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DECOMPOSITION OF RECURRENT AND H-PROJECTIVE CURVATURE TENSOR FIELDS IN A KAEHLERIAN SPACE OF FIRST ORDER

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ABSTRACT

In this paper, we have studied decomposition of recurrent and H-Projective curvature tensor fields in a Kaehlerian space of first order by considering the decomposition of curvature tensor field in terms of a non- zero vector and tensor field. Also, several theorems have been derived.

Key Words: *Kaehlerian space, Projective, Recurrent, Curvature tensor.*

2010 MSC: *32C15, 46A13, 53B35, 53C55.*

1. INTRODUCTION

A 2n-dimensional Kaehlerian space $\mathbf{K}_n^c$ is a Riemannian space which admits a tensor field an almost complex structure $\mathbf{F}_i^h$ satisfying the relation (Yano 1965).

$$F_j^i F_i^h = - A_j^h, \tag{1.1}$$

$$F_s^t F_i^s g_{ts} = g_{ji} \quad \text{and} \quad F_i^h{}_{,j} = 0 \tag{1.2}$$

$$F_{ji} = - F_{ij} \tag{1.3}$$

$$F_{ji} = F_j^t g_{ti} \tag{1.4}$$

And finally has the property that the skew-symmetric tensor F_{ih} is a killing tensor, then

$$F_{ih,j} + F_{jh,i} = 0 \tag{1.5}$$

$$F_i^h{}_{,j} + F_j^h{}_{,i} = 0 \tag{1.6}$$

$$F_i = - F^j_{i,j} \tag{1.7}$$

Where the comma (,) followed by an index denotes the operator of covariant differentiation with respect to the metric tensor g_{ji} of the Riemannian space.

The Riemannian curvature tensor field is defined by

$$R_{ijk}^h = \partial_I \left\{ {h \atop jk} \right\} - \partial_j \left\{ {h \atop ik} \right\} + \left\{ {h \atop ia} \right\}\left\{ {a \atop jk} \right\} - \left\{ {h \atop ja} \right\}\left\{ {a \atop ik} \right\} \tag{1.8}$$

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The Ricci tensor and scalar curvature are respectively given by

$$R_{ij} = R^{a}_{aij} \quad \text{and} \quad R = g^{ij} R_{ij} \tag{1.9}$$

It is well known that these tensors satisfy the following identities

$$R^{a}_{ijk} = R_{jk,I} - R_{ik,j} \tag{1.10}$$

$$R_{,i} = 2 R^{a}_{i,a} \tag{1.11}$$

$$F_i^{a} R_{aj} = - R_{ia} F_j^{a} \tag{1.12}$$

$$F_i^{a} R^{j}_{a} = R^{a}_{i} F^{i}_{a} \tag{1.13}$$

The holomorphically projective curvature tensor P^{h}_{ijk} is defined by (Sinha, 1973)

$$P^{h}_{ijk} = R^{h}_{ijk} + \frac{1}{(n+2)} (R_{ik}\delta^{h}_{j} - R_{jk}\, \delta^{h}_{i} + S_{ik} F^{h}_{j} - S_{jk} F_i^{h} + 2 S_{ij} F_k^{h}) \tag{1.14}$$

Where $S_{ij} = F_i^{a} R_{aj}$

The Bianchi identities are given by (Takano,1967).

$$R^{h}_{ijk} + R^{h}_{jki} + R^{h}_{kij} = 0 \tag{1.15}$$

$$R^{h}_{ijk,a} + R^{h}_{ika,j} + R^{h}_{iaj,k} = 0 \quad , \tag{1.16}$$

The Commutative formulae for the curvature tensor fields are given as follows:

$$N^{i}_{,jk} - N^{i}_{kj} = N^{a} R^{i}_{ajk} \tag{1.17}$$

$$N_i^{h}{}_{,ml} - N_i^{h}{}_{,1m} = N_i^{a} R^{h}_{aml} - N_a^{h} R^{a}_{iml} \tag{1.18}$$

Definition (1.1): A Kaehlerian space is said to be recurrent, if we have (Singh 1971)

$$R^{h}{}_{ijk,a} = \lambda_a R^{h}{}_{ijk} , \tag{1.19}$$

for some non-zero recurrence vector λ_a, and is called semi-recurrent (or Ricci-recurrent), if it satisfies

$$R_{ij,a} = \lambda_a R_{ij} , \tag{1.20}$$

Multiplying the above equation by g^{ij}, we get

$$R_{,a} = \lambda_a R. \tag{1.21}$$

Remark (1.1): From (1.2) it follows that every Kaehlerian recurrent space is Kaehlerian Ricci-recurrent space but the converse is not necessarily true.

2. DECOMPOSITION OF RECURRENT CURVATURE TENSOR FIELDS IN A KAEHLERIAN SPACE OF FIRST ORDER.

We Consider the decomposition of recurrent curvature tensor field R^{h}_{ijk} in the following form:

$$R^{h}_{ijk} = X'^{h} \; Y_{ij,k} \tag{2.1}$$

Where two vectors X'^{h} and a tensor field $Y_{ij,k}$ such that

$$\lambda_h X'^{h} = 1 \tag{2.2}$$

Theorem 2.1: Under the decomposition (2.1), the Bianchi identity for R^{h}_{ijk} take the forms

$$Y_{ij,k} + Y_{jk,i} + Y_{ki,j} = 0 \tag{2.3}$$

and

$$\lambda_a Y_{ij,k} + \lambda_j Y_{ik,a} + \lambda_k Y_{ia,j} = 0 \tag{2.4}$$

Proof: From (1.15) and (2.1), we have

$$X'^{h} Y_{ij,k} + X'^{h} Y_{jk,I} + X'^{h} Y_{ki,j} = 0 \tag{2.5}$$

Multiplying (2.5) by λ_h and using (2.2), we obtain the required result (2.3)

Again, using (1.16), (1.19) and (2.1) we have

$$X'^{h} (\lambda_a Y_{ij,k} + \lambda_j Y_{ik,a} + \lambda_k Y_{ia,j}) = 0 \tag{2.6}$$

Multiplying (2.6) by λ_h and using (2.2), we get the required result (2.4).

Theorem 2.2: Under the decomposition (2.1), the tensor fields R^{h}_{ijk}, R_{ij} and $Y_{ij,k}$ satisfy relation

$$\lambda_a R^{a}_{ijk} = \lambda_i R_{jk} - \lambda_j R_{ik} = Y_{ij,k} \tag{2.7}$$

Proof: With the help of (1.10), (1.19) and (1.20), we have

$$\lambda_a R^{a}_{ijk} = \lambda_i R_{jk} - \lambda_j R_{ik} \tag{2.8}$$

Multiplying (2.1) by λ_h and using relation (2.2), we have

$$\lambda_h R^{h}_{ijk} = Y_{ij,k} \tag{2.9}$$

From (2.8) and (2.9), we get the required relation (2.7).

Theorem 2.3: Under the decomposition (2.1), the quantities λ_a and X'^h behave like the recurrent vectors. The recurrent form of these quantities are given by

$$\lambda_{a,m} = \mu_m \lambda_a \tag{2.10}$$

and

$$X'^{h}_{,m} = - \mu_m X'^h \tag{2.11}$$

Proof: Differentiating (2.8) covariantly w.r.t. x^m and using (2.1) and (2.7), we have

$$\lambda_{a,m} X'^a Y_{ij,k} = \lambda_{i,m} R_{jk} - \lambda_{j,m} R_{ik} \tag{2.12}$$

Multiplying (2.13) by λa and using (2.1) and (2.9), we have

$$\lambda_{a,m} (\lambda_i R_{jk} - \lambda_j R_{ik}) = \lambda_a (\lambda_{i,m} R_{jk} - \lambda_{j,m} R_{ik}) \tag{2.13}$$

Now multiplying (2.13) by λ_h, we have

$$\lambda_{a,m} (\lambda_i R_{jk} - \lambda_j R_{ik}) \lambda_a = \lambda_a \lambda_h (\lambda_{i,m} R_{jk} - \lambda_{j,m} R_{ik}) \tag{2.14}$$

Since the expression on the right hand side of the above equation is symmetric in a and h, therefore

$$\lambda_{a,m} \lambda_h = \lambda_{h,m} \lambda_a \tag{2.15}$$

Provided $\lambda_i R_{jk} - \lambda_j R_{ik} \neq 0$

The vector field λ_a being a non-zero, we can choose a proportional vector field μ_m such that

$$\lambda_{a,m} = \mu_m \lambda_a \tag{2.16}$$

Further, differentiating (2.2) covariantly w.r.t. x^m and using (2.16), we have

$$X'^{h}_{,m} = - \mu_m X'^h$$

Theorem 2.4: Under the decomposition (2.1), the vector X'^h and the tensor $Y_{ij,k}$ satisfy the relation

$$(\lambda_m + \mu_m) Y_{ij,k} = Y_{ij,km} \tag{2.17}$$

Proof: Differentiating (2.1) covariantly w.r.t. x^m and using (1.19), (2.1) and (2.11), we get the required result (2.17).

3. DECOMPOSITION OF H-PROJECTIVE CURVATURE TENSOR FIELDS IN A KAEHLERIAN SPACE OF FIRST ORDER.

Theorem 2.5: Under the decomposition (2.1), the curvature tensor and holomorphically projective curvature tensor are equal iff

$$(Y_{ik,m} \delta^{h}_{j} - Y_{jk,m} \delta^{h}_{i}) + Y_{ik,m} (F^{I}_{i} F^{h}_{j} - F^{I}_{j} F^{h}_{i}) + 2 F^{I}_{i} Y_{ij,m} F^{h}_{k} = 0 \tag{2.18}$$

Proof: The equation (1.14) may be written in the form

$$P^{h}_{ijk} = R^{h}_{ijk} + D^{h}_{ijk} \tag{2.19}$$

Where

$$D^{h}_{ijk} = \frac{1}{(n+2)} (R_{ik} \delta^{h}_{j} - R_{jk} \delta^{h}_{i} - S_{ik} F^{h}_{j} - S_{jk} F^{h}_{i} + 2S_{ij} F_k^{\ h}) \tag{2.20}$$

Contracting indices h and k in (2.1), we have

$$R_{ij} = X'^k Y_{ij,k} \tag{2.21}$$

In view of (2.21), we have

$$S_{ij} = F_i^{\ 1} X'^m Y_{ij,m} \tag{2.22}$$

Making use of (2.21) and (2.22) in (2.20), we obtain

$$D^h_{ijk} = \frac{1}{(n+2)} [X^m (Y_{ij,m}\delta^h_j - Y_{jk,m}\,\delta^h_i) + X^m Y_{Ik,m} (F^I_i F^h_j - F^I_j F^h_i) + 2 F^I_i Y_{ij,m} F^h_k] \qquad (2.23)$$

From equation (2.19), it is clear that

$$P^h_{ijk} = R^h_{ijk} \text{ iff } D^h_{ijk} = 0, \text{ which in view of (2.23) becomes}$$

$$X_m\{(Y_{ij,m}\,\delta^h_j - Y_{jk,m}\,\delta^h_i) + X^m Y_{Ik,m} (F^I_i F^h_j - F^I_j F^h_i)\} + 2 F^I_i Y_{ij,m} F^h_k = 0 \qquad (2.24)$$

Multiplying (2.24) by λ_m and using (2.2), we obtain the required result (2.18).

Theorem 2.6: Under the decomposition (2.1), the scalar curvature R, satisfy the relation

$$\lambda_k R = g^{ij} Y_{ij,k} \qquad (2.25)$$

Proof: Contracting indices h and k in (2.1), we get

$$R_{ij} = X^{'k} Y_{ij,k} \qquad (2.26)$$

Multiplying (2.26) by g^{ij} on both sides, we have

$$g^{ij} R_{ij} = g^{ij} X^{'k} Y_{ij,k} \quad \text{or} \quad R = g^{ij} X^{'k} Y_{ij,k} \qquad (2.27)$$

Now, multiplying (2.27) by $\lambda_{k,}$ then using (2.2), we have

$$\lambda_K R = g^{ij} Y_{ij,k} \quad \text{or} \quad R_{,K} = g^{ij} Y_{ij,k}$$

Which completes the proof of the theorem.

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