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ON VAGUE COMPLETELY BETA GENERALIZED CONTINUOUS MAPPINGS IN VAGUE TOPOLOGICAL SPACES

N. GAYATHRI*[1] AND Dr. Sr. HELEN[2]

[1]Research Scholar, Nirmala College for Women, Coimbatore, India.

[2]Department of Mathematics, Nirmala College for Women, Coimbatore. India.

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ABSTRACT

The aim of this paper is to introduce the idea of new Closed sets in Vague Topological Spaces namely, Vague Beta Generalized Closed sets and to investigate the properties of completely vague Beta generalized continuous mapping and Vague nearly compact. The concepts about vague points are also discussed.

Keywords: *Vague Beta Generalized Closed set, Vague Beta Generalized open set, Vague Neighborhood, Vague Completely Beta generalized continuous mapping, Vague Point, Vague nearly compact, Vague product topological space.*

INTRODUCTION

s a generalization of Fuzzy sets [11], W.L.Gau and D.J.Buehrer [4] have introduced the Vague sets. Vague set theory is characterized with Interval membership whereas the Fuzzy set theory is characterized with Point membership. Fuzzy set theory has been introduced to handle inexact and imprecise data in the real world. The interval-based membership generalization in Vague Set is more expressive in capturing vagueness of data. Bustince.H and Burillo.P [2] have shown that Vague sets are equivalent to that of Intuitionistic Fuzzy Sets which have been developed by Atanassov.K. [1]. In this paper, we introduce a new Class of Closed sets namely, Vague Beta Generalized Closed sets.

PRELIMINARIES

Definition 2.1[4]: A vague set A in the universe of discourse U is characterized by two membership functions given by:

a) A true membership function t_A: U → [0, 1] and
b) A false membership function f_A: U → [0, 1],

where $t_A(x)$ is a lower bound on the grade of membership of x derived from the evidence for x, $f_A(x)$ is a lower bound on the negation of x derived from the evidence for x, and $t_A(x) + f_A(x) \leq 1$. Thus the grade of membership of u in the vague set A is bounded by a sub-interval $[t_A(x), 1 - f_A(x)]$ of [0, 1].

This indicates that if the actual grade of membership of x is μ(x), then, $t_A(x) \leq \mu$(x) $\leq f_A(x)$. The vague set A is written as A = {< x, [$t_A(x)$, $1 - f_A(x)$>, x∈ X} where the interval $[t_A(x), 1 - f_A(x)]$ is called the vague value of x in A, denoted by $V_A(x)$.

Definition 2.2[4]: Let A, B be two vague sets in the universe U = {x_1, x_2, ... x_n}; then the union, intersection, and complement of vague sets are defined as follows:

a) A ∪ B = {(x_i, [t_A (x_i) ∨ $t_B(x_i)$, (1 - $f_A(x_i)$) ∨ (1 - f_B(x_ii))])| x_i ∈ U},
b) A ∩ B = {(x_i, [t_A (x_i) ∧ $t_B(x_i)$, (1 - $f_A(x_i)$) ∧ (1 - $f_B(x_i)$)])| x_i ∈ U},
c) A^C = {(x_i, [f_A (x_i), 1 - $f_A(x_i)$])| x_i ∈ U}

Corresponding Author: N. Gayathri*[1],
[1]Research Scholar, Nirmala College for Women, Coimbatore, India.

Definition 2.3[4]: A vague topology (VT in short) on X is a family τ of Vague Sets in X satisfying the following axioms.

- $0,1 \in \tau$
- $G_1 \wedge G_2 \in \tau$, for any $G_1, G_2 \in \tau$
- $\vee\, G_i \in \tau$ for any family $\{G_i / \text{ i} \in \text{J}\} \subseteq \tau$

The pair (X,τ) is called a Vague Topological Space and their elements are called open sets. The complement of any Vague open set A is called a Vague Closed set A^C.

Definition 2.4[4]: Let (X, τ) be a Vague Topological Space and A be a Vague set in (X, τ). Then,

- Vague Interior of A Vint(A) = ⋁{G: G is an Vague Open Set and G ⊆ A}.
- Vague Closure of A VCl(A) =⋀{K: K is an Vague Closed Set and A ⊆ K}.

Definition 2.5[4]: A Vague Set A = $<$ x, [t_A (x), 1 - f_A(x)]$>$ of the VTS (X, τ) is said to be a,

1. a Vague regular-Closed[7] if A = VCl(Vint(A))
2. a Vague semi-Closed[7] if Vint(VCl(A)) ⊆ A.
3. a Vague α −Closed[7] if VCl(Vint(VCl(A))) ⊆ A.
4. a Vague pre-Closed[8] if VCl(Vint(A)) ⊆ A.
5. a vague β −Closed if Vint(VCl(Vint(A))) ⊆ A.

Definition 2.6: A Vague Set A = $<$ x, [t_A (x), 1 - f_A(x)]$>$ of the VTS is said to be a,

1. a Vague generalized Closed[5] if VCl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).

3. VAGUE BETA GENERALIZED CLOSED SETS

Definition 3.1: A Vague Set A in a VTS (X, τ) is said to be a Vague β - generalized Closed set, if VβCl(A) ⊆ U, whenever A ⊆ U and U is Vague β -open in (X, τ).

Proposition 3.2: For any Vague Topological Space (X, τ), we have the following results.
(i) Every vague Closed set is Vague β -generalized Closed set in (X, τ).

Proof: Let A be a vague Closed set and A ⊆ U and U is vague β - open set in (X, τ). Since A is vague Clsoed, VCl(A) = Cl(A). Since, βCl(A) ⊆ Cl(A) = A ⊆ U, ⇒ U whenever βCl(A) ⊆ U. Therefore, βCl(A) ⊆ U whenever A ⊆ U and U is vague β - open set in (X, τ). Hence, A is a Vague β - generalized Closed set in (X, τ). But the converse is not true for each preposition, which is given by the examples.

Example 3.3: Let X = {a,b} and let τ = {0, G, 1 } be the vague topology on X, where G = {< x, [0.5, 0.4], [0.4, 0.3]>}. Let A = {< x, [0.4, 0.3], [0.2, 0.6]}> be any vague set in (X, τ). Now, VβCl(A) = G^c, whenever A ⊆ G^c, for all Vague open sets G in X. Therefore, A is a Vague β-Generalized Closed set in (X, τ). But it not a Vague Closed set.

(i) Every Vague regular-Closed set is Vague β-generalized Closed set in (X, τ).
(ii) Every Vague semi-Closed set is Vague β -generalized Closed set in (X, τ).
(iii) Every Vague α-Closed set is Vague β-generalized Closed set in (X, τ).
(iv) Every Vague pre-Closed set is Vague β-generalized Closed set in (X, τ).
(v) Every Vague β-Closed set is Vagueβ-generalized Closed set in (X, τ).

Example 3.4: Let X = {a,b} and let τ = {0, G, 1 } be the vague topology on X, where G = {< x, [0.5, 0.4], [0.5, 0.6] >}. Let A = {< x, [0.4, 0.3], [0.6, 0.7]}> be any vague set in (X, τ). Here, A is a VβGCS but not VRC, since VCl(Vint(A)) = 0 ≠ A.

Example 3.5: From Example [3.3], A is a VβGCS but not VSCS, since Vint(VCl(A))) = 1 ⊈ A.

Example 3.6: From Example [3.3], A is a VβGCS but not VαCS, since VCl(Vint(VCl(A)))) = 1 ⊈ A.

Example 3.7: From Example [3.3], A is a VβGCS but not VPC, since VCl(Vint(A)) = 1 ⊈ A.

Example 3.8: From Example [3.3], A is a VβGCS but not VβCS, since Vint(VCl(Vint(A))) = 1 ⊈ A.

Theorem 3.9: Let (X, τ) be a Vague topological Space. Then, for every A ∈ Vβ-GC(X) and for every β ∈ VOS(X), A ⊆ B ⊆ Vβ-Cl(A) ⇒ β ∈ Vβ-GC(X).

Proof: Let U be a vague open set in (X, τ) and let B ⊆ U. Since, A ⊆ B, we have A ⊆ U. By hypothesis, B ⊆ VβCl(A), VβCl(B) ⊆ VβCl(VβCl(A)). Then, Vβ Cl(B) ⊆ Vβ Cl(A) ∈ U. ⇒ VβCl(A) ∈ U, whenever A ⊆ U and U is vague open in X. (i.e) B is a vague β- generalized Closed set in (X, τ).

Theorem 3.10: Let (X, τ) be a Vague topological Space. Then every Vague Set is Vβ-GCS iff VβOS = VβCS in (X, τ).

Proof: Necessity Part: Suppose that every Vague set is βGCS. Let U be an Vague β-open set. Since, every vague Closed set is Vague β-Closed set, βCl(U) = U. Therefore, U is Vague β-Closed set in (X, τ). Hence, Vague β-open set ⊆ Vague β-Closed set.

Let A is an Vague β-Closed set, then A^c is an Vague β-open set.
⇒ A^c is an Vague β-Closed set.
⇒ A is an Vague β-open set. Hence A is both Vague β-Closed set and Vague β-open set.
Therefore, VβOS = VβCS.

Sufficiency Part: Suppose, Vβ OS(X) = Vβ CS(X). Let A be any Vague Closed set where A ⊆ U and U be an vague β-open set. Then U is vague β-Closed set also. Now, A ⊆ U ⇒ β Cl(A) ⊆ β Cl(U) = U. Therefore, β Cl(A) ⊆ U, whenever A ⊆ U and U is Vague β-open set. Therefore, A is a Vague β-generalized Closed set.

Theorem 3.11: For any Vague Set A, the following conditions are equivalent:
(i) A is an vague open set and an Vague β-generalized Closed set.
(ii) A is an vague regular open set.

Proof: (i) ⇒ (ii): Let A be an vague open set and an Vague β-generalized Closed set. Then Vβ Cl(A) ⊆ A and A ⊆ Vβ Cl(A). Now, Vβ Cl(A) = A. Therefore, A is Vague β Closed set. Since A is vague open set, A is vague pre open set. (i.e) A ⊆ Vint(VCl(A)). Therefore, Vint(VCl(A)) = A.(i.e) A is vague regular open set.

(ii) ⇒ (i): Let A be a vague regular open set.(i.e) A = Vint(VCl(A)). Since every vague regular open set is vague open set and A ⊆ A, we have Vint(VCl(A)) ⊆ A. Now, A is Vague β Closed set and A is a vague open set. Therefore, A is Vague β-generalized Closed set.

4. COMPLETELY VAGUE BETA GENERALIZED CONTINUOUS MAPPING

Definition 4.1: A mapping f: $(X, \tau) \to (Y, \sigma)$ is called Completely Vague Beta Generalized Continuous Mapping (Vβ GC mapping) if $f^{-1}(B)$ is a Vague regular Closed set in (X, τ) for every Vague β-generalized Closed set B of (Y, σ).

Definition 4.2: An VTS (X, τ) is called Vague nearly compact iff every vague open cover of has a finite sub-collection such that the interior of Closures of VS's in this sub-collection covers X.

Definition 4.3: Let (X, τ) and (Y, σ) be VTS's. The vague product topological space (VPTS, for short) of (X, τ) and (Y, σ) is the Cartesian product $X \times Y$ of VS's and together with the VT ζ of $X \times Y$ which is generated by the family $\{(P_1^{-1}(A_i), P_2^{-1}(B_j)): A_i \in \tau, B_j \in \sigma\}$ and P_1, P_2 are projections of $X \times Y$ onto X and Y, respectively}. [(i.e.,) the family $\{(P_1^{-1}(A_i), P_2^{-1}(B_j)): A_i \in \tau, B_j \in \sigma\}$ is sub base for VT ζ of $X \times Y$].

Theorem 4.4: If A is an VS of X and B is an VS of Y, then
(i) $(A \times (1, 0)) \cap ((1, 0) \times B) = A \times B$.

Proof: Given that, if A is an VS of X and B is an VS of Y.
(i): Let $A = < x, (t_A(x), (1 - f_A)(x))>$ and $B = < x, (t_B(x), (1 - f_B)(x))>$

Since, $A \times (1, 0) = < x, \min(t_A(x),1), \max((1 - f_A)(x), 0) > = < x, t_A(x), (1 - f_A)(x) > = A$.

Similarly, $(1, 0) \times B = < x, \min(1, t_B(x)), \max(0, (1 - f_B)(x)) > = < x, t_B(x), (1 - f_B)(x) > = B$.

We have, $(A \times (1, 0)) \cap ((1, 0) \times B) = < x, t_A(x), (1 - f_A)(x) > \cap < x, t_B(x), (1 - f_B)(x) > = < x, (t_A(x) \wedge t_B(x)), ((1 - f_A)(x) \vee (1 - f_B)(x)) > = A \times B$.

Definition 4.5: Let $u, v \in (0, 1)$ and $u + v \leq 1$. A vague point p^X of X a vague set of X,
(i.e) $p^x = < x, (t_p, 1 - f_p) > \forall x \in X$

$$t_p(y) = \begin{cases} u \text{ if } y = x \\ 0 \text{ if } y \neq x \end{cases}$$

$$1 - f_p(y) = \begin{cases} \beta \text{ if } y = x \\ 0 \text{ if } y \neq x \end{cases}$$

In this case, x is called the support of p^X. A Vague point p^X is said to belong to a VS A = < x, (t_A, 1-f_A) > of X, defined by c(u,v) ∈ A if $u \leq t_A(x)$ and $V \leq 1 - f_A(x)$.

Definition 4.6: Let c(u, v) be a VP of a VTS (X, τ). A VS A of X is called a vague neighborhood (VN in short) of c(u,v) if there is a VOS B in X such that c(u,v) ∈ B ⊆ A.

Theorem 4.7: For any two vague completely β generalized continuous mapping, f_1, f_2: (X, τ) →(Y, σ), the function (f_1, f_2): (X, τ) (Y × Y, σ × σ) is also a vague completely β generalized continuous mapping, where $(f_1, f_2)(x) = (f_1(X), f_2(x))$ ∀ x in X.

Proof: Let A × B be any VOS in Y ×Y. Then,

$(f_1, f_2)^{-1} (A\times \times B)(x) = (A \times B)(x)(f_1(X), f_2(x))$

$= < x, \min(t_A (f_1(x)), t_B(f_2(x))), \max(((1 - f_A)(f_1(x)), (1 - f_B)(f_2(x))>.$

$= < x, \min(f_1^{-1}(t_A)(x),f_2^{-1}(t_B)(x)), \max(f_1^{-1}(1 - f_A)(x), f_1^{-1}(1 - f_B)(x))>$

$= (f_1^{-1} (A) \cap f_2^{-1}(B))(x)$. Now, we know that, $f_1^{-1}(A)$ and $f_1^{-1}(B)$ are VROS in X.

Therefore, $(f_1^{-1} (A) \cap f_2^{-1}(B))$ (x) is also a VROS. Hence, (f_1,f_2) is vague completely β generalized continuous mapping.

Theorem 4.8: Let f: (X, τ) →(Y, σ) is a Vague completely β generalized continuous mapping. Then the following statements hold:
(i) f(VβGCl(A)) ⊆ VCl(f(A)), for every vague set A in X.
(ii) VβGCl(f^{-1}(B)) ⊆ f^{-1}(VCl(B), for every vague set B in X.

Proof:
(i) Let A∈X. Then VCl(f(A) is vague Closed in Y. Since f is Vague completely β generalized continuous mapping, f-1(VCl(f(A)) is vague β generalized Closed set in X. Since A ⊆ f^{-1}(f(A)) ⊆ f^{-1}(VCl(f(A))) and f^{-1}(VCl(f(A))) is VβGCS, which implies VβGCl(A) ⊆ f^{-1}(VCl(f(A))). Therefore, f(VβGCl(A)) ⊆ VCl(f(A)), for every vague set A in X.
(ii) When A = f^{-1}(B), in (i), we have, f(VβGCl(f^{-1}(B))) ⊆ VCl(f(f^{-1}(B))) ⇒f(VβGCl(f^{-1}(B))) ⊆ VCl(B). Hence, VβGCl(f^{-1}(B)) ⊆ f^{-1}(VCl(B)) for every vague set B in X.

Theorem 4.9: A mapping f: (X, τ) →(Y, σ) is a Vague completely β generalized continuous mapping if for every VP c(u,v) ∈ X and for every VN A of f(c(u, v)), there exists a VROS, B ⊆ X such that c(u,v) ∈ B ⊆ f^{-1}(A).

Proof: Let c(u,v) ∈ X and let A be a VN of f(c(u, v)). Then there exists a VOS U in Y such that f(c(u,v)) ∈ U ⊆ A. Since every VOS is a VβGCS, U is a VβGCS in Y. Hence by hypothesis f^{-1}(U) is a VROS in X and c(u,v) ∈ f^{-1}(U). Let B= f^{-1}(U). Therefore, c(u,v) ∈ B ⊆ f^{-1}(A).

Theorem 4.10: A mapping f: (X, τ) →(Y, σ) is a Vague completely β generalized continuous if and only if f^{-1}(A) is a VROS in X for every VβGOS A in Y.

Proof - Necessity Part: Let A be a VβGOS in Y. This implies A^c is a VβGCS in Y. Since f is a vague completely β generalized continuous mapping, $f^{-1}(A^c)$ is a vague regular Closed set in X. Hence $f^{-1}(A^c) = (f^{-1}(A))^c$, f^{-1}(A) is a vague regular open set in X.

Proof - Sufficiency Part: Let A be a VβGCS in Y. Then, A^c is a Vβ GOS in Y. By hypothesis, $f^{-1}(A^c)$ is a VROS in X, since $f^{-1}(A^c) = (f^{-1}(A))^c$, f^{-1}(A) is a vague regular Closed set in X. Hence f is a Vague completely β generalized continuous mapping.

Theorem 4.11: Let f: (X, τ) →(Y, σ) be a mapping. Then the following are equivalent.
(i) f is a vague completely β generalized continuous mapping.
(ii) f^{-1}(B) is a vague regular open set in X for every vague β generalized open set B in Y.
(iii) For every vague point c(u, v) ∈ X and for every vague β generalized open set B in Y such that if f(c(u, v)) ∈ B there exists a vague regular open set A in X such that c(u, v) ∈ A and f(A) ⊆ B.

Proof: (i) ⇒ (ii): The proof is obvious by definition [4.1].

(ii) ⇒ (iii): Let c(u, v) ∈ X. Let B be a vague β generalized open set and f^{-1}(B) is a vague regular open set in X. Let f(c(u,v)) ∈ B and let A = f^{-1}(B). Then f^{-1}(f(c(u,v))) ∈ f^{-1}(B) →c(u, v) ∈ A. And f(A) = f(f^{-1}(B)) ⊆ B, which implies f(A) ⊆ B.

(iii) ⇒(i): Let be a vague β generalized open set B in Y and let c(u,v) ∈ A and f(c(u, v)) ⊆ B. Then by hypothesis, there exists a vague regular open set E such that c(u, v) ∈ E and f(E) ⊆ B. Now, c(u, v) ⊆ f^{-1}(B). But E ⊆ f^{-1}(B). That is, $f^{-1}(B) = \cup_{c(u,v) \subseteq f^{-1}(B)} E \in f^{-1}(B)$. This implies $f^{-1}(B) = \cup_{c(u,v) \subseteq f^{-1}(B)} E$, where E is a vague regular open set and hence f^{-1}(B) is a vague regular open set in X. Hence f^{-1}(B) is a vague completely β generalized continuous mapping.

Theorem 4.12: Let f: (X, τ) →(Y, σ) and g: (Y, σ) → (Z, η) be a mapping. Then the following statements hold.

(i) f be a vague completely β generalized continuous mapping and g be a vague continuous mapping. Then the composition g ° f: (X, τ) → (Z, η) is vague completely β-generalized continuous mapping.

(ii) f be a vague completely β generalized continuous mapping and g be a vague semi-continuous mapping. Then the composition g ° f: (X, τ) → (Z, η) is vague completely β-generalized continuous mapping.

(iii) f be a vague completely β generalized continuous mapping and g be a vague α continuous mapping. Then the composition g ° f: (X, τ) → (Z, η) is vague completely β - generalized continuous mapping.

(iv) f be a vague completely β generalized continuous mapping and g be a vague β continuous mapping. Then the composition g ° f: (X, τ) → (Z, η) is vague completely β generalized continuous mapping.

(v) f be a vague completely β generalized continuous mapping and g be a vague pre-continuous mapping. Then the composition g ° f: (X, τ) →(Z, η) is vague completely β generalized continuous mapping.

Proof: (i): Let A be a vague Closed set in (Z, η). Then G^{-1}(A) a Closed set in (Y, σ). By hypothesis, every vague Closed set is vague β generalized Closed set. Therefore, G^{-1}(A) is a vague β generalized Closed set. Since f is vague completely β generalized continuous, $f^{-1}(g^{-1}(A))$ is a vague regular Closed set in X. (i.e) $(g \circ f)^{-1}(A)$ is a vague regular Closed set in X. Hence, g ° f: (X, τ) →(Z, η) is vague completely β generalized continuous mapping.
Proofs of (ii)-(v) are similar to (i).

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