LOGARITHMIC MEAN LABELING OF PATH RELATED GRAPHS

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ABSTRACT

A function \( f \) is called a logarithmic mean labeling of a graph \( G(V, E) \) with \( p \) vertices and \( q \) edges if \( f : V(G) \rightarrow \{1,2,3,...,q+1\} \) is injective and the induced function \( f^*: E(G) \rightarrow \{1,2,3,...,q\} \) defined as

\[
f^*(uv) = \left\lfloor \frac{f(v)-f(u)}{\ln(f(v))-\ln(f(u))} \right\rfloor, \text{ for all } uv \in E(G),
\]

is bijective. A graph that admits a logarithmic mean labeling is called a logarithmic mean graph. In this paper, we have discussed the logarithmic mean labeling of the graphs path \( P_n \), the star graph \( S_n \), \( P_n \circ K_1 \), \( TW(P_n) \), the graph \( P_n \circ S_m \), the graph \( P_n \circ S_m \), square graph of a path, total graph of a path, middle graph of a path, the graph \( S(P_n \circ K_1) \) and the arbitrary subdivision of \( S_3 \).

Keywords: labeling, logarithmic mean labeling, logarithmic mean graph.

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1. INTRODUCTION

Throughout this paper, by a graph we mean a finite, undirected and simple graph. Let \( G(V, E) \) be a graph with \( p \) vertices and \( q \) edges. For notations and terminology, we follow [4]. For a detailed survey on graph labeling we refer to [3].

Path on \( n \) vertices is denoted by \( P_n \). A tree is a connected acyclic graph. \( P_n \) is a tree obtained from a path on \( n \) vertices by attaching \( X_i \) pendant vertices at each \( i \)-th vertex of the path, for \( 1 \leq i \leq n \). A Twig \( TW(P_n) \), \( n \geq 3 \) is a graph obtained from a path by attaching exactly two pendant vertices to each internal vertices of the path. If \( v_1^{(i)}, v_2^{(i)}, v_3^{(i)}, ..., v_{m+1}^{(i)} \) and \( u_1,u_2,...,u_n \) be the vertices of the star graph \( S_m \) and the path \( P_n \), then the graph \( [P_n,S_m] \) is obtained from \( n \) copies of \( S_m \) and the path \( P_n \) by joining \( u_i \) with the central vertex \( v_1^{(i)} \) of the \( i \)-th copy of \( S_m \) by means of an edge, for \( 1 \leq i \leq n \). The corona \( G_1 \circ G_2 \) is a graph obtained by taking one copy of \( G_1 \) of order \( p_1 \) and \( p_1 \) copies of \( G_2 \) and then joining the \( i \)-th vertex of \( G_1 \) with every vertex in the \( i \)-th copy of \( G_2 \). The graph \( G_1 \circ K_2 \) is called as comb. When \( G_2 = G_1^{m} \), then the graph \( G_1 \circ G_2 \) is denoted as \( G_1 \circ S_m \). Square of a graph \( G \), denoted by \( G^2 \), has the vertex set as \( V \) and two vertices are adjacent in \( G^2 \) if they are at a distance either 1 or 2 apart in \( G \). The total graph \( T(G) \) of a graph \( G \) is the graph whose vertex set is \( V(G) \cup E(G) \) and two vertices are adjacent if and only if either they are adjacent vertices of \( G \) or adjacent edges of \( G \) or one is a vertex of \( G \) and the other one is an edge incident on it. The middle graph \( M(G) \) of a graph \( G \) is the graph whose vertex set is \( \{v: v \in V(G)\} \cup \{e: e \in E(G)\} \) and the edge set is \( \{e_1e_2: e_1,e_2 \in E(G) \text{ and } e_1 \text{ and } e_2 \text{ are adjacent edges of } G\} \cup \{ve: v \in V(G), e \in E(G) \text{ and } v \text{ is incident with } e\} \). For a graph \( G \) the graph \( S(G) \) is obtained by subdividing each edge of \( G \) by a vertex. An arbitrary subdivision of a graph \( G \) is a graph obtained from \( G \) by a sequence of elementary subdivisions forming edges into paths through new vertices of degree 2. An arbitrary super subdivision \( P(m_1,m_2,...,m_{n-1}) \) of a path \( P_n \) is a graph obtained by replacing each \( i \)-th edge of \( P_n \) by identifying its end vertices of the edge with a partition of \( K_{2,m_i} \) having \( 2 \) elements, where \( m_i \) is any positive integer.

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The concept of mean labeling was first introduced by S. Somasundaram and R. Ponraj [5] and it was developed in [6]. Similarly the concept of $F$-geometric mean labeling was first introduced by A. Durai Baskar et al. [1] and it was developed in [2].

Motivated by the works of so many authors in the area of graph labeling, we introduced a new type of labeling called logarithmic mean labeling. A function $f$ is called a logarithmic mean labeling of a graph $G(V,E)$ if $f:V(G) \rightarrow \{1, 2, 3, \ldots, q + 1\}$ is injective and the induced function $f^*: (V(G)) \rightarrow \{1, 2, 3, \ldots, q\}$ defined as

$$f^*(uv) = \frac{f(v) - f(u)}{\ln f(v) - \ln f(u)}$$

is bijective. A graph that admits a logarithmic mean labeling is called a logarithmic mean graph.

Figure-1.1: A logarithmic mean graph

In this paper, we have obtained the logarithmic meanness of the graphs path $P_n$, the star graph $S_n$, $P_n(X_1,X_2,\ldots,X_n)$, $TW(P_n)$, the graph $P_n + S_m$, the graph $[P_n;S_m]$, square graph of a path, total graph of a path, middle graph of a path, the graph $P(1,2,3,\ldots,n-1)$, the graph $S(P_n + K_1)$ and the arbitrary subdivision of $S_3$.

2. MAIN RESULTS

Theorem 2.1: Every path is a logarithmic mean graph.

Proof: Let $v_1,v_2,\ldots,v_n$ be the vertices of the path $P_n$. We define $f:V(P_n) \rightarrow \{1,2,\ldots,n\}$ as follows $f(v_i) = i$, for $1 \leq i \leq n$. The induced edge labeling is as follows $f^*(v_iv_{i+1}) = i$, for $1 \leq i \leq n-1$. Hence $f$ is a logarithmic mean labeling of the path $P_n$. Thus the path $P_n$ is a logarithmic mean graph.

Figure-2.1: A logarithmic mean labeling of $P_7$.

Theorem 2.2: Union of any two trees is not a logarithmic mean graph.

Proof: Let $G$ be the union of two trees $S$ and $T$, then $|V(G)| = |V(S)| + |V(T)|$ and $|E(G)| = |E(S)| + |E(T)| = |V(S)| + |V(T)| - 2$. Since $|V(G)| > |E(G)| + 1$, a logarithmic mean labeling does not exist on $G$.

Corollary 2.3: Any forest is not a logarithmic mean graph.

Proof: By the above Theorem 2.2, the result follows.

Theorem 2.4: The star graph $S_n$ is a logarithmic mean graph if and only if $n \leq 3$.

Proof: The number of vertices and edges of $S_n$ are $n + 1$ and $n$ respectively. If $f$ is a logarithmic mean labeling of $S_n$ then it is a bijective function from $V(S_n)$ to $\{1, 2, \ldots, n + 1\}$. As we have to label 1 for an edge, the vertex labels of its pair of adjacent vertices are either 1 and 2 or 1 and 3. So, the central vertex of $S_n$ is labeled as either 1 or 2 or 3. 1 cannot be a label for the central vertex in case of $n \geq 2$, since two of the pendant vertices of $S_n$ are labeled as 2 and 3. When $n \geq 3$, 2 cannot be the label for the central vertex, since two of its pendant vertices having the labels 3 and 4. When $n \geq 4$, the pendant vertices are labeled to be 4 and 5 if the label of central vertex is 3.

Figure-2.2: A logarithmic mean labeling of $S_n, n \leq 3$. 

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Theorem 2.5: \( P_n(X_1,X_2,...,X_n) \) is a logarithmic mean graph, for \( 1 \leq X_i \leq 3 \) and \( |X_i - X_{i+1}| \leq 1 \), for \( 1 \leq i \leq n \).

Proof: Let \( u_1,u_2,\ldots,u_n \) be the vertices of the path \( P_n \). Let \( v_1^{(i)},v_2^{(i)},\ldots,v_{n+1}^{(i)} \) be the pendant vertices attached at \( u_j \) for \( 1 \leq j \leq n \).

Define \( f:V(P_n(X_1,X_2,...,X_n)) \to \{1,2,3,\ldots,\sum_{i=1}^{n} X_i + n\} \) as follows:

\[
f(v_i^{(i)}) = \begin{cases} 
  f(v_i^{(i)}) + 2 & X_i = 2 \text{ and } j = 2 \\
  f(v_i^{(i)}) + 1 & X_i = 3 \text{ and } j = 2 \\
  f(v_i^{(i)}) + 3 & X_i = 3 \text{ and } j = 3
\end{cases}
\]

for \( 2 \leq i \leq n \),

\[
f(u_i) = \begin{cases} 
  f(v_i^{(i)}) + 1 & X_i = 1 \\
  f(v_i^{(i)}) + 2 & X_i = 2
\end{cases}
\]

The induced edge labeling is as follows:

\[
f^*(u_iu_{i+1}) = \begin{cases} 
  f(u_i) & X_i = 1 \text{ and } j = 2 \\
  f(u_i) - 1 & X_i = 3 \text{ and } j = 2 \\
  f(u_i) & X_i = 3 \text{ and } j = 3
\end{cases}
\]

for \( 1 \leq i \leq n \),

\[
f^*(v_i^{(j)}u_i) = \begin{cases} 
  f(u_i) & X_i = 1 \text{ and } j = 2 \\
  f(u_i) - 1 & X_i = 3 \text{ and } j = 2 \\
  f(u_i) & X_i = 3 \text{ and } j = 3
\end{cases}
\]

Hence, \( f \) is a logarithmic mean labeling of \( P_n(X_1,X_2,...,X_n) \). Thus the graph \( P_n(X_1,X_2,...,X_n) \) is a logarithmic mean graph for \( 1 \leq X_i \leq 3 \) and \( |X_i - X_{i+1}| \leq 1 \).

Corollary 2.6: \( TW(P_n) \) is a logarithmic mean graph for \( m \leq 3 \).

Corollary 2.7: \( P_n \circ S_m \) is a logarithmic mean graph for \( m \leq 3 \).

Theorem 2.8: \([P_n,S_m]\) is a logarithmic mean graph, for \( m \leq 2 \) and \( n \geq 1 \).

Proof: Let \( u_1,u_2,\ldots,u_n \) be the vertices of the path \( P_n \) and Let \( v_1^{(i)},v_2^{(i)},\ldots,v_{m+1}^{(i)} \) be the vertices of the star graph \( S_m \) such that \( v_1^{(i)} \) is the central vertex of \( S_m \), for \( 1 \leq i \leq n \).

Case-1. \( m = 1 \).

Define \( f:V([P_n;S_m]) \to \{1,2,3,\ldots,3n\} \) as follows:

\[
f(u_i) = \begin{cases} 
  3i & 1 \leq i \leq n \text{ and } i \text{ is odd} \\
  3i - 2 & 1 \leq i \leq n \text{ and } i \text{ is even}
\end{cases}
\]

\[
f(v_1^{(i)}) = \begin{cases} 
  3i - 1 & 1 \leq i \leq n \text{ and } i \text{ is odd} \\
  3i & 1 \leq i \leq n \text{ and } i \text{ is even}
\end{cases}
\]

The induced edge labeling is as follows:

\[
f^*(u_iu_{i+1}) = \begin{cases} 
  3i & 1 \leq i \leq n - 1, \\
  3i - 1 & 1 \leq i \leq n \text{ and } i \text{ is odd} \\
  3i - 2 & 1 \leq i \leq n \text{ and } i \text{ is even}
\end{cases}
\]

\[
f^*(u_iv_1^{(i)}) = \begin{cases} 
  3i - 1 & 1 \leq i \leq n \text{ and } i \text{ is odd} \\
  3i - 2 & 1 \leq i \leq n \text{ and } i \text{ is even}
\end{cases}
\]
Case-ii. \( m = 2 \).
Define \( f : V([P_n; S_m]) \to \{1, 2, 3, \ldots, 4n\} \) as follows:
\[
f(u_i) = \begin{cases} 
4i & 1 \leq i \leq n \text{ and } i \text{ is odd} \\
4i - 2 & 1 \leq i \leq n \text{ and } i \text{ is even},
\end{cases}
\]
\[
f(v_1^{(0)}) = 4i - 1, \text{ for } 1 \leq i \leq n,
\]
\[
f(v_2^{(0)}) = 4i - 3, \text{ for } 1 \leq i \leq n \text{ and}
\]
\[
f(v_3^{(0)}) = \frac{4i}{4}, \text{ for } 1 \leq i \leq n \text{ and } i \text{ is even}.
\]

The induced edge labeling is as follows:
\[
f^*(u_i u_{i+1}) = 4i, \text{ for } 1 \leq i \leq n-1
\]
\[
f^*(u_i v_1^{(0)}) = \frac{4i}{4}, \text{ for } 1 \leq i \leq n \text{ and } i \text{ is odd}
\]
\[
f^*(v_1^{(0)} v_2^{(0)}) = 4i - 3, \text{ for } 1 \leq i \leq n \text{ and}
\]
\[
f^*(v_1^{(0)} v_3^{(0)}) = \frac{4i}{4}, \text{ for } 1 \leq i \leq n \text{ and } i \text{ is even}.
\]

Hence, \( f \) is a logarithmic mean labeling of \([P_n; S_m]\). Thus the graph \([P_n; S_m]\) is a logarithmic mean graph, for \( m \leq 2 \) and \( n \geq 1 \).

**Theorem 2.9:** \( P_n^2 \) is a logarithmic mean graph for every \( n \geq 3 \).

**Proof:** Let \( v_1, v_2, \ldots, v_n \) be the vertices of the path \( P_n \).
Define \( f : V(P_n^2) \to \{1, 2, 3, \ldots, 2(n-1)\} \) as follows:
\[
f(v_i) = 2i - 1, \text{ for } 1 \leq i \leq n - 1
\]
\[
f(v_n) = 2(n-1).
\]

The induced edge labeling is as follows:
\[
f^*(v_i v_{i+1}) = 2i - 1, \text{ for } 1 \leq i \leq n - 1
\]
\[
f^*(v_i v_{i+2}) = 2i, \text{ for } 1 \leq i \leq n - 2.
\]

Hence, \( f \) is a logarithmic mean labeling of the graph \( P_n^2 \). Thus the graph \( P_n^2 \) is a logarithmic mean graph, for \( n \geq 3 \).

**Theorem 2.10:** \( T(P_n) \) is a logarithmic mean graph, for any \( n \geq 2 \).

**Proof:** Let \( V(P_n) = \{v_1, v_2, \ldots, v_n\} \) and \( E(P_n) = \{e_i = v_i v_{i+1}; 1 \leq i \leq n-1\} \) be the vertex set and edge set of the path \( P_n \). Then
\[
V(T(P_n)) = \{v_1, v_2, \ldots, v_n, e_1, e_2, \ldots, e_{n-1}\}
\]
\[
E(T(P_n)) = \{e_i v_{i+1}, e_i v_{i+2}, e_i v_{i+3}; 1 \leq i \leq n-1\} \cup \{e_i e_{i+1}; 1 \leq i \leq n-2\}.
\]

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Define \( f: V(T(P_n)) \to \{1,2,3,...,A(n-1)\} \) as follows:
\[
\begin{align*}
  f(v_i) &= 4i - 3, \quad \text{for } 1 \leq i \leq n-1, \\
  f(v_n) &= 4n - 4 \quad \text{and} \\
  f(e_i) &= 4i - 1, \quad \text{for } 1 \leq i \leq n-1.
\end{align*}
\]

The induced edge labeling is as follows:
\[
\begin{align*}
  f^*(v_iv_{i+1}) &= 4i - 2, \quad \text{for } 1 \leq i \leq n-1, \\
  f^*(e_ie_{i+1}) &= 4i, \quad \text{for } 1 \leq i \leq n-2, \\
  f^*(e_iv_i) &= 4i - 3, \quad \text{for } 1 \leq i \leq n-1 \quad \text{and} \\
  f^*(e_i v_{i+1}) &= 4i - 1, \quad \text{for } 1 \leq i \leq n-1.
\end{align*}
\]

Hence, \( f \) is a logarithmic mean labeling of the graph \( T(P_n) \). Thus the graph \( T(P_n) \) is a logarithmic mean graph, for \( n \geq 2 \).

---

**Theorem 2.11:** The middle graph of a path is a logarithmic mean graph.

**Proof:** Let \( V(P_n) = \{v_1, v_2, ..., v_n\} \) and \( E(P_n) = \{e_i = v_i v_{i+1}; 1 \leq i \leq n-1\} \) be the vertex set and edge set of the path \( P_n \).

Then \( V(M(P_n)) = \{v_1, v_2, ..., v_n, e_1, e_2, ..., e_{n-1}\} \) and \( E(M(P_n)) = \{v_i e_i, e_i v_{i+1}; 1 \leq i \leq n-1\} \cup \{e_i e_{i+1}; 1 \leq i \leq n-2\} \).

Define \( f: V(M(P_n)) \to \{1,2,...,3n-3\} \) as follows:
\[
\begin{align*}
  f(v_i) &= 3i - 2, \quad \text{for } 1 \leq i \leq n-1, \\
  f(v_n) &= 3n - 3 \quad \text{and} \\
  f(e_i) &= 3i - 1, \quad \text{for } 1 \leq i \leq n-1.
\end{align*}
\]

The induced edge labeling is as follows:
\[
\begin{align*}
  f^*(v_i e_i) &= 3i - 2, \quad \text{for } 1 \leq i \leq n-1, \\
  f^*(e_i e_{i+1}) &= 3i - 1, \quad \text{for } 1 \leq i \leq n-1 \quad \text{and} \\
  f^*(e_i v_{i+1}) &= 3i, \quad \text{for } 1 \leq i \leq n-2.
\end{align*}
\]

Hence, \( f \) is a logarithmic mean labeling of the middle graph \( M(P_n) \). Thus the middle graph \( M(P_n) \) is a logarithmic mean graph.

---

**Theorem 2.12:** For any \( n \geq 2, P(1,2,3,...,n-1) \) is a logarithmic mean graph.

**Proof:** Let \( v_1, v_2, ..., v_n \) be the vertices of the path \( P_n \) and let \( u_{ij} \) be the vertices of the partition of \( K_{m_i} \) with cardinality \( m_i, 1 \leq i \leq n-1 \) and \( 1 \leq j \leq m_i \).

Define \( f: V(P(1,2,...,n-1)) \to \{1,2,3,...,n(n-1) + 1\} \) as follows:
\[
\begin{align*}
  f(v_i) &= i(i-1) + 1, \quad \text{for } 1 \leq i \leq n \quad \text{and} \\
  f(u_{ij}) &= i(i-1) + 2j, \quad \text{for } 1 \leq j \leq i \quad \text{and} \quad 1 \leq i \leq n-1.
\end{align*}
\]

The induced edge labeling is as follows:
\[
\begin{align*}
  f^*(v_i u_{ij}) &= i(i-1) + j, \quad \text{for } 1 \leq j \leq i \quad \text{and} \quad 1 \leq i \leq n-1 \quad \text{and} \\
  f^*(u_{ij} v_{i+1}) &= i^2 + j, \quad \text{for } 1 \leq j \leq i \quad \text{and} \quad 1 \leq i \leq n-1.
\end{align*}
\]

Hence, \( f \) is a logarithmic mean labeling of the graph \( P(1,2,...,n-1) \). Thus the graph \( P(1,2,...,n-1) \) is a logarithmic mean graph.
Theorem 2.13: \( S(P_n \circ K_1) \) is a logarithmic mean graph, for \( n \geq 2 \).

Proof: Let \( V[P_n \circ K_1] = \{u_i, v_i; 1 \leq i \leq n\} \). Let \( x_i \) be the vertex which divides the edge \( u_iv_i \), for \( 1 \leq i \leq n \) and \( y_i \) be the vertex which divides the edge \( u_{i+1}v_i \), for \( 1 \leq i \leq n - 1 \). Then \( V[S(P_n \circ K_1)] = \{u_i, v_i, x_i, y_i; 1 \leq i \leq n, 1 \leq j \leq n - 1\} \) and \( E[S(P_n \circ K_1)] = \{u_ix_i, v_ix_i; 1 \leq i \leq n\} \cup \{u_iy_i, y_iu_{i+1}; 1 \leq i \leq n - 1\} \).

We define \( f: V[S(P_n \circ K_1)] \rightarrow \{1, 2, 3, \ldots, 4n - 1\} \) as follows:

\[
\begin{align*}
  f(u_i) &= 4i - 1, \text{ for } 1 \leq i \leq n, \\
  f(v_i) &= 4i + 1, \text{ for } 1 \leq i \leq n - 1, \\
  f(x_i) &= 4i - 2, \text{ for } 1 \leq i \leq n \text{ and} \\
  f(y_i) &= \begin{cases} 
    4i - 4, & \text{if } i = 1 \\
    2, & \text{if } 2 \leq i \leq n.
  \end{cases}
\end{align*}
\]

The induced edge labeling is as follows:

\[
\begin{align*}
  f^*(u_iv_i) &= 4i - 1, \text{ for } 1 \leq i \leq n - 1 \\
  f^*(y_iu_{i+1}) &= 4i + 1, \text{ for } 1 \leq i \leq n - 1 \\
  f^*(u_ix_i) &= 4i - 2, \text{ for } 1 \leq i \leq n \text{ and} \\
  f^*(v_ix_i) &= \begin{cases} 
    4i - 4, & \text{if } i = 1 \\
    2, & \text{if } 2 \leq i \leq n.
  \end{cases}
\end{align*}
\]

Hence \( f \) is a logarithmic mean labeling of \( S(P_n \circ K_1) \). Thus the graph \( S(P_n \circ K_1) \) is a logarithmic mean graph, for \( n \geq 2 \).

Theorem 2.14: Arbitrary subdivision of \( S_3 \) is a logarithmic mean graph.

Proof: Let \( G \) be a graph of arbitrary subdivision of \( S_3 \). Let \( v_0, v_1, v_2 \) and \( v_3 \) be the vertices of \( G \) in which \( v_0 \) is the central vertex and \( v_1, v_2 \) and \( v_3 \) are the pendant vertices of \( S_3 \). Let the edges \( v_0v_1, v_0v_2 \) and \( v_0v_3 \) of \( S_3 \) be subdivided by \( p_1, p_2 \) and \( p_3 \) number of vertices respectively.

Let \( v_0, v_1^{(1)}, v_2^{(1)}, v_3^{(1)}, \ldots, v_{p_1+1}^{(1)} (= v_1), v_0, v_1^{(2)}, v_2^{(2)}, v_3^{(2)}, \ldots, v_{p_2+1}^{(2)} (= v_2) \) and \( v_0, v_1^{(3)}, v_2^{(3)}, v_3^{(3)}, \ldots, v_{p_3+1}^{(3)} (= v_3) \) be the vertices of \( G \) and \( v_0 = v_0^{(1)} \), for \( 1 \leq i \leq 3 \).

Let \( e_j^{(i)} = v_j^{(i)}v_j^{(i)}, 1 \leq j \leq p_1 + 1 \) and \( 1 \leq i \leq 3 \) be the edges of \( G \) and it has \( p_1 + p_2 + p_3 + 4 \) vertices and \( p_1 + p_2 + p_3 + 3 \) edges with \( p_1 \leq p_2 \leq p_3 \).
Define $f: V(G) \rightarrow \{1, 2, 3, \ldots, p_1 + p_2 + p_3 + 4\}$ as follows:

$f(v_0) = p_1 + p_2 + 3$,

$f(v_1^{(1)}) = p_1 + p_2 + 4 - 2i$, for $1 \leq i \leq p_1 + 1$,

$f(v_2^{(2)}) = \begin{cases} p_1 + p_2 + 3 - 2i, & 1 \leq i \leq p_1 + 1 \\ p_2 + 2 - i, & p_1 + 2 \leq i \leq p_2 + 1 \end{cases}$ and

$f(v_3^{(3)}) = p_1 + p_2 + 3 + i$, for $1 \leq i \leq p_3 + 1$.

The induced edge labeling is as follows:

$f^*(v_1^{(1)}v_i^{(1)}) = p_1 + p_2 + 2 - 2i$, for $1 \leq i \leq p_1$,

$f^*(v_2^{(2)}v_i^{(2)}) = \begin{cases} p_1 + p_2 + 1 - 2i, & 1 \leq i \leq p_1 \\ p_2 + 1 - i, & p_1 + 1 \leq i \leq p_2 \end{cases}$,

$f^*(v_3^{(3)}v_i^{(3)}) = p_1 + p_2 + 3 + i$, for $1 \leq i \leq p_3$,

$f^*(v_0v_1^{(1)}) = p_1 + p_2 + 2$,

$f^*(v_0v_2^{(2)}) = p_1 + p_2 + 1$ and

$f^*(v_0v_3^{(3)}) = p_1 + p_2 + 3$.

Hence $f$ is a logarithmic mean labeling of $G$. Thus the arbitrary subdivision of $S_3$ is a logarithmic mean graph.

**REFERENCES**


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