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SOME NEW TOPOLOGICAL INDICES OF GRAPHS

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ABSTRACT

In 2019, Basavanagoud et al. introduced new degree based topological indices called Kulli-Basava indices and studied their mathematical and chemical properties which have good response with mean isomer degeneracy. In this paper, we propose the first and second hyper Kulli-Basava indices and their polynomials of a graph and compute exact formulae for complete graphs, wheel graphs, gear graphs and helm graphs. Also we determine the Kulli-Basava indices of gear graphs and helm graphs.

Key words: *hyper Kulli-Basava indices, polynomials, graphs.*

Mathematics Subject Classification: *05C07, 05C76, 92E10.*

1. INTRODUCTION

In Chemical Sciences, topological indices have been found to be useful in chemical documentation, isomer discrimination, structure property relationships, structure activity relationships. There has been considerable interest in the general problem of determining topological indices.

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of edges incident to v. The edge connecting the vertices u and v will be denoted by uv. The degree of an edge $e = uv$ in a graph G is defined by $d_G(e) = d_G(u) + d_G(v) - 2$. The set of all vertices adjacent to v is called the open neighborhood of v and denoted by $N_G(v)$. Let $S_G(v)$ denote the sum of the degrees of all vertices adjacent to a vertex v. The set of all edges incident to v is called the edge neighborhood of v and denoted by $N_e(v)$. Let $S_e(v)$ denote the sum of the degrees of all edges incident to a vertex v. We refer to [1] for undefined term and notation.

The first and second Zagreb indices are the degree based topological indices, introduced by Gutman and Trinajstić in [2]. These indices have many applications in Chemistry. Recently, Basavanagoud and Jakkannavar introduced [3] the following Kulli-Basava indices:

The first Kulli-Basava index of a graph G is defined as

$$KB_1(G) = \sum_{uv \in E(G)} \left[S_e(u) + S_e(v)\right].$$

The modified first Kulli-Basava index of a graph G is defined as

$$KB_1^*(G) = \sum_{u \in V(G)} S_e(u)^2.$$

The second Kulli-Basava index of a graph G is defined as

$$KB_2(G) = \sum_{uv \in E(G)} S_e(u) S_e(v).$$

The third Kulli-Basava index of a graph G is defined as

$$KB_3(G) = \sum_{uv \in E(G)} \left|S_e(u) - S_e(v)\right|.$$

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Considering the above Kulli-Basava indices, we define the following polynomials:

The first Kulli-Basava polynomial of a graph G is defined as

$$KB_1(G,x)=\sum_{uv\in E(G)} x^{[S_e(u)+S_e(v)]}.$$

The modified first Kulli-Basava polynomial of a graph G is defined as

$$KB_1^*(G,x)=\sum_{u\in V(G)} x^{S_e(u)^2}.$$

The second Kulli-Basava polynomial of a graph G is defined as

$$KB_2(G,x)=\sum_{uv\in E(G)} x^{S_e(u)S_e(v)}.$$

The third Kulli-Basava polynomial of a graph G is defined as

$$KB_3(G,x)=\sum_{uv\in E(G)} x^{|S_e(u)-S_e(v)|}.$$

In a recent years, the Zagreb index [4], F-index [5], sum connectivity index [6], reverse indices [7], Revan indices [8], Dakshayani index [9] were introduced and extensively studied.

The hyper Zagreb index of a graph G was introduced by Shirdel et al. in [10] and it is defined as

$$HM_1(G)=\sum_{uv\in E(G)} \left[d_G(u)+d_G(v)\right]^2.$$

In [11], Gao et al. proposed the second hyper Zagreb index, defined as

$$HM_2(G)=\sum_{uv\in E(G)} \left[d_G(u)d_G(v)\right]^2.$$

We now introduce the first and second hyper Kulli-Basava indices of a graph G, defined as

$$HKB_1(G)=\sum_{uv\in E(G)} \left[S_e(u)+S_e(v)\right]^2,$$

$$HKB_2(G)=\sum_{uv\in E(G)} \left[S_e(u)S_e(v)\right]^2.$$

Considering the first and second hyper Kulli-Basava indices, we propose the first and second hyper Kulli-Basava polynomials of a graph, defined as

$$HKB_1(G,x)=\sum_{uv\in E(G)} x^{[S_e(u)+S_e(v)]^2},$$

$$HKB_2(G,x)=\sum_{uv\in E(G)} x^{[S_e(u)S_e(v)]^2}.$$

2. COMPLETE GRAPHS

Let K_n be a complete graph with n vertices. Then the degree of each vertex of K_n is $n-1$ and the number of edges in K_n is $\dfrac{n(n-1)}{2}$.

Lemma 1: Let K_n be a complete graph with n vertices, $n \geq 2$. Then

(i) $V_1 = \{u \in V(K_n) \mid S_e(u) = 2(n-1)(n-2)\}$, $|V_1| = n$.

(ii) $E_1 = \{uv \in V(K_n) \mid S_e(u) = S_e(v) = 2(n-1)(n-2)\}$, $|E_1| = \dfrac{n(n-1)}{2}$.

Theorem 2: If K_n is a complete graph with $n \geq 2$ vertices, then

$$KB_1(K_n,x)=\frac{n(n-1)}{2}x^{4(n-1)(n-2)}$$

$$KB_1^*(K_n,x)=nx^{4(n-1)^2(n-2)^2}$$

$$KB_2(K_n,x)=\frac{n(n-1)}{2}x^{4(n-1)^2(n-2)^2}$$

$$KB_3(K_n,x)=\frac{n(n-1)}{2}x^0.$$

Proof: By using definitions and Lemma 1, we obtain

$$KB_1(K_n,x)=\sum_{uv\in E(K_n)}x^{S_e(u)+S_e(v)}=|E_1|x^{4(n-1)(n-2)}=\frac{n(n-1)}{2}x^{4(n-1)(n-2)}$$

$$KB_1^*(K_n,x)=\sum_{u\in V(K_n)}x^{S_e(u)^2}=|V_1|x^{4(n-1)^2(n-2)^2}=nx^{4(n-1)^2(n-2)^2}$$

$$KB_2(K_n,x)=\sum_{u\in E(K_n)}x^{S_e(u)S_e(v)}=|E_1|x^{4(n-1)^2(n-2)^2}=\frac{n(n-1)}{2}x^{4(n-1)^2(n-2)^2}$$

$$KB_3(K_n,x)=\sum_{uv\in E(K_n)}x^{|S_e(u)-S_e(v)|}=|E_1|x^0=\frac{n(n-1)}{2}x^0.$$

Theorem 3: The first hyper Kulli-Basava index and its polynomial of a complete graph are given by

(i) $HKB_1(K_n)=8n(n-1)^3(n-2)^2$

(ii) $HKB_1(K_n,x)=\dfrac{n(n-1)}{2}x^{16(n-1)^2(n-2)^2}$.

Proof:

(i) Let K_n be a complete graph with $n \geq 2$ vertices. By using definition and Lemma 1, we derive

$$HKB_1(K_n)=\sum_{uv\in E(K_n)}[S_e(u)+S_e(v)]^2=|E_1|[4(n-1)(n-2)]^2$$
$$=8n(n-1)^3(n-2)^2.$$

(ii) From definition and by using Lemma 1, we obtain

$$HKB_1(K_n,x)=\sum_{uv\in E(K_n)}x^{[S_e(u)+S_e(v)]^2}=|E_1|x^{[4(n-1)(n-2)]^2}$$
$$=\frac{n(n-1)}{2}x^{16(n-1)^2(n-2)^2}.$$

Theorem 4: The second hyper Kulli-Basava index and its polynomial of a complete graph are given by

(i) $HKB_2(K_n)=8n(n-1)^5(n-2)^4$.

(ii) $HKB_2(K_n,x)=\dfrac{n(n-1)}{2}x^{16(n-1)^4(n-2)^4}$.

Proof:

(i) Let K_n be a complete graph with $n \geq 2$ vertices. Then by using definition and Lemma 1, we deduce

$$HKB_2(K_n)=\sum_{uv\in E(K_n)}[S_e(u)S_e(v)]^2=|E_1|[4(n-1)^2(n-2)^2]^2$$
$$=8n(n-1)^5(n-2)^4.$$

(ii) By using definition and Lemma 1, we obtain

$$HKB_2(K_n,x)=\sum_{uv\in E(K_n)}x^{[S_e(u)S_e(v)]^2}=|E_1|x^{[4(n-1)^2(n-2)^2]^2}$$
$$=\frac{n(n-1)}{2}x^{16(n-1)^4(n-2)^4}.$$

3. WHEEL GRAPHS

A wheel W_n is the join of K_1 and C_n. Clearly W_n has $n+1$vertices and $2n$ edges. A graph W_n is depicted in Figure 1. The vertices of C_n are called rim vertices and the vertex of K_1 is called apex.

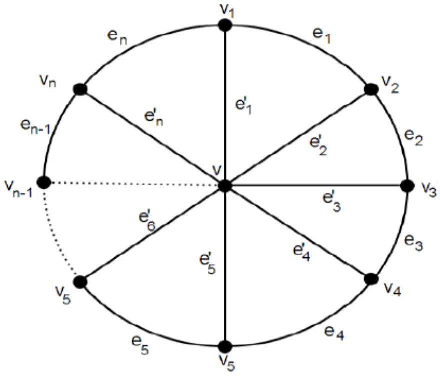

Figure-1: Wheel W_n

Lemma 5: Let W_n be a wheel with $n+1$ vertices. Then W_n has two types of vertices as given below:

$V_1 = \{u \in V(W_n) \mid S_e(u) = n(n+1)\}$, $|V_1| = 1$.

$V_2 = \{u \in V(W_n) \mid S_e(u) = n+9\}$, $|V_2| = n$.

Lemma 6: Let W_n be a wheel with $2n$ edges. Then W_n has two types of edges as given below:

$E_1 = \{uv \in E(W_n) \mid S_e(u) = n+9, S_e(v) = n(n+1)\}$, $|E_1| = n$.

$E_2 = \{uv \in E(W_n) \mid S_e(u) = n+9, S_e(v) = n+9\}$, $|E_2| = n$.

Theorem 7: Let W_n be a wheel with $n+1$ vertices and $2n$ edges, $n \geq 3$. Then

$$KB_1(W_n, x) = nx^{n^2+2n+9} + nx^{2n+18}$$

$$KB_1^*(W_n, x) = x^{n^2(n+1)^2} + nx^{(n+9)^2}$$

$$KB_2(W_n, x) = nx^{n(n+1)(n+9)} + nx^{(n+9)^2}$$

$$KB_3(W_n, x) = nx^{|n^2-9|} + nx^0$$

Proof: By using definitions and Lemmas 5 and 6, we deduce

$$KB_1(W_n, x) = \sum_{uv \in E(W_n)} x^{S_e(u)+S_e(v)} = |E_1|x^{n+9+n(n+1)} + |E_2|x^{n+9+n+9}$$

$$= nx^{n^2+2n+9} + nx^{2n+18}$$

$$KB_1^*(W_n, x) = \sum_{u \in V(W_n)} x^{S_e(u)^2} = |V_1|x^{n^2(n+1)^2} + |V_2|x^{(n+9)^2}$$

$$= x^{n^2(n+1)^2} + nx^{(n+9)^2}$$

$$KB_2(W_n, x) = \sum_{uv \in E(W_n)} x^{S_e(u)S_e(v)} = |E_1|x^{(n+9)n(n+1)} + |E_2|x^{(n+9)(n+9)}$$

$$= nx^{n(n+1)(n+9)} + nx^{(n+9)^2}$$

$$KB_3(W_n, x) = \sum_{uv \in E(W_n)} x^{|S_e(u)-S_e(v)|} = |E_1|x^{|n+9-n(n+1)|} + |E_2|x^{|n+9-n-9|}$$

$$= nx^{|n^2-9|} + nx^0$$

Theorem 8: The first hyper Kulli-Basava index and its polynomial of a wheel W_n are given by

(i) $HKB_1(W_n) = n(n^2 + 2n + 9)^2 + n(2n + 18)^2$.

(ii) $HKB_1(W_n, x) = nx^{(n^2+2n+9)^2} + nx^{(2n+18)^2}$.

Proof:

(i) Let W_n be a wheel with $n \geq 3$ vertices. Then by using definition and Lemma 6, we deduce

$$HKB_1(W_n) = \sum_{uv \in E(W_n)} [S_e(u) + S_e(v)]^2 = |E_1|[n+9+n(n+1)]^2 + |E_2|(n+9+n+9)^2$$

$$= n(n^2+2n+9)^2 + n(2n+18)^2.$$

(ii) $$HKB_1(W_n, x) = \sum_{uv \in E(W_n)} x^{[S_e(u)+S_e(v)]^2}$$

$$= nx^{(n^2+2n+9)^2} + nx^{(2n+18)^2}.$$

Theorem 9: The second hyper Kulli-Basava index and its polynomial of a wheel W_n are given by

(i) $HKB_2(W_n) = n(n+9)^2 \left[n^2(n+1)^2 + (n+9)^2\right].$

(ii) $HKB_2(W_n, x) = nx^{n^2(n+1)^2(n+9)^2} + nx^{(n+9)^4}.$

Proof:

(i) By using definition and Lemma 6, we derive

$$HKB_2(W_n) = \sum_{uv \in E(W_n)} [S_e(u)S_e(v)]^2 = |E_1|[(n+9)n(n+1)]^2 + |E_2|[(n+9)(n+9)]^2$$

$$= n(n+9)^2\left[n^2(n+1)^2 + (n+9)^2\right].$$

(ii) $$HKB_2(W_n, x) = \sum_{uv \in E(W_n)} x^{[S_e(u)S_e(v)]^2}$$

$$= nx^{n^2(n+1)^2(n+9)^2} + nx^{(n+9)^4}.$$

4. GEAR GRAPHS

A graph is a gear graph obtained from W_n by adding a vertex between each pair of adjacent rim vertices and it is denoted by G_n. Clearly G_n has $2n$+1 vertices and $3n$ edges. A graph G_n is shown in Figure 2.

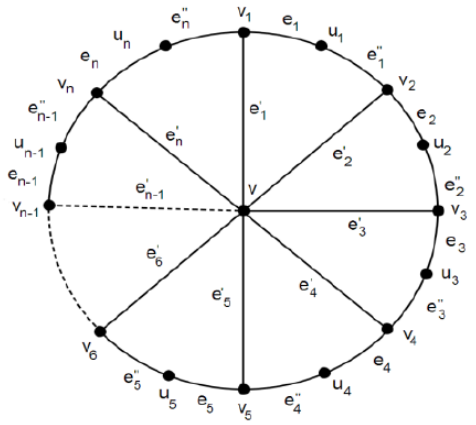

Figure-2: Gear graph G_n

Lemma 10: Let G_n be a gear graph with $2n$+1 vertices. Then G_n has three types of vertices as given below.

$V_1 = \{u \in V(G_n) \mid S_e(u) = n(n+1)\}$, $|V_1| = 1$.
$V_2 = \{u \in V(G_n) \mid S_e(u) = n+7)\}$, $|V_2| = n$.
$V_3 = \{u \in V(G_n) \mid S_e(u) = 6)\}$, $|V_3| = n$.

Lemma 11: Let G_n be a gear graph with $3n$ edges. Then G_n has two types of edges as shown below.

$E_1 = \{uv \in E(G_n) \mid S_e(u) = n(n+1), S_e(v) = n+7\}$, $|E_1| = n$.
$E_2 = \{uv \in E(G_n) \mid S_e(u) = n+7, S_e(v) = 6\}$, $|E_2| = 2n$.

Theorem 12: Let G_n be a gear graph with $2n$+1 vertices and $3n$ edges. Then

$$KB_1(G_n) = n^3 + 4n^2 + 33n.$$

$$KB_1^*(G_n) = n^4 + 3n^3 + 15n^2 + 85n.$$

$$KB_2(G_n) = n(n+7)(n^2+n+12).$$
$$KB_3(G_n) = n|n^2-7|+2n(n+1).$$

Proof: By using definitions and Lemmas 10 and 11, we deduce

$$KB_1(G_n) = \sum_{uv\in E(G_n)} \left[S_e(u)+S_e(v)\right]$$
$$= n\left[n(n+1)+n+7\right]+2n(n+7+6)$$
$$= n^3+4n^2+33n.$$
$$KB_1^*(G_n) = \sum_{u\in V(G_n)} S_e(u)^2$$
$$= \left[n(n+1)\right]^2+n(n+7)^2+n6^2$$
$$= n^4+3n^3+15n^2+85n.$$
$$KB_2(G_n) = \sum_{uv\in E(G_n)} S_e(u)S_e(v)$$
$$= n\left[n(n+1)(n+7)\right]+2n\left[(n+7)6\right]$$
$$= n(n+7)(n^2+n+12).$$
$$KB_3(G_n) = \sum_{uv\in E(G_n)} \left|S_e(u)-S_e(v)\right|$$
$$= n\left|n(n+1)-(n+7)\right|+2n\left|n+7-6\right|$$
$$= n\left|n^2-7\right|+2n(n+1).$$

Theorem 13: Let G_n be a gear graph with $2n+1$ vertices and $3n$ edges. Then

$$KB_1(G_n,x) = nx^{n^2+2n+7}+2nx^{n+13}.$$
$$KB_1^*(G_n,x) = x^{n^2(n+1)^2}+nx^{(n+7)^2}+nx^{36}.$$
$$KB_2(G_n,x) = nx^{n(n+1)(n+7)}+2nx^{6(n+7)}.$$
$$KB_3(G_n,x) = nx^{|n^2-7|}+2nx^{n+1}.$$

Proof: By using definitions and Lemmas 10 and 11, we derive

$$KB_1(G_n,x) = \sum_{uv\in E(G_n)} x^{S_e(u)+S_e(v)} = |E_1|x^{n(n+1)+n+7}+|E_2|x^{n+7+6}$$
$$= nx^{n^2+2n+7}+2nx^{n+13}$$
$$KB_1^*(G_n,x) = \sum_{u\in V(G_n)} x^{S_e(u)^2} = |V_1|x^{n^2(n+1)^2}+|V_2|x^{(n+7)^2}+|V_3|x^{6^2}$$
$$= x^{n^2(n+1)^2}+nx^{(n+7)^2}+nx^{36}.$$
$$KB_2(G_n,x) = \sum_{uv\in E(G_n)} x^{S_e(u)S_e(v)} = |E_1|x^{n(n+1)(n+7)}+|E_2|x^{6(n+7)}$$
$$= nx^{n(n+1)(n+7)}+2nx^{6(n+7)}$$
$$KB_3(G_n,x) = \sum_{uv\in E(G_n)} x^{|S_e(u)-S_e(v)|} = |E_1|x^{|n(n+1)-(n+7)|}+|E_2|x^{|n+7-6|}$$
$$= nx^{|n^2-7|}+2nx^{n+1}.$$

Theorem 14: The first hyper Kulli-Basava index and its polynomial of a gear graph G_n are given by

(i) $HKB_1(G_n) = n(n^2+2n+7)^2+2n(n+13)^2.$

(ii) $HKB_1(G_n,x) = nx^{(n^2+2n+7)^2}+2nx^{(n+13)^2}.$

Proof: Let G_n be a gear graph with $2n$+1 vertices and $3n$ edges.

(i) By using definition and Lemma 11, we obtain

$$HKB_1(G_n) = \sum_{uv \in E(G_n)} \left[S_e(u) + S_e(v)\right]^2 = |E_1|\left[n(n+1) + n + 7\right]^2 + |E_2|(n+7+6)^2$$

$$= n(n^2 + 2n + 7)^2 + 2n(n+13)^2.$$

(ii) $$HKB_1(G_n, x) = \sum_{uv \in E(G_n)} x^{\left[S_e(u)+S_e(v)\right]^2}$$

$$= nx^{(n^2+2n+7)^2} + 2nx^{(n+13)^2}.$$

Theorem 15: The second hyper Kulli-Basava index and its polynomial of a gear graph G_n are given by

(i) $HKB_2(G_n) = n(n+7)^2\left[n^2(n+1)^2 + 72\right].$

(ii) $HKB_2(G_n, x) = nx^{n^2(n+1)^2(n+7)^2} + 2nx^{36(n+7)^2}.$

Proof:

(i) By using definition and Lemma 11, we have

$$HKB_2(G_n) = \sum_{uv \in E(G_n)} \left[S_e(u)S_e(v)\right]^2 = |E_1|\left[n(n+1)(n+7)\right]^2 + |E_2|\left[(n+7)6\right]^2$$

$$= n(n+7)^2\left[n^2(n+1)^2 + 72\right].$$

(ii) $$HKB_2(G_n, x) = \sum_{uv \in E(G_n)} x^{\left[S_e(u)S_e(v)\right]^2}$$

$$= nx^{n^2(n+1)^2(n+7)^2} + 2nx^{36(n+7)^2}.$$

5. HELM GRAPHS

A helm graph H_n is a graph obtained from W_n by attaching an end edge to each rim vertex. Clearly H_n has $2n$+1 vertices and 3n edges, see Figure 3.

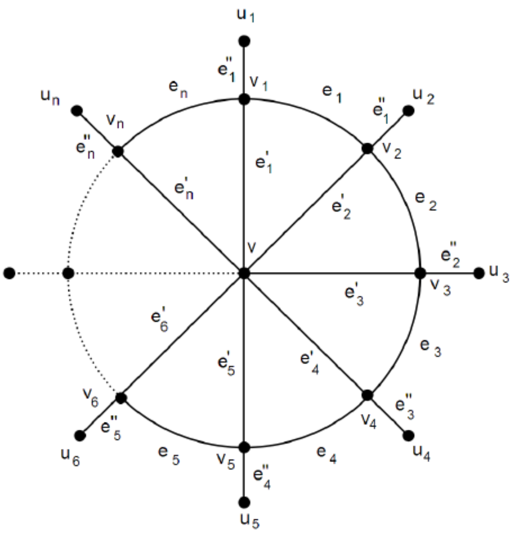

Figure-3: Helm graph H_n

Lemma 16: Let H_n be a helm graph with $2n$+1 vertices.
Then H_n has three types of vertices as follows:

$V_1 = \{u \in V(H_n) \mid S_e(u) = n(n+2)\}$, $|V_1| = 1$.
$V_2 = \{u \in V(H_n) \mid S_e(u) = n+17\}$, $|V_2| = n$.
$V_3 = \{u \in V(H_n) \mid S_e(u) = 3\}$, $|V_3| = n$.

Lemma 17: Let H_n be a helm graph with $3n$ edges. Then H_n has three types of edges as follows.

$E_1 = \{uv \in E(H_n) \mid S_e(u) = n(n+2), S_e(v) = n+17\}$, $|E_1| = n$.
$E_2 = \{uv \in E(H_n) \mid S_e(u) = S_e(v) = n+17\}$, $|E_2| = n$.
$E_3 = \{uv \in E(H_n) \mid S_e(u) = n+17, S_e(v) = 3\}$, $|E_2| = n$.

Theorem 18: Let H_n be a helm graph with $2n + 1$ vertices and $3n$ edges. Then

$$KB_1(H_n) = 3n^3 + 4n^2 + 71n.$$
$$KB_1^*(H_n) = n^4 + 5n^3 + 38n^2 + 298n.$$
$$KB_2(H_n) = n(n+7)(n^2 + 3n + 20).$$
$$KB_3(H_n) = n\left|n^2 + n - 17\right| + n(n+14).$$

Proof: By using definitions and Lemmas 16 and 17, we obtain

$$KB_1(H_n) = \sum_{uv\in E(H_n)} \left[S_e(u) + S_e(v)\right]$$
$$= n\left[n(n+2) + n + 17\right] + n\left[n+17+n+17\right] + n(n+17+3)$$
$$= 3n^3 + 4n^2 + 71n.$$
$$KB_1^*(H_n) = \sum_{u\in V(H_n)} S_e(u)^2$$
$$= \left[n(n+2)\right]^2 + n(n+17)^2 + n\times 3^2$$
$$= n^4 + 5n^3 + 38n^2 + 298n.$$
$$KB_2(H_n) = \sum_{uv\in E(H_n)} S_e(u)S_e(v)$$
$$= n\left[n(n+2)(n+17)\right] + n\left[(n+17)(n+17)\right] + n\left[(n+17)3\right]$$
$$= n(n+17)(n^2 + 3n + 20).$$
$$KB_3(H_n) = \sum_{uv\in E(H_n)} \left|S_e(u) - S_e(v)\right|$$
$$= n\left|n(n+2) - (n+17)\right| + n\,|(n+17) - (n+17)| + n\left|n+17-3\right|$$
$$= n\left|n^2 + n - 17\right| + n(n+14).$$

Theorem 19: Let H_n be a helm graph with $2n+1$ vertices and $3n$ edges. Then

$$KB_1(H_n, x) = nx^{n^2+3n+17} + nx^{2(n+17)} + nx^{n+20}.$$
$$KB_1^*(H_n, x) = x^{[n(n+2)]^2} + nx^{(n+17)^2} + nx^9.$$
$$KB_2(H_n, x) = nx^{n(n+2)(n+17)} + nx^{(n+17)^2} + nx^{3(n+17)}.$$
$$KB_3(H_n, x) = nx^{\left|n^2+n-17\right|} + nx^0 + nx^{n+14}.$$

Proof: By using definitions and Lemmas 16 and 17, we deduce

$$KB_1(H_n, x) = \sum_{uv\in E(H_n)} x^{S_e(u)+S_e(v)}$$
$$= nx^{n(n+2)+n+17} + nx^{n+17+n+17} + nx^{n+17+3}.$$
$$= nx^{n^2+3n+17} + nx^{2(n+17)} + nx^{n+20}.$$
$$KB_1^*(H_n, x) = \sum_{u\in V(H_n)} x^{S_e(u)^2}$$
$$= x^{[n(n+2)]^2} + nx^{(n+17)^2} + nx^9.$$
$$KB_2(H_n, x) = \sum_{uv\in E(H_n)} x^{S_e(u)S_e(v)}$$
$$= nx^{n(n+2)(n+17)} + nx^{(n+17)^2} + nx^{3(n+17)}.$$
$$KB_3(H_n, x) = \sum_{uv\in E(H_n)} x^{\left|S_e(u)-S_e(v)\right|}$$
$$= nx^{\left|n(n+2)-(n+17)\right|} + nx^{\left|(n+17)-(n+17)\right|} + nx^{\left|n+17-3\right|}.$$
$$= nx^{\left|n^2+n-17\right|} + nx^0 + nx^{n+14}.$$

Theorem 20: The first hyper Kulli-Basava index and its polynomial of a helm graph H_n are

(i) $HKB_1(H_n) = n(n^2+3n+17)^2 + 4n(n+17)^2 + n(n+20)^2$.

(ii) $HKB_1(H_n, x) = nx^{(n^2+3n+17)^2} + nx^{(2n+34)^2} + nx^{(n+20)^2}$.

Proof: Let H_n be a helm graph with $2n + 1$ vertices and $3n$ edges.

(i) By using definition and Lemma 17, we deduce

$$HKB_1(H_n) = \sum_{uv\in E(H_n)} [S_e(u)+S_e(v)]^2 = n[n(n+2)+n+17]^2 + n[n+17+n+17]^2 + n(n+17+3)^2$$

$$= n(n^2+3n+17)^2 + 4n(n+17)^2 + n(n+20)^2.$$

(ii) $$HKB_1(H_n, x) = \sum_{uv\in E(H_n)} x^{[S_e(u)+S_e(v)]^2}$$

$$= nx^{(n^2+3n+7)^2} + nx^{(2n+34)^2} + nx^{(n+20)^2}.$$

Theorem 21: The second hyper Kulli-Basava index or its polynomial of a helm graph H_n are

(i) $HKB_2(H_n) = n(n+17)^2\left[n^2(n+2)^2 + (n+17)^2 + 9\right]$.

(ii) $HKB_2(H_n, x) = nx^{n^2(n+2)^2(n+17)^2} + nx^{(n+17)^4} + 2nx^{9(n+17)^2}$.

Proof: Let H_n be a helm graph with $2n+1$ vertices and $3n$ edges.

(i) $$HKB_2(H_n) = \sum_{uv\in E(H_n)} [S_e(u)S_e(v)]^2 = n[n(n+2)(n+17)]^2 + n[(n+17)(n+17)]^2 + n[(n+17)3]^2$$

$$= n(n+7)^2\left[n^2(n+2)^2 + (n+17)^2 + 9\right].$$

(ii) $$HKB_2(H_n, x) = \sum_{uv\in E(H_n)} x^{[S_e(u)S_e(v)]^2}$$

$$= nx^{[n(n+2)(n+17)]^2} + nx^{(n+17)^4} + nx^{9(n+17)^2}.$$

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