International Journal of Mathematical Archive-10(7), 2019, 1-5
IJMA Available online through www.ijma.info ISSN 2229 – 5046

DOMINATION OF SEMI REGULAR GRAPHS USING WIENER INDEX AND DISTANCE MATRIX

M. SAKTHI*
Assistant Professor in Mathematics,
Vivekanandha Arts and Science College for Women, Sankari, India.

(Received On: 18-05-19; Revised & Accepted On: 30-06-19)

ABSTRACT

Basically domination depends with distance of vertices in graph theory. In this paper, domination of semi regular graphs using wiener index and distance matrix is discussed.

Key words: *semi regular graphs, domination, wiener index, distance matrix.*

Subject classification: *05C69.*

1. INTRODUCTION

Domination is the most useful concept in many fields like as networks , project planning, computer etc. In this paper, it is explained that the method of finding domination sets and domination number of semi regular graphs using wiener index method and also distance matrix.

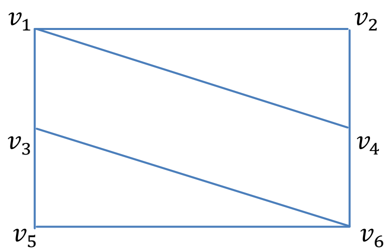
2. WIENER INDEX OF A VERTEX

The *Wiener index* of a vertex v in a graph G, denoted by $W_G(v)$ is the sum of distances between v and all others.
A *dominating set* for a graph G = (V, E) is a subset D of V such that every vertex not in D (every vertex in V- D) is joined to at least one member of D by some edge. (i.e.) A set D of vertices in a graph G is called a dominating set of G if every vertex in V-D is adjacent to some vertex in D. The *domination number* γ (G) is the number of vertices in a smallest dominating set for G. (The cardinality of minimum dominating set)

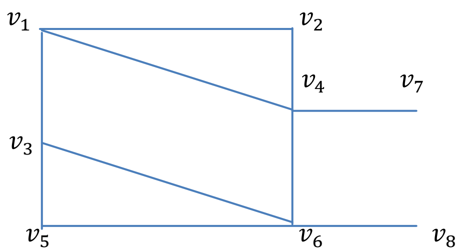
3. FINDING DOMINATION NUMBER AND SETS

1. Find $W_G(v_j)$ where j=1,2,…n.
2. Write $D_i = \{v_j / W_G(v_j) has\ same\ value\ for\ j\}$ where i=1,2,….
3. If $i = 2$ then $\gamma(G) = |\ D_i|$,. where D_i is the set of minimum value of $W_G(v_j)$ and $D = D_1 and\ D_2$.Otherwise $i > 2$ then $D = D_i$, where D_i is the set of minimum value of $W_G(v_j)$ of v_j $\forall j$ and $\gamma(G) = |D|$.

$fig1: 1\ semi\ regular$



$fig2: 2\ semi\ regular$



$fig3: 3\ semi\ regular$

fig 1: $W_G(v_1) = 4, W_G(v_2) = 4, W_G(v_3) = 6, W_G(v_4) = 6\ then\ D_1 = \{v_1, v_2\}, D_2 = \{v_3, v_4\}$
$we\ get\ \gamma(G) = |D| = 2, D(G) = \{D_1, D_2\}$

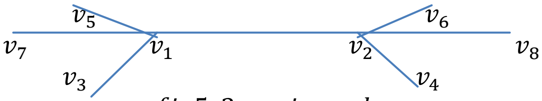
Corresponding Author: M. Sakthi*
Assistant Professor in Mathematics,
Vivekanandha Arts and Science College for Women, Sankari, India.

$fig2$: $W_G(v_1) = 7, W_G(v_2) = 9, W_G(v_3) = 7, W_G(v_4) = 7, W_G(v_5) = 9, W_G(v_6) = 7$ *then* $D_1 = \{v_1, v_3, v_4, v_6\}$, $D_2 = \{v_2, v_5\}$ *we get* $\gamma(G) = |D| = 2, D(G) = D_1 \,\&\, D_2$

$fig\ 3$: $W_G(v_1) = 12, W_G(v_2) = 14, W_G(v_3) = 12, W_G(v_4) = 10, W_G(v_5) = 14, W_G(v_6) = 10, W_G(v_7) = 16$, $W_G(v_8) = 16$ *then* $D_1 = \{v_1, v_3\}, D_2 = \{v_2, v_5\}, D_3 = \{v_4, v_6\}, D_4 = \{v_7, v_8\}$ *we get* $\gamma(G) = |D| = 2$

Similarly adding two vertices v_i, v_j by an edge on v_4, v_6 respectively we get next semi regular graph for all $i \neq j$. And also get $\gamma(G) = 2, D(G) = \{v_4, v_6\}$

$fig4: 2\ semi\ regular$

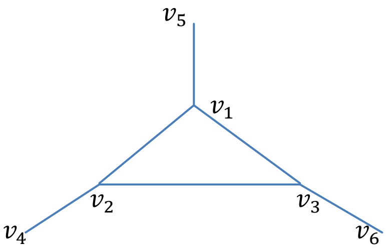


$fig5: 3\ semi\ regular$

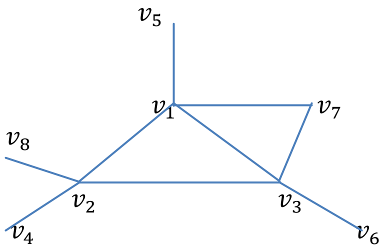
$fig\ 4$: $W_G(v_1) = 7, W_G(v_2) = 7, W_G(v_3) = 11, W_G(v_4) = 11, W_G(v_5) = 11, W_G(v_6) = 11$ *then* $D_1 = \{v_1, v_2\}$, $D_2 = \{v_3, v_4, v_5 v_6\}$, *we get* $\gamma(G) = |D| = 2, D(G) = D_1 \,\&\, D_2$

$fig\ 5$: $W_G(v_1) = 10, W_G(v_2) = 10, W_G(v_3) = 16, W_G(v_4) = 16, W_G(v_5) = 16, W_G(v_6) = 16, W_G(v_7) = 16$, $W_G(v_8) = 16$ *then* $D_1 = \{v_1, v_2\}, D_2 = \{v_3, v_4, v_5, v_6, v_7, v_8\}$ *we get* $\gamma(G) = min|D_i| = 2, D(G) = \{D_1, D_2\}$

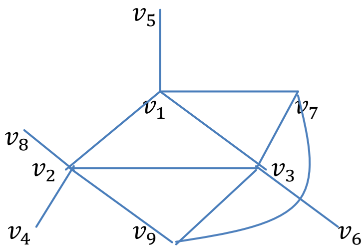
Similarly adding two vertices v_i, v_j by an edge on v_1, v_2 respectively we get next semi regular graph for all $i \neq j$. And also get $\gamma(G) = 2, D(G) = \{v_1, v_2\}$.



$fig\ 6: 2\ semi\ regular$



$fig7: 3\ semi\ regular$

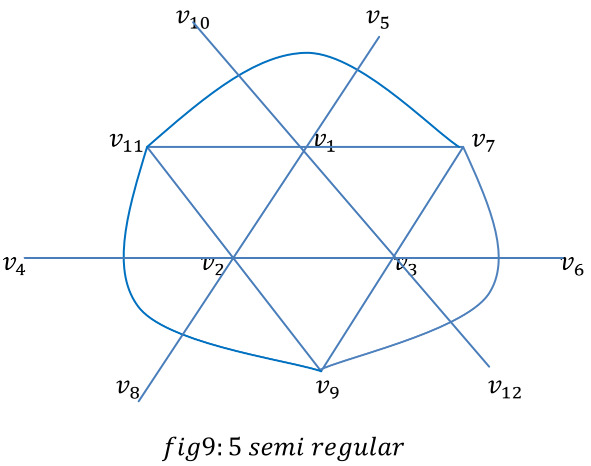


$fig8: 4\ semi\ regular$

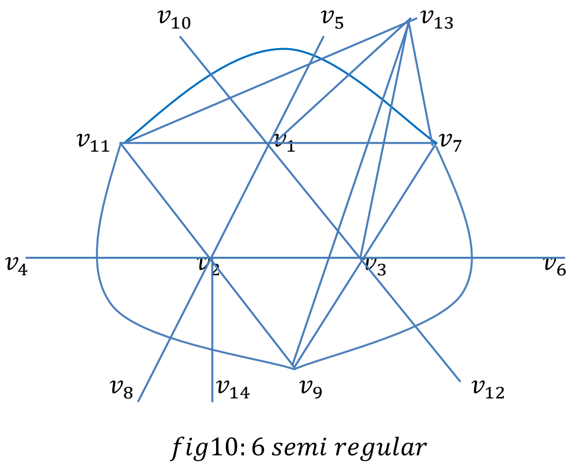
$fig\ 6$: $W_G(v_1) = 7, W_G(v_2) = 7, W_G(v_3) = 7, W_G(v_4) = 11, W_G(v_5) = 11, W_G(v_6) = 11$ *then* $D_1 = \{v_1, v_2, v_3\}$, $D_2 = \{v_4, v_5 v_6\}$, *we get* $\gamma(G) = |D| = 3, D(G) = D_1 \,\&\, D_2$.

$fig\ 7$: $W_G(v_1) = 10, W_G(v_2) = 10, W_G(v_3) = 10, W_G(v_4) = 16, W_G(v_5) = 16, W_G(v_6) = 16, W_G(v_7) = 14$, $W_G(v_8) = 16$ *then* $D_1 = \{v_1, v_2, v_3\}, D_2 = \{v_4, v_5, v_6, v_8\}, D_3 = \{v_7\}$ *we get* $\gamma(G) = |D| = 3, D(G) = \{D_1\}$

$fig\ 8$: $W_G(v_1) = 13, W_G(v_2) = 13, W_G(v_3) = 13, W_G(v_4) = 21, W_G(v_5) = 21, W_G(v_6) = 21, W_G(v_7) = 17$, $W_G(v_8) = 21, W_G(v_9) = 17, W_G(v_{10}) = 21$ *then* $D_1 = \{v_1, v_2, v_3\}, D_2 = \{v_4, v_5, v_6, v_8, v_{10}\}, D_3 = \{v_7, v_9\}$ *we get* $\gamma(G) = |D| = 3, D(G) = \{D_1\}$



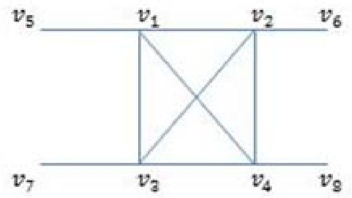
$fig9: 5\ semi\ regular$



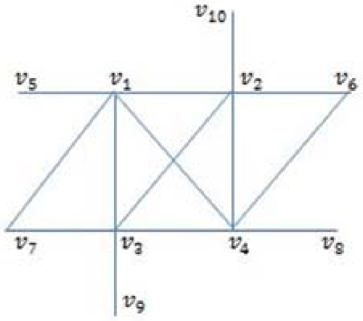
$fig10: 6\ semi\ regular$

$fig\ 9$: $W_G(v_1) = 16, W_G(v_2) = 16, W_G(v_3) = 16, W_G(v_4) = 21, W_G(v_5) = 21, W_G(v_6) = 21, W_G(v_7) = 22$, $W_G(v_8) = 21, W_G(v_9) = 22, W_G(v_{10}) = 21, W_G(v_{11}) = 22, W_G(v_{12}) = 21$ *then* $D_1 = \{v_1, v_2, v_3\}$, $D_2 = \{v_7, v_9, v_{11}\}, D_3 = \{v_4, v_5, v_6, v_8, v_{10}, v_{12}\}$ *we get* $\gamma(G) = |D| = 3, D = \{v_1, v_2, v_3\}$

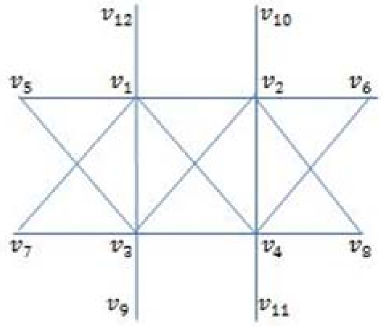
$fig\ 10{:}\ W_G(v_1) = 19, W_G(v_2) = 19, W_G(v_3) = 19, W_G(v_4) = 31, W_G(v_5) = 31, W_G(v_6) = 31, W_G(v_7) = 24,$
$W_G(v_8) = 31, W_G(v_9) = 23, W_G(v_{10}) = 31, W_G(v_{11}) = 23, W_G(v_{12}) = 31, W_G(v_{13}) = 24,$
$W_G(v_{14}) = 31\ then\ D_1 = \{v_1, v_2, v_3\}, D_2 = \{v_4, v_5, v_6, v_8, v_{10}, v_{12}, v_{14}\}, D_3 = \{v_7, v_{13}\}, D_4 = \{v_9, v_{11}\}$
$we\ get\ \gamma(G) = |D| = 3, D = \{v_1, v_2, v_3\}$



$fig11{:}\ 3\ semi\ regular$



$fig12{:}\ 4\ semiregular$

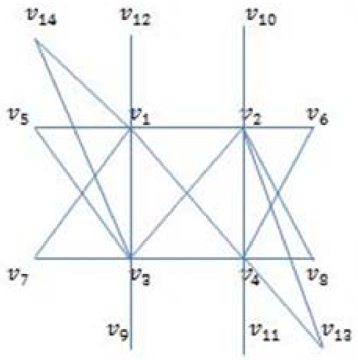


$fig13{:}\ 5\ semi\ regular$

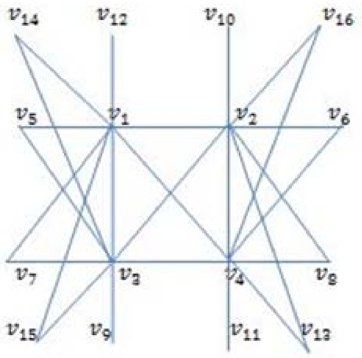
$fig\ 11{:}\ W_G(v_1) = 10, W_G(v_2) = 10, W_G(v_3) = 10, W_G(v_4) = 10, W_G(v_5) = 16, W_G(v_6) = 16, W_G(v_7) = 16,$
$W_G(v_8) = 16\ then\ D_1 = \{v_1, v_2, v_3, v_4\}, D_2 = \{v_5, v_6, v_7, v_8\}\ we\ get\ \gamma(G) = |D| = 4, D(G) = D_1 \&\ D_2$

$fig\ 12{:}\ W_G(v_1) = 13, W_G(v_2) = 13, W_G(v_3) = 13, W_G(v_4) = 13, W_G(v_5) = 21, W_G(v_6) = 19, W_G(v_7) = 19,$
$W_G(v_8) = 21, W_G(v_9) = 21, W_G(v_{10}) = 21\ then\ D_1 = \{v_1, v_2, v_3, v_4\}, D_2 = \{v_5, v_8, v_9, v_{10}\}, D_3 = \{v_6, v_7\}$
$we\ get\ \gamma(G) = |D| = 4, D(G) = \{v_1, v_2, v_3, v_4\}$

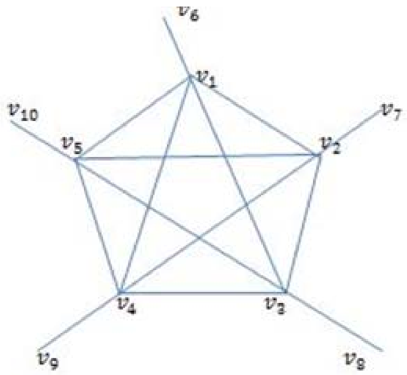
$fig\ 13{:}\ W_G(v_1) = 16, W_G(v_2) = 16, W_G(v_3) = 16, W_G(v_4) = 16, W_G(v_5) = 24, W_G(v_6) = 24, W_G(v_7) = 24,$
$W_G(v_8) = 24, W_G(v_9) = 26, W_G(v_{10}) = 26, W_G(v_{11}) = 26, W_G(v_{12}) = 26\ then\ D_1 = \{v_1, v_2, v_3, v_4\},$
$D_2 = \{v_5, v_6, v_7, v_8\}, D_3 = \{v_9, v_{10}, v_{11}, v_{12}\}\ we\ get\ \gamma(G) = |D| = 4, D = \{v_1, v_2, v_3, v_4\}.$



$fig14{:}\ 6\ semi\ regular$



$fig15{:}\ 7\ semi\ regular$

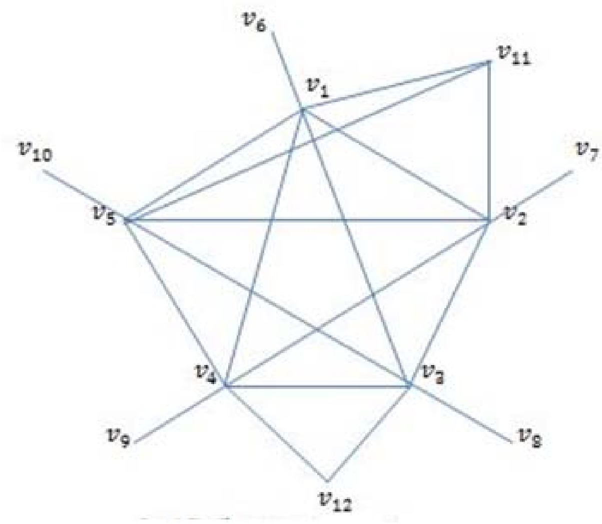


$fig\ 16{:}\ 4\ semi\ regular$

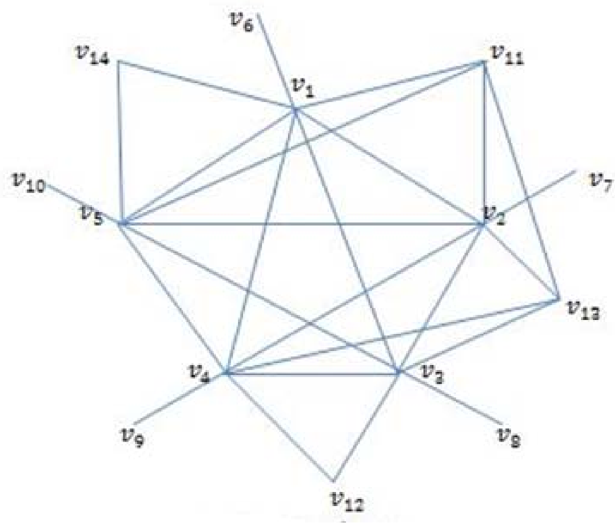
$fig\ 14{:}\ W_G(v_1) = 19, W_G(v_2) = 19, W_G(v_3) = 19, W_G(v_4) = 19, W_G(v_5) = 29, W_G(v_6) = 29, W_G(v_7) = 29,$
$W_G(v_8) = 29, W_G(v_9) = 31, W_G(v_{10}) = 31, W_G(v_{11}) = 31, W_G(v_{12}) = 31, W_G(v_{13}) = 29,$
$W_G(v_{14}) = 29\ then\ D_1 = \{v_1, v_2, v_3, v_4\}, D_2 = \{v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}\}, D_3 = \{v_{13}, v_{14}\},$
$we\ get\ \gamma(G) = |D| = 4, D = \{v_1, v_2, v_3, v_4\}$

$fig\ 15{:}\ W_G(v_1) = 22, W_G(v_2) = 22, W_G(v_3) = 22, W_G(v_4) = 22, W_G(v_5) = 34, W_G(v_6) = 34, W_G(v_7) = 34,$
$W_G(v_8) = 34, W_G(v_9) = 36, W_G(v_{10}) = 36, W_G(v_{11}) = 36, W_G(v_{12}) = 36, W_G(v_{13}) = 34,$
$W_G(v_{14}) = 34\ , W_G(v_{15}) = 34, W_G(v_{16}) = 34\ then\ D_1 = \{v_1, v_2, v_3, v_4\}, D_2 = \{v_9, v_{10}, v_{11}, v_{12}\},$
$D_3 = \{v_5, v_6, v_7, v_8, v_{13}, v_{14}, v_{15}, v_{16}\}, D_4 = \{v_{11}, v_{12}\}\ we\ get\ \gamma(G) = |D| = 4, D = \{v_1, v_2, v_3, v_4\}$

$fig\ 16{:}\ W_G(v_1) = 13, W_G(v_2) = 13, W_G(v_3) = 13, W_G(v_4) = 13, W_G(v_5) = 13, W_G(v_6) = 21, W_G(v_7) = 21,$
$W_G(v_8) = 21, W_G(v_9) = 21, W_G(v_{10}) = 21, then\ D_1 = \{v_1, v_2, v_3, v_4, v_5\}, D_2 = \{v_6, v_7, v_8, v_9, v_{10}\}\ we\ get$
$\gamma(G) = |D| = 5, D(G) = D_1\ \&\ D_2.$



$fig17: 5\ semi\ regular$



$fig18: 6\ semi\ regular$

$fig17: W_G(v_1) = 16, W_G(v_2) = 16, W_G(v_3) = 16, W_G(v_4) = 16, W_G(v_5) = 16, W_G(v_6) = 26, W_G(v_7) = 25,$ $W_G(v_8) = 26, W_G(v_9) = 26, W_G(v_{10}) = 26, W_G(v_{11}) = 22, W_G(v_{12}) = 24\ then\ D_1 = \{v_1, v_2, v_3, v_4, v_5\},$ $D_2 = \{v_6, v_7, v_8, v_9, v_{10}\}, D_3 = \{v_{11}, v_{12}\}\ we\ get\ \gamma(G) = |D| = 5, D = \{v_1, v_2, v_3, v_4, v_5\}.$

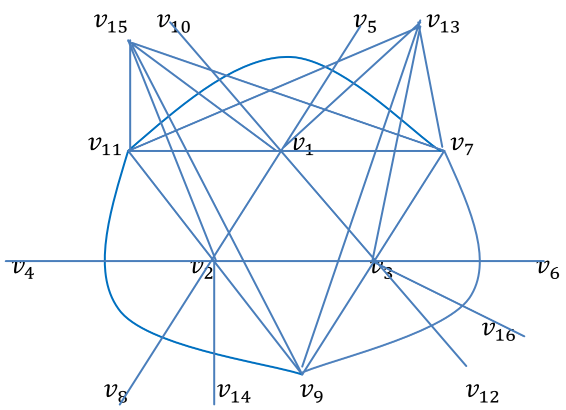
$fig\ 18: W_G(v_1) = 19, W_G(v_2) = 19, W_G(v_3) = 19, W_G(v_4) = 19, W_G(v_5) = 19, W_G(v_6) = 31, W_G(v_7) = 31,$ $W_G(v_8) = 31, W_G(v_9) = 31, W_G(v_{10}) = 31, W_G(v_{11}) = 25, W_G(v_{12}) = 29, W_G(v_{13}) = 25,$ $W_G(v_{14}) = 29\ then\ D_1 = \{v_1, v_2, v_3, v_4, v_5\}, D_2 = \{v_6, v_7, v_8, v_9, v_{10}\}, D_3 = \{v_{11}, v_{13}\}, D_4 = \{v_{12}, v_{14}\}$ $we\ get\ \gamma(G) = |D| = 5, D = \{v_1, v_2, v_3, v_4, v_5\}.$

4. DISTANCE MATRIX

Distance matrix is a square matrix (two dimensional array) containing the distances, taken pairwise , between the elements of set.(i.e.) the square matrix of distances between each pair of vertices in graphs.

5. FINDING DOMINATION NUMBER AND SETS

1. Find the shortest distances of each pair of vertices and table it. (i.e.) Find distance matrix of given graph.
2. Find the total distance of each rows and columns .
   $\sum r_j = a_{j1} + a_{j2} + \cdots + a_{jn}, \sum c_j = a_{1j} + a_{2j} + \cdots + a_{nj}, \sum r_j = \sum c_j = W_G(v_j)\ \forall j = 1,2, \dots n$
3. Write $D_i = \{v_i / W_G(v_j) has\ same\ value\ for\ j\}$ where i, j =1,2,….n
4. If $i = 2$ then $\gamma(G) = |D|$ where D is the set of minimum value of $W_G(v_j)$ and $D = D_1 and\ D_2$.Otherwise $i > 2$ then $D = D_i$, where D_i is the set of minimum value of $W_G(v_j)$ of v_j $\forall j$ and $\gamma(G) = |D|$.



$fig19: 7\ semi\ regular$

$Wiener\ index\ of\ fig\ 19: 7\ semi\ regular\ graph$

$W_G(v_1) = 22, W_G(v_2) = 22, W_G(v_3) = 22,$
$W_G(v_4) = 36, W_G(v_5) = 36, W_G(v_6) = 36,$
$W_G(v_7) = 27, W_G(v_8) = 36, W_G(v_9) = 27,$
$W_G(v_{10}) = 36, W_G(v_{11}) = 27, W_G(v_{12}) = 36,$
$W_G(v_{13}) = 28, W_G(v_{14}) = 36, W_G(v_{15}) = 28,$
$W_G(v_{16}) = 36\ then\ D_1 = \{v_1, v_2, v_3\},$
$D_2 = \{v_7, v_9, v_{11}\},$
$D_3 = \{v_4, v_5, v_6, v_8, v_{10}, v_{12}, v_{14}, v_{16}\},$
$D_4 = \{v_{13}, v_{15}\}\ we\ get$
$\gamma(G) = |D| = 3,$
$D = \{v_1, v_2, v_3\}$

c/r	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}	v_{11}	v_{12}	v_{13}	v_{14}	v_{15}	v_{16}	$\sum r_j$
v_1	0	1	1	2	1	2	1	2	2	1	1	2	1	2	1	2	22
v_2	1	0	1	1	2	2	2	1	1	2	1	2	2	1	1	2	22
v_3	1	1	0	2	2	1	1	2	1	2	2	1	1	2	2	1	22
v_4	2	1	2	0	3	3	3	2	2	3	2	3	3	2	2	3	36
v_5	1	2	2	3	0	3	2	3	3	2	2	3	2	3	2	3	36
v_6	2	2	1	3	3	0	2	3	2	3	3	2	2	3	3	2	36
v_7	1	2	1	3	2	2	0	3	1	2	1	2	1	3	1	2	27
v_8	2	1	2	2	3	3	3	0	2	3	2	3	3	2	2	3	36
v_9	2	1	1	2	3	2	2	2	0	3	1	2	1	2	1	2	27
v_{10}	1	2	2	3	2	3	1	3	3	0	2	3	2	3	2	3	36
v_{11}	1	1	2	2	2	3	1	2	1	2	0	3	1	2	1	3	27
v_{12}	2	2	1	3	3	2	2	3	2	3	3	0	2	3	3	2	36
v_{13}	1	2	1	3	2	2	1	3	1	2	1	2	0	3	2	2	28
v_{14}	2	1	2	2	3	3	3	2	2	3	2	3	3	0	2	3	36
v_{15}	1	1	2	2	2	3	1	2	1	2	1	3	2	2	0	3	28
v_{16}	2	2	1	3	3	2	2	3	2	3	3	2	2	3	3	0	36
$\sum c_j$	22	22	22	36	36	36	27	36	27	36	27	36	28	36	28	36	491

$\boldsymbol{Table-1}$: $distance\ matrix\ of\ fig19\ \ (7\ semi\ regular)$

6. CONCLUSION

In this paper, it is discussed about the relations between distance and domination. And also explained some concepts for domination with domination number using wiener index and domination matrix based on distance.

7. REFERENCES

1. Allison Northup, A Study on semiregular graphs, Stetson University (2002).
2. N.Murugesan and Deepa S. Nair, The Domination and Independence of Some Cubic Bipartite Graphs, Int. J. Contemp. Math Sciences, Vol.6, 2011, No.13, 611-618.
3. N.Murugesan, R.Anitha ,International Journal of Mathematical Archive-5(4), 2014, 182-189 Available online through www.ijma.info ISSN 2229 – 5046, "some properties of semiregular graphs".

Source of support: Nil, Conflict of interest: None Declared.