FIXED POINT THEOREMS USING WEAK COMPATIBILITY OWC IN FUZZY METRIC SPACE

MEENU[1], DR. VINOD KUMAR[2] AND DR. SUSHMA[3*]

[1]Research Scholar, Dept. of Mathematics, Baba Mastnath University, Rohtak, India.
&
Asst. Prof., Dept. of Mathematics, A.I.J.H.M. College, Rohtak, India.

[2]Professor, Department of Mathematics, Baba Mastnath University, Rohtak, India.

[3]Asst. Professor, Dept. of Mathematics, Kanya Mahavidyalaya, Kharkhoda (Sonepat), India.

ABSTRACT

In this paper, a fixed point theorem for six self mappings is presented by using the concept of weak compatible maps also presents some common fixed point theorems for occasionally weakly compatible mapping in fuzzy metric space.

Keywords: Common fixed point, Fuzzy metric space, occasionally weakly compatible mappings.

1. INTRODUCTION

The fixed point theory has been studied and generalized in different spaces. Fuzzy set theory is one of uncertainty approaches where in topological structure are basic tools to develop mathematical models compatible to concrete real life situation. Fuzzy set was defined by Zadeh [27]. Kramosil and Michalek [15] introduced fuzzy metric space, George and Veermani [7] modified the notion of fuzzy metric spaces with the help of continuous t−norms. Many researchers have obtained common fixed point theorem for mapping satisfying different types of commutativity conditions. Vasuki [26] proved fixed point theorems for R−weakly commuting mapping. Pant [19, 20, 21] introduced the new concept reciprocally continuous mappings and established some common fixed point theorems. Balasubramaniam [5] have show that Rhoades [23] open problem on the existence of contractive definition which generates a fixed point but does not force the mapping to be continuous at the fixed point, posses an affirmative answer. Recent literature on fixed point in fuzzy metric space can be viewed in [1, 2, 3, 10, 17].

Jain and Singh [29] proved a fixed point theorem for six self maps in a fuzzy metric space. In this paper, a fixed point theorem for six self maps has been established using the concept of weak compatibility of pairs of self maps in fuzzy metric space. Also presents some common fixed point theorems for more general commutative condition i.e. occasionally weakly compatible mappings in fuzzy metric space.

For the sake of completeness, we recall some definition and known results in fuzzy metric space.

2. PRELIMINARY NOTES

Definition 2.1: A fuzzy set A in X is a function with domain X and values in [0, 1].

Definition 2.2: A binary operation * : [0, 1] × [0, 1] → [0, 1] is a continuous t−norms if * is satisfying conditions
  (i)   * is an commutative and associative ;
  (ii) * is continuous ;
  (iii) a * 1 = a for all a ∈ [0, 1] ;
  (iv) a * b ≤ c * d whenever a ≤ c and b ≤ d, and a, b, c, d ∈ [0, 1].

Corresponding Author: Dr. Sushma[3]*
[3]*Asst. Professor, Dept. of Mathematics, Kanya Mahavidyalaya, Kharkhoda (Sonepat), India.*

Definition 2.3: A 3–tuple (X, M, *) is said to be a fuzzy metric space if X is an arbitrary set, * is a continuous t–norm and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions, for all x, y, z ∈ X, s, t > 0,

(F1)   M(x, y, t) > 0 ;
(F2)   M(x, y, t) = 1 if and only if x = y
(F3)   M(x, y, t) = M(y, x, t) ;
(F4)   M(x, y, t) * M(y, z, s) ≤ M(x, z, t + s) ;
(F5)   M(x, y, •) : (0, ∞) → (0, 1]is continuous

Then M is called a fuzzy metric on X. Then M(x, y, t) denotes the degree of nearness between x and y with respect to t.

Example 2.1: (Induced fuzzy metric) Let (X, d) be a metric space. Denote a * b = ab for all a, b ∈ [0, 1] and let M_d be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows

$$M_d(x, y, t) = \frac{t}{t + d(x + y)}$$

Then $(X, M_d, *)$ is a fuzzy metric space. We call this fuzzy metric induced by a metric d as the standard intuitionistic fuzzy metric.

Definition 2.4: Let (X, M, *) be a fuzzy metric space. Then
  (a) a sequence $\{x_n\}$ in X is said to converges to x in X if for each ∈ > 0 and each t > 0, there exists $n_0 ∈ N$ such that $M(x_n, x, t) > 1 − ∈$ for all $n ≥ n_0$.
  (b) a sequence $\{x_n\}$ in X is said to be Cauchy if for each ∈ > 0 and each t > 0, there exists $n_0 ∈ N$ such that $M(x_n, x_m, t) > 1 − ∈$ for all $n, m ≥ n_0$.
  (c) a fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.5: A pair of self mapping (f, g) of a fuzzy metric space (X, M, *) is said to be
  (i) weakly commuting if M(fgx, gfx, t) ≥ M(fx, gx, t) for all x ∈ X and t > 0
  (ii) R−weakly commuting if there exists some R > 0 such that M(fgx, gfx, t) ≥ M(fx, gx, t/R) for all x ∈ X and t > 0.

Definition 2.6: Two self mapping f and g of a fuzzy metric space (X, M, *) are called compatible if $\lim_{n\to\infty} M(fgx_n, gfx_n, t) = 1$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x$ for some x in X.

Definition 2.7: Two self maps f and g of a fuzzy metric space (X, M, *) are called reciprocally continuous on X if $\lim_{n\to\infty} fgx_n = fx$ and $\lim_{n\to\infty} gfx_n = gx$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x$ for some x in X.

Definition 2.8: Let X be a set, f, g self maps of X. A point x in X is called a coincidence point of f and g iff fx = gx. We shall call w = fx = gx a point of coincidence of f and g.

Definition 2.9: A pair of maps S and T is called weakly compatible pair if they commute at coincidence points i.e. if Sx = Tx for some x ∈ X then STx = TSx.

Definition 2.10: Two self maps f and g of a set X are occasionally weakly compatible (OWC) iff there is a point x in X which is a coincidence point of f and g at which f and g commute.

Al−Thagafi and Nasur Shahzad [4] shown that occasionally weakly is weakly compatible but converse is not true.

Example 2.2: Let R be the usual metric space. Define S, T : R → R by Sx = 2x and Tx = x^2 for all x ∈ R. Then Sx = Tx for x = 0, 2 but STO = TSO and ST2 ≠ TS2. S and T are occasionally weakly compatible self maps but not weakly compatible.

Proposition 2.1: Self mapping A and S of a fuzzy metric space (X, M, *) are compatible.

Proof: Suppose $A_p = S_p$ for some p in X. Consider a sequence $\{P_n\} = P$. Now $\{Ap_n\} → A_p$ and $\{Sp_n\} → S_p(A_p)$. As A and S are compatible we have $M(ASp_n, SAp_n, t) → 1$ for all t > 0 as n → ∞. Thus $ASp_n = SAp_n$ and we get that (A, S) is weakly compatible. The following is an example of pair of self maps in a fuzzy metric space which are weakly compatible but not compatible.

Example 2.3: Let (X, M, *) be a fuzzy metric space where X = [0, 2], t–norm is defined by a * b = min {a, b} for all

a, b $\in$ [0, 1] and M(x, y, t) = $e^{-\frac{|x-y|}{t}}$ for all x, y $\in$ X. Define self maps A and S on X as follows

$$A_x = \begin{cases} 2-x & \text{if } 0 \le x \le 1 \\ 2 & \text{if } 1 \le x \le 2 \end{cases} \quad \text{and} \quad S_x = \begin{cases} x & \text{if } 0 \le x < 1 \\ 2 & \text{if } 1 \le x \le 2 \end{cases}$$

Taking $\quad x_n = 1 - \dfrac{1}{n}, \; n = 1, 2, 3, ...$

Then $\quad x_n \to x_n < 1$ and $2 - x_n > 2$ for all.

Also $\quad Ax_n, Sx_n \to 1$ as $n \to \infty$

$$M(ASx_n, SAx_n, t) = e^{-\frac{1}{2}} \ne 1 \text{ as } n \to \infty$$

Hence the pair (A, S) is not compatible. Also set of coincidence points of A and S is [1, 2]. Now for any x $\in$ [1, 2]. Ax = Sx = 2 and AS(x) = A(2) = 2 = S(2) = SA(x). Thus A and S are weakly compatible but not compatible. From the above example, it is obvious that the concept of weak compatibility is more general than that of compatibility.

Proposition 2.2: In a fuzzy metric space (X, M, *) limit of a sequence is unique.

Lemma 2.1: Let (X, M, *) be a fuzzy metric space. Then for all x, y $\in$ X, M(x, y, $\bullet$) is a non–decreasing function.

Lemma 2.2: Let (X, M, *) be a fuzzy metric space. If there exists q $\in$ (0, 1) such that for all x, y $\in$ X, M(x, y, qt) $\ge$ M(x, y, t), for all t > 0, then x = y.

Lemma 2.3: Let {x_n} be a sequence in a fuzzy metric space (X, M, *). If there exists a number q $\in$ (0, 1) such that M(x_{n+2}, x_{n+1}, qt) $\ge$ M(x_{n+1}, x_n, t) for all t > 0 and n $\in$ N. Then {x_n} is a Cauchy sequence in X.

Lemma 2.4: Let X be a set, f, g owc self maps of X. If f and g have a unique point of coincidence, w = fx = gx, then w is the unique common fixed point of f and g.

Lemma 2.5: The only t–norm * satisfying r * r $\ge$ r for all r $\in$ [0, 1] is the minimum t–norm, that is a * b = min {a, b} for all a, b $\in$ [0, 1].

3. MAIN RESULTS

Theorem 3.1: Let (X, M, *) be a complete fuzzy metric space and let A, B, S, T, P and Q be mappings from X into itself such that the following conditions are satisfied
   (a) P(X) $\subset$ ST(X), Q(X) $\subset$ AB(X) :
   (b) AB = BA, ST = TS, PB = BP, QT = TQ;
   (c) either AB or P is continuous.
   (d) (P, AB) is compatible and (Q, ST) is weakly compatible;
   (e) there exists q $\in$ (0, 1) such that for every x, y $\in$ X and t > 0
      M(Px, Qy, qt) $\ge$ min {M(ABx, STy, t), M(Px, ABx, t), M(Qy, STy, t), M(Px, STy, t)}

Proof: Let $x_0 \in$ X. From (a) there exists x_1, $x_2 \in$ X such that $Px_0 = STx_1$ and $Qx_1 = ABx_2$. Inductively, we can construct sequence {x_n} and {y_n} in X such that $Px_{2n-2} = STx_{2n-1} = y_{2n-1}$ and $Qx_{2n-1} = ABx_{2n} = y_{2n}$ for n = 1, 2, 3, …

Step-1: Put x = x_{2n} and y = x_{2n+1} in (e), we get
$$M(Px_{2n}, Qx_{2n+1}, qt) \ge \min\{M(ABx_{2n}, STx_{2n+1}, t), M(Px_{2n}, ABx_{2n}, t),$$
$$M(Qx_{2n+1}, STx_{2n+1}, t), M(Px_{2n}, STx_{2n+1}, t)\}$$
$$= \min\{M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n}, t), M(y_{2n+2}, y_{2n+1}, t),$$
$$M(y_{2n+1}, y_{2n+1}, t)\} \ge M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n+2}, t)$$
Then from Lemma 2.1 and 2.2, we have

$$M(y_{2n+1}, y_{2n+2}, qt) \ge M(y_{2n}, y_{2n+1}, t)$$

Similarly, $\quad M(y_{2n+2}, y_{2n+3}, qt) \ge M(y_{2n+1}, y_{2n+2}, t)$

Thus, $\quad M(y_{n+1}, y_{n+2}, qt) \geq M(y_n, y_{n+1}, t)$ for all n = 1, 2, …

$$M(y_n, y_{n+1}, t) \geq M(y_n, y_{n+1}, \frac{t}{q}) \geq M(y_{n-2}, y_{n-1}, \frac{t}{q^2})$$

$$... \geq M(y_1, y_2, \frac{t}{q^n}) \rightarrow 1 \text{ as } n \rightarrow \infty$$

and hence $M(y_n, y_{n+1}, t) \rightarrow 1$ as $n \rightarrow \infty$ for any t > 0. For each $\in$ > 0 and t > 0, we can choose $n_0 \in N$ such that $M(y_n, y_{n+1}, t) > 1 - \in$ for all $n > n_0$. For m, n $\in$ N, we suppose m > n. Then we have

$$M(y_n, y_m, t) \geq \min\{M(y_n, y_{n+1}, \frac{t}{m-n}), M(y_{n+1}, y_{n+2}, \frac{t}{m-n}), ..., M(y_{m-1}, y_m, \frac{t}{m-n})\}$$

$$\geq \min\{(1-\in)(1-\in), ..., (1-\in)(m-n) \text{times}\}$$

$$\geq (1-\in) \text{ and hence } \{y_n\} \text{ is a Cauchy sequence in X.}$$

Since (X, M, *) is complete, $\{y_n\}$ converges to some point z $\in$ X. Also its subsequences converges to the some point z $\in$ X i.e.

$$\{Qx_{2n+1}\} \rightarrow z \text{ an } \{STx_{2n+1}\} \rightarrow z \tag{1}$$

$$\{Px_{2n}\} \rightarrow z \text{ and } \{ABx_{2n}\} \rightarrow z \tag{2}$$

Case-I: Suppose AB is continuous since AB is continuous, we have $(AB)^2 x_{2n} \rightarrow ABz$ and $ABPx_{2n} \rightarrow ABz$. As (P, AB) is compatible pair, then $PABx_{2n} \rightarrow ABz$.

Step-2: Put $x = ABx_{2n}$ and $y = x_{2n+1}$ in (e), we get
$$M(PABx_{2n}, Qx_{2n+1}, qt) \geq \min \{(M(ABABx_{2n}, STx_{2n+1}, t),$$
$$M(PABx_{2n}, ABABx_{2n}, t), M(Qx_{2n+1}, STx_{2n+1}, t), M(PABx_{2n}, STx_{2n+1}, t)\}$$

Taking n $\rightarrow \infty$, we get
$$M(ABz, z, qt) \geq \min\{(M(ABz, z, t), M(ABz, ABz, t), M(z, z, t), M(ABz, z, t)\}$$
$$= M(ABz, z, t)$$
i.e. $\quad M(ABz, z, qt) \geq M(ABz, z, t)$

Therefore, by using Lemma 2.2, we get
$$ABz = z \tag{3}$$

Step:3: Put x = z and $y = x_{2n+1}$ in (e), we have
$$M(Pz, Qx_{2n+1}, qt) \geq \min \{M(ABz, STx_{2n+1}, t), M(Pz, ABz, t), M(Qx_{2n+1}, STx_{2n+1}, t), M(Pz, STx_{2n+1}, t)\}$$

Taking n $\rightarrow \infty$ and using equation (1), we get
$$M(Pz, z, qt) \geq \min\{M(z, z, t), M(Pz, z, t), M(z, z, t), M(Pz, z, t)\}$$
$$= M(Pz, z, t),$$
i.e. $\quad M(Pz, z, qt) \geq M(Pz, z, t)$.

Therefore by using Lemma 2.2, we get Pz = z. Therefore ABz = Pz = z.

Step-4: Putting x = Bz and $y = x_{2n+1}$ in condition (e), we get
$$M(PBz, Qx_{2n+1}, qt) \geq \min \{(M(ABBz, STx_{2n+1}, t), M(PBz, ABBz, t),$$
$$M(Qx_{2n+1}, STx_{2n+1}, t), M(PBz, STx_{2n+1}, t)\}.$$
As BP = PB, AB = BA, so we have P(Bz) = B(Pz) = Bz and (AB)(Bz) = (BA)(Bz) = B(ABz) = Bz. Taking n $\rightarrow \infty$ and using (1), we get
$$M(Bz, z, qt) \geq \min \{M(Bz, z, t), M(Bz, Bz, t), M(z, z, t), M(Bz, z, t)\}$$
$$= M(Bz, z, t)$$
i.e. $\quad M(Bz, z, qt) \geq M(Bz, z, t)$.

Therefore, by using Lemma 2.2, we get Bz = z and also we have ABz = Az = z.
Therefore
$$Az = Bz = Pz = z \tag{4}$$

Step-5: As $P(X) \subset ST(X)$, there exists $u \in X$ such that $z = Pz = STu$. Putting $x = x_{2n}$ and $y = u$ in (e), we get
$$M(Px_{2n}, Qu, qt) \geq \min\{M(ABx_{2n}, STu, t), M(Px_{2n}, ABx_{2n}, t), M(Qu, STu, t), M(Px_{2n}, STu, t)\}$$

Taking $n \to \infty$ and using (1) and (2), we get
$$M(z, Qu, qt) \geq \min\{M(z, z, t), M(z, z, t), M(Qu, z, t), M(z, z, t)\}$$
$$= M(Qu, z, t)$$
i.e. $\qquad M(z, Qu, qt) \geq M(z, Qu, t)$

Therefore by using lemma 2.2., we get $Qu = z$. Hence $STu = z = Qu$. Since (Q, ST) is weak compatible.

Therefore, we have $QSTu = STQu$. Thus $Qz = STz$.

Step-6: Putting $x = x_{2n}$ and $y = z$ in (e), we get
$$M(Px_{2n}, Qz, qt) \geq \min \{M(ABx_{2n}, STz, t), M(Px_{2n}, ABx_{2n}, t), M(Qz, STz, t), M(Px_{2n}, STz, t)\}.$$

Taking $n \to \infty$ and using (2) and step 5, we get
$$M(z, Qz, qt) \geq \min \{M(z, Qz, t), M(z, z, t), M(Qz, Qz, t), M(z, Qz, t)\}$$
$$= M(z, Qz, t)$$
i.e. $\qquad M(z, Qz, qt) \geq M(z, Qz, t)$

Therefore, by using lemma 2.2., we get $Qz = z$.

Step-7: Putting $x = x_{2n}$ and $y = Tz$ in (e), we get
$$M(Px_{2n}, QTz, qt) \geq \min \{M(ABx_{2n}, STTz, t), M(Px_{2n}, ABx_{2n}, t)$$
$$M(QTz, STTz, t), M(Px_{2n}, STTz, t)\}.$$
As $QT = TQ$ and $ST = TS$, we have $QTz = TQz = Tz$ and $ST(Tz) = T(STz) = TQz = Tz$.

Taking $n \to \infty$, we get
$$M(z, Tz, qt) \geq \min\{M(z, Tz, t), M(z, z, t), M(Tz, Tz, t), M(z, Tz, t)\}$$
$$= M(z, Tz, t)$$
i.e. $\qquad M(z, Tz, qt) \geq M(z, Tz, t)$

Therefore by using lemma 2.2, we get $Tz = z$. Now $STz = Tz = z$ implies $Sz = z$. Hence
$$Sz = Tz = Qz = z \qquad\qquad\qquad\qquad\qquad (5)$$

Combining (4) and (5), we get
$$Az = Bz = Pz = Qz = Tz = Sz = z.$$ Hence, z is the common fixed point of A, B, S, T, P and Q.

Case-II: Suppose P is continuous, As P is continuous,
$$P^2 x_{2n} \to Pz \text{ and } P(AB)x_{2n} \to Pz.$$
As (P, AB) is compatible, we have $(AB)Px_{2n} \to Pz$.

Step-8: Putting $x = Px_{2n}$ and $y = x_{2n+1}$ in condition (e), we have
$$M(PPx_{2n}, Qx_{2n+1}, qt) \geq \min \{M(ABPx_{2n}, STx_{2n+1}, t),$$
$$M(PPx_{2n}, ABPx_{2n}, t), M(Qx_{2n+1}, STx_{2n+1}, t), M(PPx_{2n}, STx_{2n+1}, t)\}$$

Taking $n \to \infty$, we get
$$M(Pz, z, qt) \geq \min \{M(Pz, z, t), M(Pz, Pz, t), M(z, z, t), M(Pz, z, t)\}$$
$$= M(Pz, z, t)$$
i.e. $\qquad M(Pz, z, qt) \geq M(Pz, z, t)$

Therefore by using Lemma 2.2., we have $Pz = z$, further using steps 5, 6, 7, we get $Qz = STz = Sz = Tz = z$.

Step-9: As $Q(X) \subset AB(X)$, there exists $w \in X$ such that $z = Qz = ABw$. Put $x = w$ and $y = x_{2n+1}$ in (e), we have
$$M(Pw, Qx_{2n+1}, qt) \geq \min \{M(ABw, STx_{2n+1}, t), M(Pw, ABw, t),$$
$$M(Qx_{2n+1}, STx_{2n+1}, t), M(Pw, STx_{2n+1}, t)\}$$

Taking $n \to \infty$, we get
$$M(Pw, z, qt) \geq \min \{M(z, z, t), M(Pw, z, t), M(z, z, t), M(Pw, z, t)\}$$
$$= M(Pw, z, t)$$

i.e. $\qquad$ M(Pw, z, qt) $\geq$ M(Pw, z, t)

Therefore, by using Lemma 2.2, we get Pw = z.

Therefore ABw = Pw= Z. As (P, AB) is compatible, we have Pz = ABz. Also, from step 4, we get Bz = z.

Thus Az = Bz = Pz = z and we see that z is the common fixed point of the six maps in this case also.

Uniqueness: let u be another common fixed point of A, B, S, T, P and Q. Then Au = Bu = Pu = Qu = Su = Tu = u. Put x = z and y = u, in (e), we get
$\qquad$ M(Pz, Qu, qt) $\geq$ min. {M(ABz, STu, t), M(Pz, ABz, t), M(Qu, STu, t), M(Pz, STu, t)}.

Taking n $\to \infty$, we get
$\qquad$ M(z, u, qt) $\geq$ min{M(z, u, t), M(z, z, t), M(u, u, t), M(z, u, t)}
$\qquad\qquad$ = M(z, u, t)
i.e. $\qquad$ M(z, u, qt) $\geq$ M(z, u, t).

Therefore by using lemma 2.2, we get z = u. Therefore z is the unique fixed point of self–maps A, B, S, T, P and Q.

Corollary 3.1: Let (X, M, *) be a complete fuzzy metric space and let A, S, P and Q be mapping from X into itself such that the following conditions are satisfied :
  (a) P(X) $\subset$ S(X), Q(X) $\subset$ A(X) ;
  (b) either A or P is continuous ;
  (c) (P, A) is compatible and (Q, S) is weakly compatible ;
  (d) there exists q $\in$ (0, 1) such that for every x, y $\in$ X and t > 0
  (e) M(Px, Qy, qt) $\geq$ min{M(Ax, Sy, t), M(Px, Ax, t), M(Qy, Sy, t), M(Px, Sy,t)}
Then A, S, P and Q have a unique fixed point in X.

Theorem 3.2: Let (X, M, *) be a complete fuzzy metric space and let A, B, S, T, P and Q be self mappings of X. Let Pair {A, S}, {B, T} and {P, Q} be owc. If there exists q $\in$ (0, 1) such that
$\qquad$ M(Px, Qy, qt) $\geq$ min {M(ABx, STy, t), M(Px, ABx, t), M(Qy, STy, t), M(Px, STy, t)} $\qquad$ (1)
for all x, y $\in$ X and for all t > 0, then there exists a unique point w $\in$ X such that ABw = Pw = w and a unique point z $\in$ X such that STy = Qy = z. Moreover, z = w, so that there is a unique common fixed point of A, B, S, T, P and Q.

Proof: Let the pair {A, S}, {B, T} and {P, Q} be owc, so there are points x, y $\in$ X such that ABx = Px and STy = Qy, we claim that ABx = STy. If not, by inequality (1)
$\qquad$ M(Px, Qy, qt) $\geq$ min{M(ABx, STy, t), M(Px, ABx, t), M(Qy, STy, t), M(Px, STy, t)}
$\qquad\qquad$ = min{M(Px, Qy, t), M(Px, Px, t), M(Qy, Qy, t), M(Px, Qy, t)}
$\qquad\qquad$ = M(Px, Qy, t)
Therefore ABx = STy i.e. ABx = Px = STy = Qy.

Suppose that there is a another point z such that ABz = STz then By (1), we have ABz = Pz = STy = Qy so ABx = ABz and w = ABx = STx is the unique point of coincidence of A and S by Lemma 2.4, w is the only common fixed point of A and S. Similarly there is a unique point z $\in$ X such that z = Bz = Tz and z = Pz = Qz.

Assume that w $\neq$ z, we have
$\qquad$ M(w, z, qt) = M(Pw, Qw, qt)
$\qquad\qquad$ = min{M(ABw, STz, t), M(Pw, ABw, t), M(Qz, STz, t), M(Pw, STz, t)
$\qquad\qquad$ = min{M(w, z, t), M(w, w, t), M(z, z, t),M(w, z, t)}
$\qquad\qquad$ = M(w, z, t)
Therefore, we have z = w by lemma 2.4, and z is a common fixed point of A, B, S, T and Q. The uniqueness of the fixed point holds from (1).

Theorem 3.3: Let (X, M, *) be a complete fuzzy metric space and let A, B, S, T, P and Q be self mappings of X. Let the pair {A, S}, {B, T} and {P, Q} be owc. If there exists q $\in$ (0, 1) such that
$\qquad$ M(Px, Qy, qt) $\geq$ ϕ(min{M(ABx, STy, t), M(Px, ABx, t), M(Qy, STy, t), M(Px, STy, t)}) $\qquad$ (2)
for all x, y $\in$ X and ϕ : [0, 1] $\to$ [0, 1] such that ϕ(t) > 1 for all 0 < t < 1, then there exists a unique common fixed point of A, B, S, T, P and Q.

Proof: The proof follows theorem 3.2.

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