4.0	3.0
D	3.9	2.3	0	1.0	0.7
E	0	0	0	0	0

Step-7: Make initial assignment.

	M1	M2	M3	M4	M5	
A	3.9	2.3	5.1	$\boxed{0}$	1.9	
B	1.3	$\boxed{0}$	2.3	1.1	2.5	
C	2.4	1.9	$\boxed{0}$	4.0	3.0	√
D	3.9	2.3	$\cancel{0}$	1.0	0.7	√
E	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	$\cancel{0}$	$\cancel{0}$	
			√			

Here, 4th row and 5th column do not have any assignment. Thus the solution is not optimum and we go to next.

Step-8: Find out the smallest number (ie, 0.7) which does not have any line passing through it. Select the smallest number to subtract from all the numbers that do not have any lines passing through them and add to all those numbers that have two lines passing through them. Keep the rest of them the same.

	M1	M2	M3	M4	M5
A	3.9	2.3	5.8	$\boxed{0}$	1.9
B	1.3	$\boxed{0}$	3.0	1.1	2.5
C	1.7	1.2	$\boxed{0}$	3.3	2.3
D	3.2	1.6	$\cancel{0}$	0.3	$\boxed{0}$
E	$\boxed{0}$	$\cancel{0}$	0.7	$\cancel{0}$	$\cancel{0}$

Here we see that all zeros are either assigned or crossed out. That is, the total assigned zero's is 5 which is equal to the number of rows or columns.

The optimum assignment is: $A \rightarrow M4,\ B \rightarrow M2,\ C \rightarrow M3,\ D \rightarrow M4,\ E \rightarrow M1.$
∴ The total maximum assignment profit = $10.10 + 8.40 + 11.10 + 8.00 + 0 = Rs. 37.60$

(ii) Revised One's Assignment Method (ROA) Method.

Consider the problem of assigning four jobs to five machines. The assignment cost are given below.

		Machines				
		M1	M2	M3	M4	M5
	A	6.20	7.80	5.00	10.10	8.20
	B	7.10	8.40	6.10	7.30	5.90
Jobs	C	8.70	9.20	11.10	7.10	8.10
	D	4.80	6.40	8.70	7.70	8.00

Step-1: The given problem is an unbalanced (ie, the number of rows is not equal to the number of columns) assignment problem then we have to convert it into a balanced assignment problem by introducing a dummy row with costs one (ie, 1).

	M1	M2	M3	M4	M5
A	6.20	7.80	5.00	10.10	8.20
B	7.10	8.40	6.10	7.30	5.90
C	8.70	9.20	11.10	7.10	8.10
D	4.80	6.40	8.70	7.70	8.00
E	1	1	1	1	1

Step-2: Find the maximum element of each row in the assignment matrix and write it on the right hand side of the row matrix.

	M1	M2	M3	M4	M5	Max
A	6.20	7.80	5.00	10.10	8.20	10.10
B	7.10	8.40	6.10	7.30	5.90	8.40
C	8.70	9.20	11.10	7.10	8.10	11.10
D	4.80	6.40	8.70	7.70	8.00	8.70
E	1	1	1	1	1	1

Step-3: Divide the row maximum from each row.

	M1	M2	M3	M4	M5
A	0.61	0.77	0.50	1	0.81
B	0.85	1	0.73	0.87	0.70
C	0.78	0.83	1	0.64	0.73
D	0.55	0.74	1	0.89	0.92
E	1	1	1	1	1

Step-4: Find the maximum element of each column in the assignment matrix and write it below the column matrix.

	M1	M2	M3	M4	M5
A	0.61	0.77	0.50	1	0.81
B	0.85	1	0.73	0.87	0.70
C	0.78	0.83	1	0.64	0.73
D	0.55	0.74	1	0.89	0.92
E	1	1	1	1	1
Max	1	1	1	1	1

Step-5: Divide the column maximum from each column from the reduced matrix.

	M1	M2	M3	M4	M5
A	0.61	0.77	0.50	1	0.81
B	0.85	1	0.73	0.87	0.70
C	0.78	0.83	1	0.64	0.73
D	0.55	0.74	1	0.89	0.92
E	1	1	1	1	1

Step-6:: Make initial assignment.

	M1	M2	M3	M4	M5	
A	0.61	0.77	0.50	$\boxed{1}$	0.81	
B	0.85	$\boxed{1}$	0.73	0.87	0.70	
C	0.78	0.83	$\boxed{1}$	0.64	0.73	√
D	0.55	0.74	$\cancel{1}$	0.89	0.92	√
E	$\boxed{1}$	$\cancel{1}$	$\cancel{1}$	$\cancel{1}$	$\cancel{1}$	
			√			

Here, 4th row and 5th column do not have any assignment. Thus the solution is not optimum and we go to next.

Step-7: Find out the largest number (ie,0.92) which does not have any line passing through it. Select the smallest number to divide from all the numbers that do not have any lines passing through them and multiply to all those numbers that have two lines passing through them. Keep the rest of them the same.

	M1	M2	M3	M4	M5
A	0.61	0.77	0.46	$\boxed{1}$	0.81
B	0.85	$\boxed{1}$	0.67	0.87	0.70
C	0.85	0.90	$\boxed{1}$	0.70	0.79
D	0.60	0.80	$\cancel{1}$	0.97	$\boxed{1}$
E	$\boxed{1}$	$\cancel{1}$	0.92	$\cancel{1}$	$\cancel{1}$

Here we see that all ones are either assigned or crossed out. That is, the total assigned one's is 5 which is equal to the number of rows or columns.

The optimum assignment is : $A \to M4,\ B \to M2,\ C \to M3,\ D \to M5,\ E \to M1.$
$\therefore$ The total maximum assignment profit = $10.10 + 8.40 + 11.10 + 8.00 + 1 = Rs. 38.60$

Step-8: Finally in our optimum solution the dummy row cost$(E, M1)$ take it 1 as 0.
$\therefore$ The total maximum assignment profit = $10.10 + 8.40 + 11.10 + 8.00 + 0 = Rs. 37.60$

Comparison of Optimal Values of two Methods

Example	HA- method	ROA-method	Optimum Value
2	37.60	37.60	37.60

CONCLUSION

In this paper, we presented a new method for solving unbalanced assignment problem and also same it as balanced assignment problem. And this method can be used for maximize as well as minimized objective functions of Assignment problem. Also in our numerical example shows that the optimal solution of HA-method and ROA-method are the same. So we can easy to solve the given Assignment problem using this different (ie, ROA-method) approach.

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