International Journal of Mathematical Archive-10(1), 2019, 21-25
IJMA Available online through www.ijma.info ISSN 2229 – 5046

TOTAL EFICIENT DOMINATION IN JUMP GRAPHS

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(Received On: 14-12-18; Revised & Accepted On: 05-01-19)

ABSTRACT

A set D of vertices of a jump graph J(G) is a total efiient dominating set, if every vertex in V(J(G)) is adjacent to exactly one vertex in D. Total efficient domination number $\gamma_{te}J(G))$ of J(G) is the minimum cardinality of a total efficient dominating set of J(G). In this paper the exact values of γ_{te} (J(G)) for some standared graphs are found and some bounds are obtained .Also a Nordhus-Gadumm type result is obtained . In addition the total efficient domatic number $d_{te}(J(G))$ of J(G) is defined to be maximum order of a partition of the vertex set of J(G) into total efficient dominating set of J(G). Also a relation between (J(G) and $d_{te}(J(G))$ is established.

Keywords; *Efficient dominating set, total dominating set, total efficient dominating set, total efficient domination number.*

Mathematics subject classification: *05C.*

INTRODUCTION

By a graph we mean a finite, undirected without loops multiple edges and isolated vertices Terms undefined here may be found in Kulli [1]

A set D of vertices in ajump graph J(G) = (V,E) is called dominating set if every vertex in V-D is adjacent to some vertex in D. The domination number γ(J(G)) of graph J(G) is the minimum cardinality of a dominating set of J(G). Recently many new domination parameter given by Venkangoud et.al.,

A dominating set D of J(G) is an efficient dominating set if every vertex in V – D ia adjacent to exactly one vertex in D. The efficient domination number γ_e(J(G)) of J(G) is the minimum cardinality of an efficient dominating set of J(G).

Kulli and Patwari [28] introduced the concept of total domination.

A set D of vertices in a graph J(G) is a total efficient dominating set of J(G) if every vertex in V is adjacent to exactly one vertex in D. The total efficient domination number γ_{te}(J(G)) of J(G) is the minimum cardinality of a total efficient dominating set of J(G).

A γ_{te}-set is a minimum total efficient dominating set. Let Δ(J(G) (δ(J(G))) denote the maximum 9minimum) dgree mong the vertices of J(G), let $\lceil x \rceil$ denote the least integer greater than or equal to x.

We note that γ_t(J(GH)) and γ_{te}(J(G)) are only defined for J(G) with $\delta(J(G)) \geq 1$

2. TOTAL EFFICIENT DOMINATION NUMBER

We list exact values of the total efficient domination number for some standard graphs**.**

Proposition 1: If P_p is a path with p vertices, then

$$\gamma_{te}(J(P_p)) = \lceil \tfrac{p}{2} \rceil \text{ where } p \equiv 0 \pmod 4 \text{ and } p \equiv 3 \pmod 4$$

Proposition 2: If C_p is a cycle with p vertices, then

$$\gamma_{te}(J(C_p)) = \tfrac{p}{2} \text{ when } p \equiv 0 \pmod 4$$

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Proposition 3: If $K_{m,n}$ is a complete biparatite graph $1 \leq m \leq n$, then

$\gamma_{te}(J(K_{m,n})) = 2$

Remark 4: Every graph J(G) without isolated vertices does not contain a total efficient dominating set. It implies that $\gamma_{te(}J(G))$ does not exist.

Proposition 5: If K_p is a complete graph with $p \geq 3$ vertices, then $\gamma_{te}(J(K_p))$ does not exist.

Proposition 6: If $\gamma_{te}(J(G))$ exists, then

$\gamma_t(J(G)) \leq \gamma_{te}(J(G))$ and this bound is sharp.

Proof: Clearly every efficient total dominating set is an efficient dominating set, thus the above inequality holds

The complete bipartite graphs $K_{m,n}$ $1 \leq m \leq n$ achieve this bound.

Proposition 7: If $\gamma_{te}(J(G))$ exists, then

$\gamma_t(J(G)) \leq \gamma_{te}(J(G))$ and this bound is sharp.

Proof: Clearly every efficient total dominating set is an efficient dominating set, thus the above inequality holds
The complete bipartite graphs $K_{m,n}$ $2 \leq m \leq n$ achieve this bound.

Theorem 8: Let J(G) be a (p, q) connected graph with $p \geq 2$ vertices, Then

$2(p - q) \leq \gamma_{te}(J(G))$.

Furthermore inequality holds if and only if J(G) is a tree with exactly one cut vertex or exactly two cut vertices.

Proof: Let D be a –set of J(G). Then for each vertex $u \in V - D$, there exists a vertex v in D such that $uv \in E$ Also for each vertex $x \in D$, there exists unique vertex $y \in D$ such that $xy \in E$. Then

$q \geq \frac{|D|}{2} + |V - D|$
or $2q \geq |D| + 2|v - D|$
or $2q \geq \gamma_{te}(J(G)) + 2p - 2\gamma_{te(}(J(G))$
or $2(q - P) \leq \gamma_{te}(J(G))$

We prove the second part.

Suppose J(G) is a tree with exactly one cut vertex or two cut vertices.
Then $\gamma_{te}(J(G)) = 2 = 2(p - q)$, since $p - q = 1$

Conversely suppose $\gamma_{te}(J(G)) = 2(p - q)$,. We now prove that J(G) is a tree with at most two cut vertices. Clearly for any graph without isolated vertices, $\gamma_{te}(J(G)) \geq 2$

Suppose $p < q$. Then $2(p - q)$ is negative, which is a contradiction.

Suppose $p = q$. Then $2(p - q)$ is zero, which is contradiction.

Suppose $p>q$. Since J(G) is connected, it implies that J(G) is a tree with exactly 3 vertices, then t Remark 4 $\gamma_{te}(J(G))$ does not exist. If J(G) is a tree with at least 4 cut vertices then $\gamma_{te}(J(G)) \geq 4 \neq 2(p - q)$, since $p - q=1$. Thus we conclude that J(G) is a tree with at most two cut vertices.

Next we characterize graphs for which $\gamma_{te}(J(G))=p$.

Theorem 9: Let J(G) be a graph without isolated vertices and with $p \geq 2$vertices. Thus $\gamma_{te}(J(G)) =p$ if and only if $J(G) = mK_2$, $m \geq 1$.

Proof: Suppose $J(G)= mK_2$, $m\geq 1$. Obviously $\gamma_{te}(J(G)) = p$

Conversely Suppose $\gamma_{te}(J(G)) =p$ We now prove that $J(G)= mK_2$, $m \geq 1$. Assume $J(G) \neq mK_2$. Then $deg_G\ u \geq 2$ .Let D be a $\gamma_{te}(J(G))$ –set of J(G). Since $\gamma_{te}(J(G)) =p$, it implies that $|V - D| = \phi$. Hence $u \in D$. Since $deg_G u \geq 2$, it implies that u is adjacent with at least two vertices in D, which is a contradiction. Suppose $deg_G\ u < 1$. Then u is an isolated vertex, again a contradiction.

Thus $deg_G\ u = 1$ Since u is arbitrary6, it follow that $J(G)= mK_2$, $m\geq 1$.

The following theorem gives a lower bound for γ_{te}(J(T)).

Theorem 10 : Let J(T) be a tree with p ≥ 3 vertices, If γ_{te}(J(T)) exists, then

$$\gamma_{te}(J(T)) \leq \lceil \frac{m}{2} \rceil + 1$$

Where m is the number of cut vertex of J(T).

Proof: Let J(T) be a tree with p ≥ 3 vertices Suppose γ_{te}(J(T)) exists. We now prove that $\gamma_{te}(J(T)) \leq \lceil \frac{m}{2} \rceil + 1$. On the contrary, assume $\gamma_{te}(J(T)) \leq \lceil \frac{m}{2} \rceil +1$. Then there exist 3 cut vertices u, v, w in D such that uv, vw are edges of J(T) whee D is a γ_{te}-set of J(T). By remark 4 γ_{te}(J(T)) does not exist which is a contradiction. This prove that $\gamma_{te}(J(T)) \leq \lceil \frac{m}{2} \rceil + 1$.

Nordhaus-Gaddum type results were obtained for many parameters for example, in [30, 31, 32, 33, 34, 35, 36].

Now we establish Nordhaus-Gaddum type result.

Theorem 11: Let J(G) and J($\bar{G}$) have no isolated vertices. If both γ_{te}(J(G)) and γ_{te}(J($\bar{G}$)) exist, then

$$4 \leq \gamma_{te}(J(G)) + \gamma_{te}(J(\bar{G})). \leq p+3.$$

Proof: Let J(G) and J($\bar{G}$) have no isolated vertices. If both γ_{te}(J(G)) and γ_{te}(J($\bar{G}$)) exist, then $\gamma_{te}(J(G)) \geq 2$ and $\gamma_{te}(J(\bar{G})) \geq 2$ Therefore $4 \leq \gamma_{te}(J(G)) + \gamma_{te}(J(\bar{G}))$.

We have $\gamma_{te}(J(G)) \leq p - \Delta(J(G)) + 1$

Therefore $\gamma_{te}(J(G)) \leq p - \delta(J(G)) + 1$

Also we have $\gamma_{te}(J(\bar{G})) \leq p - \Delta(\bar{G}) + 1$

Thus

$$\begin{aligned} \gamma_{te}(J(G)) + \gamma_{te}(J(\bar{G})) &\leq 2p - [\delta(J(G)) + \Delta(\bar{G})] + 2 \\ &\leq p - (p - 1) + 2 \\ &\leq p + 3 \end{aligned}$$

The graph P_4 achieves the lower bound.

3. TOTAL EFFICIENT DOMATIC NUMBER

Definition 12: The total efficient dogmatic number d_{te}(J(G)) for some standard graphs.

Proposition 13: For any cycle C_{4n}, n≥ 1

$$d_{te}(J(C_{4n})) = 2$$

Proposition 14: For any complete bipartite graph $K_{m,n}$ $1 \leq m \leq n$

$$d_{te}(J(K_{m,n})) = m$$

Proposition 15: For any tree T with p ≥ 2 vertices,

$$d_{te}(J(T)) = 1$$

Proposition 16: Let J(G) be a graph without isolated vertices, If γ_{te}(J(G)) exists, then

$$d_{te}(J(G)) \leq \frac{P}{\gamma_{te}(J(G))}$$

Proposition 17: Let J(G) be a graph without isolated vertices If d_{te}(J(G)) exists, then

$$d_{te}(J(G)) \leq \delta(J(G)).$$

Proposition 18: If J(G) is a graph without isolated vertices and if γ_{te}(J(G)) exists, then

$$\gamma_{te}(J(G)) + d_{te}(J(G)) \leq p + 1.$$

Furthermore, equality holds if J(G) = mK_2 $m \geq 1$

Proof: By theorem 11 we have

$$\gamma_{te}(J(G)) \leq p - \Delta(J(G)) + 1$$

Or

$$\gamma_{te}(J(G)) \leq p - \delta(J(G)) + 1$$

By proposition 17 we have $\gamma_{te}(J(G)) \leq \delta(J(G))$.

Hence $\gamma_{te}(J(G)) + d_{te}(J(G)) \leq p + 1$.

We pove the second part.

If $J(G) = mK_2$, $m \geq 1$ then by theoem9, $\gamma_{te}(J(G)) = p$. Also $d_{te}(J(G)) = 1$

Thus $\gamma_{te}(J(G)) + d_{te}(J(G)) = p + 1$.

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Source of support: Nil, Conflict of interest: None Declared.