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ON NANO TOTALLY SEMI CONTINUOUS FUNCTIONS IN NANO TOPOLOGICAL SPACE

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ABSTRACT

The properties of new class of functions, namely Nano semi totally continuous functions and Nano totally semi continuous functions in Nano topological space are analyzed in this paper. The relation of these functions with already existing well known functions are studied.

Key Words: *Nano open set, Nano semi open set, Nano closed set, Nano clopen set.*

1. INTRODUCTION

The theory of Nano topology [3] proposed by Lellis Thivagar and Richard is an extension of set theory for the study of intelligent systems characterized by insufficient and incomplete information. The elements of a Nano topological space are called the Nano open set. It originates from the Greek word, Nanos" which means, dwarf" in its modern scientific sense, an order to magnititude-one billionth. The Topology is named as Nano topology so because of its size, since it has at most five elements. The author has defined Nano topological space in terms of Lower and upper approximations. He also introduced certain weak form of Nano open set [3] such as Nano open set, Nano semi-open sets and Nano pre open sets. Further he introduced continuity [4] which is the core concept of topology in Nano topological space.

2. PRELIMINARY

Throughout this paper (U, $\tau_R(X)$) (or X) represent Nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset H of a space (U, $\tau_R(X)$), Ncl (H) and Nint (H) denote the Nano closure of *H* and the Nan interior of H respectively. We recall the following definitions which are useful in the sequel.

Definition 2.1: [2] Let U be a non-empty finite set of objects called universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernibility with one another. The pair (U, R) is said to be the Approximation Space. Let X ⊆ U.

1. The lower approximation of X with respect to R is the set of all objects which can be certain classified as X with respect to R and is denoted by $L_R(X)$
2. $L_R(X) = \cup_{x\in U} \{R(X) / R(X) \leq X\}$
3. The upper approximation of X with respect to R is the set of objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$
4. $U_R(X) = \cup_{x\in U} \{R(X) / R(X) \cap X \neq \emptyset\}$
5. The boundary region of x with respect to R is the se of all objects which can be classified neither as X nor not X with respect to R and it is denoted by $B_R(X)$
6. $B_R(X) = U_R(X) - L_R(X)$

Definition 2.2: Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where X⊆ U. Then the property $\tau_R(X)$ satisfies the following axioms

1. U and ∅ belongs to $\tau_R(X)$.
2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
3. The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Thus $\tau_R(X)$ is a topology on U is called the NANO TOPOLORY on U with respect to X. (U, $\tau_R(X)$) as the Nano topological space. The elements of $\tau_R(X)$ are celled as Nano-open sets and complements of Nano-open sets are called Nano-closed sets.

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Definition 2.3: If (U, $\tau_R(X)$) is a Nano topological space and if H⊆ U, then

1. The NANO-INTERIOR of H is defined as the union of all Nano-open subsets of H and it is dented by Nint (H).
   i.e.) Nint (H) is the largest Nano-open set contained in H.
2. The NANO-CLOSURE of H is defined as the intersection of all Nano-closed sets containing H and it is denoted by Ncl (H).
   i.e.) Ncl (H) is the smallest Nano-closed set containing H.
3. The set H is called NANO CLOPEN if it is both Nano-open and Nano-closed and is denoted by Nco (H).

Definition 2.4: If (U, $\tau_R(X)$) is a Nano topological space with respect to X and H ⊆ U is called Nano Semi-open if H ⊆ Ncl (Nint (H)) and it is denoted by Nso (U, $\tau_R(X)$).

The complement of Nano Semi-open is called Nano Semi-closed and it is denoted by Nscl (U, $\tau_R(X)$).

Example 2.5: Let U = {a, b, c, d} with U/R = {{a}, {b}, {c, d}} and X = {a, c} ⊂ U. Then the Nano topology is $\tau_R(X)$ = {U, ∅, {a}, {c, d}, {a, c, d}}.

Definition 2.6: [5] A function f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is said to be NANO TOTALLY CONTINUOUS if $f^{-1}(V)$ is Nano-clopen in U for each Nano-open subset in V.

Example 2.7: Consider the Nano topological space (U, $\tau_R(X)$) and (V, $\tau_{R'}(Y)$) where U = V = {a, b, c, d} X = {b, c} Y = {a, c}. U/R = {{a}, {b}, {b, d}} and V/R' = {{a}, {b}, {c, d}}
Then $\tau_R(X)$ = {U, ∅, {a}, {b, c, d}} and $\tau_{R'}(Y)$ = {U, ∅, {a}} NCO (U, $\tau_R(X)$) = {U, ∅, {a}, {b, c, d}}

Define f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is given by
f (a) = a, f (b) = b, f(c) = c, f (d) = d
$f^{-1}(a) = a$, $f^{-1}(b) = b$, $f^{-1}(c) = c$, $f^{-1}(d) = d$

Here, Inverse image of Nano-open set in V is Nano-clopen in U.

Hence, f is Nano totally continuous

4. NANO SEMI TOTALLY CONTINUOUS FUNCTIONS AND THEIR BASIC PROPERTIES

In this section, Nano semi totally continuous functions are defined and some of the properties are analyzed.

Definition 3.1: A function f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is said to be NANO SEMI TOTALLY CONTINUOUS if Inverse image every Nano semi open subset of V is Nano-clopen in U.

Example 3.2: Consider the Nano topological space (U, $\tau_R(X)$) and (V, $\tau_{R'}(Y)$) where U = V = {a, b, c, d} X = {b, c} Y = {a, c}. U/R = {{a}, {b}, {b, d}} and V/R' = {{a}, {b}, {c, d}}. Then $\tau_R(X)$ = {U, ∅, {a}, {b, c, d}} and $\tau_{R'}(Y)$ = {U, ∅, {a}}NCO (U, $\tau_R(X)$) = {U, ∅, {a}, {b, c, d}} NSO (V, $\tau_{R'}(Y)$) = {U, ∅, {a}}

Define f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is given by
f (a) = a, f (b) = b, f(c) = c, f (d) = d
$f^{-1}(a) = a$, $f^{-1}(b) = b$, $f^{-1}(c) = c$, $f^{-1}(d) = d$

Here, Inverse image of Nano semi open set in V is Nano-clopen in U.

Hence, f is Nano semi totally continuous.

Theorem 3.3: A function f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Nano semi totally continuous iff inverse image of every Nano semi closed subset of V is Nano-clopen in U.

Proof: Suppose f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Nano semi totally continuous.

Let F be Nano semi closed in V. Then F^C is Nano semi open in V.

Since f is Nano totally continuous, $f^{-1}(F^C)$ is Nano clopen in U.
⇒ $f^{-1}(F)$ is Nano clopen in U.

Conversely,
Suppose inverse image of every Nano semi closed subset of V is Nano clopen in U.

Let G be Nano semi open in V. Then G^C is Nano semi closed in V.

By hypothesis, $f^{-1}(G^C) = (f^{-1}(G))^C$ is Nano clopen in U.
i.e.) $f^{-1}(G)$ is Nano clopen in U.

Hence, f is Nano semi totally continuous.

Theorem 3.4: Every Nano semi totally continuous function is Nano totally continuous function**.**

Proof: Suppose f: $(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is a Nano semi totally continuous function.

Let A be any Nano-open set in V. Since every Nano open set is Nano semi open set, A is Nano semi open in V.

Since f is Nano semi totally continuous, $f^{-1}(A)$ is Nano-clopen in U.
Thus f is Nano totally continuous function.

Theorem 3.5 Let f: $(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is Nano semi totally continuous and g: $(V, \tau_{R'}(Y)) \rightarrow (w, \tau_{R''}(z))$ is Nano semi continuous. Then g∘f is Nano totally continuous.

Proof: Let G be Nano-open in w. Since g is Nano semi continuous, $g^{-1}(G)$ is Nano semi open in V.

Also f is Nano semi totally continuous, $f^{-1}g^{-1}(G)$ is Nano clopen in U.
i.e.) $(g\circ f)^{-1}$ is Nano clopen in U. Therefore, g∘f is Nano totally continuous.

Definition 3.6: If f: $(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ and A_0 is a subset of $(U, \tau_R(X))$. We define the NANO RESTRICTION of f to A_0 be the function mapping A_0 into $(V, \tau_{R'}(Y))$ whose rule is $\{(a, f(a)) / a\ A_0\}$. It is denoted by $f|A_0$ which is read f is Nano restricted to A_0.

Theorem 3.7 (Issac's Theorem): If f: $(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is Nano semi totally continuous and A is Nano-clopen subset of $(U, \tau_R(X))$ then the Nano restriction f|A: $A \rightarrow (V, \tau_{R'}(Y))$ is Nano semi totally continuous.

Proof: Consider the function f|A and v be Nano semi open in V.

Since f is Nano semi totally continuous, $f^{-1}(V)$ is Nano clopen in U.

Since A is Nano clopen subset of U and $(f|A)^{-1}(V) = A\cap f^{-1}(V)$ is Nano clopen in A, it follow that $(f|A)^{-1}(V)$ is Nano clopen in A. Hence, f|A is Nano totally continuous.

Theorem 3.8: The composition of two Nano semi totally continuous functions is Nano semi totally continuous.

Proof: Let f: $(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ and g: $(V, \tau_{R'}(Y)) \rightarrow (w, \tau_{R''}(z))$ be any two Nano semi totally continuous function. Let V be Nano semi open in w.

Since g is Nano semi totally continuous, $g^{-1}(V)$ is Nano-clopen in V.
$\Rightarrow$ $g\text{-}^{1}(V)$ is Nano-open in V.

Since f is Nano semi totally continuous, $f^{-1}g^{-1}(V)$ is Nano-clopen in U.
i.e.) $(g\circ f)^{-1}$ is Nano-clopen in U.

Hence g∘f is Nano semi totally continuous.

Theorem 3.9: Every Nano semi totally continuous function is Nano semi continuous function.

Proof: Suppose f: $(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is a Nano semi totally continuous function.

Let A be any Nano-open set in V. Since f is Nano semi totally continuous, $f^{-1}(A)$ is Nano-clopen in U.
$\Rightarrow$ $f^{-1}(A)$ is Nano-open in U.
Thus f is Nano semi continuous function.

5. NANO TOTALLY SEMI CONTINUOUS FUNCTIONS AND THEIR BASIC PROPERTIES

In this section, Nano semi totally continuous functions are defined and some of the properties are analyzed.

Definition 5.1 A function f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is said to be NANO TOTALLY SEMI CONTINUOUS if Inverse image every Nano -open subset of V is Nano semi clopen in U.

Example 4.2 Consider the Nano topological space (U, $\tau_R(X)$) and (V, $\tau_{R'}(Y)$) where U = V = {a, b, c, d} X = {a, b} Y = {a, c}. U/R = {{a}, {c}, {b, d}} and V/R' = {{a}, {b}, {c, d}}
Then $\tau_R(X)$ = {U, ∅, {a}, {b, d}, {a, c, d}} and $\tau_{R'}(Y)$ = {U, ∅, {a}, {c, d}, {b, c, d}}
NSCO (U, $\tau_R(X)$) = {U, ∅, {a}, {a, c}, {b, d}, {b, c, d}}

Define f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is given by
f (a) = a, f (b) = c, f(c) = b, f (d) = d
$f^{-1}(a) = a$, $f^{-1}(b) = c$, $f^{-1}(c) = b$, $f^{-1}(d) = d$

Here, Inverse image of Nano-open set in V is Nano semi clopen in U.

Hence, f is Nano totally semi continuous.

Theorem 4.3 A function f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Nano totally semi continuous iff inverse image of every Nano closed subset of V is Nano semi clopen in U.

Proof: Suppose f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Nano totally semi continuous.

Let F be Nano-closed in V. Then F^C is Nano-open in V.

Since f is Nano totally semi continuous, $f^{-1}(F^C)$ is Nano semi clopen in U.
⇒ $f^{-1}(F)$ is Nano semi clopen in U.

Conversely,

Suppose inverse image of every Nano-closed subset of V is Nano semi clopen in U.

Let G be Nan-open in V. Then G^C is Nano-closed in V.

By hypothesis, $f^{-1}(G^C) = (f^{-1}(G))^C$ is Nano semi clopen in U.
i.e.) $f^{-1}(G)$ is Nano semi clopen in U. Hence, f is Nano totally semi continuous.

Theorem 4.4: Every Nano totally semi continuous function is Nano semi continuous function.

Proof: Suppose f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Nano totally semi continuous.

Let A be a Nano-open set in V.

Sine f is Nano totally semi continuous, $f^{-1}(A)$ is Nano semi clopen in U.
⇒ $f^{-1}(A)$ is Nano semi open in U. Thus f is Nano semi continuous.

Note 4.5: The converse of the above theorem need not be true.

For, Consider the Nano topological space (U, $\tau_R(X)$) and (V, $\tau_{R'}(Y)$) where U = V = {a, b, c, d} X = {a, b} Y = {a, c}. U/R = {{a}, {c}, {b, d}} and V/R' = {{a}, {b}, {c, d}}
Then $\tau_R(X)$ = {U, ∅, {a}, {b, d}, {a, b, d}} and $\tau_{R'}(Y)$ = {U, ∅, {a}, {c, d}, {a, c, d}}
NSO (U, $\tau_R(X)$) = {U, ∅, {a}, {b, d}, {a, c}, {b, c, d}, {a, b, d}}
NSCO (U, $\tau_R(X)$) = {U, ∅, {a}, {b, d}, {a, c}, {b, c, d}}
NSO (V, $\tau_{R'}(Y)$) = {U, ∅, {a}, {c, d}, {a, b}, {b, c, d}, {a, c, d}}

Define f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is given by
f (a) = a, f (b) = c, f(c) = b, f (d) = d
$f^{-1}(a) = a$, $f^{-1}(b) = c$, $f^{-1}(c) = b$, $f^{-1}(d) = d$

Here, Inverse image of Nano-open set in is V Nano semi open in U.

Hence, f is Nano semi continuous.

But, the Nano-open set {a, c, d} of V is not Nano semi clopen in U.

Therefore, f is not a Nano totally semi continuous.

Theorem 4.6: Every Nano semi totally continuous function is a Nano totally semi continuous function.

Proof: Suppose f: $(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is Nano semi totally continuous function.

Let A be a Nano-open set in V.

Since every Nano-open set is a Nano semi open set and f is Nano semi totally continuous function it follows that $f^{-1}(A)$ is Nano clopen in U.

Hence, f is a Nano totally semi continuous function.

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