DOMINATION OF SEMI REGULAR GRAPHS USING WIENER INDEX AND DISTANCE MATRIX

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ABSTRACT

Basically domination depends with distance of vertices in graph theory. In this paper, domination of semi regular graphs using wiener index and distance matrix is discussed.

Key words: semi regular graphs, domination, wiener index, distance matrix.

Subject classification: 05C69.

1. INTRODUCTION

Domination is the most useful concept in many fields like as networks, project planning, computer etc. In this paper, it is explained that the method of finding domination sets and domination number of semi regular graphs using wiener index method and also distance matrix.

2. WIENER INDEX OF A VERTEX

The Wiener index of a vertex $v$ in a graph $G$, denoted by $W_G(v)$ is the sum of distances between $v$ and all others.

A dominating set for a graph $G = (V, E)$ is a subset $D$ of $V$ such that every vertex not in $D$ (every vertex in $V - D$) is joined to at least one member of $D$ by some edge. (i.e.) A set $D$ of vertices in a graph $G$ is called a dominating set of $G$ if every vertex in $V-D$ is adjacent to some vertex in $D$. The domination number $\gamma(G)$ is the number of vertices in a smallest dominating set for $G$. (The cardinality of minimum dominating set)

3. FINDING DOMINATION NUMBER AND SETS

1. Find $W_G(v_j)$ where $j=1,2,...n$.
2. Write $D_i = \{v_j/W_G(v_j) has same value for j\}$ where $i=1,2,...$.
3. If $i = 2$ then $\gamma(G) = |D_1|$, where $D_1$ is the set of minimum value of $W_G(v_j)$ and $D = D_1 and D_2$. Otherwise $i > 2$ then $D = D_i$, where $D_i$ is the set of minimum value of $W_G(v_j)$ of $v_j \forall j$ and $\gamma(G) = |D|$.

\[
\begin{align*}
\text{fig1:1 semi regular} & \quad \text{fig2:2 semi regular} & \quad \text{fig3:3 semi regular} \\
& \quad \text{fig1: } W_G(v_1) = 4, W_G(v_2) = 4, W_G(v_3) = 6, W_G(v_4) = 6 \text{ then } D_1 = \{v_1, v_2\}, D_2 = \{v_3, v_4\} \text{ we get } \gamma(G) = |D| = 2, D(G) = \{D_1, D_2\}
\end{align*}
\]

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FIG 2: $W_G(v_1) = 7, W_G(v_2) = 9, W_G(v_3) = 7, W_G(v_4) = 7, W_G(v_5) = 9, W_G(v_6) = 7$ then $D_1 = \{v_1, v_3, v_4, v_6\}$, $D_2 = \{v_2, v_5\}$ we get $\gamma(G) = |D| = 2, D(G) = D_1 & D_2$

FIG 3: $W_G(v_1) = 12, W_G(v_2) = 14, W_G(v_3) = 12, W_G(v_4) = 10, W_G(v_5) = 14, W_G(v_6) = 10, W_G(v_7) = 16, W_G(v_8) = 16$ then $D_1 = \{v_1, v_3\}, D_2 = \{v_2, v_5\}, D_3 = \{v_4, v_6\}, D_4 = \{v_7, v_8\}$ we get $\gamma(G) = |D| = 2$

Similarly adding two vertices $v_i, v_j$ by an edge on $v_4, v_6$ respectively we get next semi regular graph for all $i \neq j$. And also get $\gamma(G) = 2, D(G) = \{v_4, v_6\}$

FIG 4: 2 semi regular

FIG 5: 3 semi regular

FIG 6: $W_G(v_1) = 7, W_G(v_2) = 7, W_G(v_3) = 11, W_G(v_4) = 11, W_G(v_5) = 11, W_G(v_6) = 11$ then $D_1 = \{v_1, v_2\}$, $D_2 = \{v_3, v_4, v_5, v_6\}$ we get $\gamma(G) = |D| = 2, D(G) = D_1 & D_2$

FIG 7: $W_G(v_1) = 10, W_G(v_2) = 10, W_G(v_3) = 16, W_G(v_4) = 16, W_G(v_5) = 16, W_G(v_6) = 16, W_G(v_7) = 16, W_G(v_8) = 16$ then $D_1 = \{v_1, v_2\}, D_2 = \{v_3, v_4, v_5, v_6, v_7\}$ we get $\gamma(G) = \min |D_i| = 2, D(G) = \{D_1, D_2\}$

Similarly adding two vertices $v_i, v_j$ by an edge on $v_1, v_2$ respectively we get next semi regular graph for all $i \neq j$. And also get $\gamma(G) = 2, D(G) = \{v_1, v_2\}$

FIG 8: 4 semi regular

FIG 9: 5 semi regular

FIG 10: 6 semi regular
fig 10: \( W_G(v_1) = 19, W_G(v_2) = 19, W_G(v_3) = 19, W_G(v_4) = 31, W_G(v_5) = 31, W_G(v_6) = 24, W_G(v_7) = 31, W_G(v_8) = 23, W_G(v_9) = 31, W_G(v_{10}) = 23, W_G(v_{11}) = 31, W_G(v_{12}) = 24, W_G(v_{13}) = 31 \) then \( D_1 = \{v_1, v_2, v_3, v_4\}, D_2 = \{v_5, v_6, v_7, v_8\} \) we get \( \gamma(G) = |D| = 3, D = \{v_1, v_2, v_3\} \)

fig 11: 3 semi regular

fig 12: 4 semi regular

fig 13: 5 semi regular

fig 14: \( W_G(v_1) = 19, W_G(v_2) = 19, W_G(v_3) = 19, W_G(v_4) = 19, W_G(v_5) = 16, W_G(v_6) = 16, W_G(v_7) = 16, W_G(v_8) = 16 \) then \( D_1 = \{v_1, v_2, v_3, v_4, v_5\}, D_2 = \{v_6, v_7, v_8\} \) we get \( \gamma(G) = |D| = 4, D(G) = D_1 \& D_2 \)

fig 15: 7 semi regular

fig 16: 4 semi regular

fig 17: 6 semi regular

fig 18: 5 semi regular

fig 19: 4 semi regular

fig 20: 3 semi regular

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5. FINDING DOMINATION NUMBER AND SETS

Elements of set. (i.e.) the square matrix of distances between each pair of vertices in graphs.

Distance matrix is a square matrix (two dimensional array) containing the distances, taken pairwise, between the elements of set. (i.e.) the square matrix of distances between each pair of vertices in graphs.

4. DISTANCE MATRIX

Distance matrix is a square matrix (two dimensional array) containing the distances, taken pairwise, between the elements of set. (i.e.) the square matrix of distances between each pair of vertices in graphs.

5. FINDING DOMINATION NUMBER AND SETS

1. Find the shortest distances of each pair of vertices and table it. (i.e.) Find distance matrix of given graph.
2. Find the total distance of each rows and columns.
   \[ \sum r_j = a_{j1} + a_{j2} + \cdots + a_{jn}, \sum c_j = a_{1j} + a_{2j} + \cdots + a_{nj}, \sum r_j = \sum c_j = W_G(v) \forall j = 1, 2, \ldots, n \]
3. Write \( D_i = \{v_i/W_G(v_j) has same value for j\} \) where \( i, j = 1, 2, \ldots, n \)
4. If \( i = 2 \) then \( \gamma(G) = |D| \) where \( D \) is the set of minimum value of \( W_G(v_j) \) and \( D = D_1 \) and \( D_2 \). Otherwise \( i > 2 \) then \( D = D_1 \), where \( D_1 \) is the set of minimum value of \( W_G(v_j) \) of \( v_j \forall j \) and \( \gamma(G) = |D| \).

Viener index of fig 19:7 semi regular graph

\[ W_G(v_1) = 22, W_G(v_2) = 22, W_G(v_3) = 22, \]
\[ W_G(v_4) = 36, W_G(v_5) = 36, W_G(v_6) = 36, \]
\[ W_G(v_7) = 27, W_G(v_8) = 27, W_G(v_9) = 27, \]
\[ W_G(v_{10}) = 36, W_G(v_{11}) = 27, W_G(v_{12}) = 36, \]
\[ W_G(v_{13}) = 28, W_G(v_{14}) = 36, W_G(v_{15}) = 28, \]
\[ W_G(v_{16}) = 36 \] then \( D_1 = \{v_1, v_2, v_3\} \), \( D_2 = \{v_7, v_9, v_{11}\} \), \( D_3 = \{v_4, v_5, v_6, v_9, v_{10}, v_{12}, v_{14}, v_{16}\} \), \( D_4 = \{v_{13}, v_{15}\} \) we get \( \gamma(G) = |D| = 3 \), \( D = \{v_1, v_2, v_3\} \)
6. CONCLUSION

In this paper, it is discussed about the relations between distance and domination. And also explained some concepts for domination with domination number using wiener index and domination matrix based on distance.

7. REFERENCES


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| 𝑐/𝑟 | 𝑣₁ | 𝑣₂ | 𝑣₃ | 𝑣₄ | 𝑣₅ | 𝑣₆ | 𝑣₇ | 𝑣₈ | 𝑣₉ | 𝑣₁₀ | 𝑣₁₁ | 𝑣₁₂ | 𝑣₁₃ | 𝑣₁₄ | 𝑣₁₅ | 𝑣₁₆ | ∑ 𝑟ᵢ |
|------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 𝑣₁  | 0  | 1  | 1  | 2  | 1  | 2  | 1  | 2  | 1  | 2  | 1  | 2  | 1  | 2  | 1  | 2  | 22 |
| 𝑣₂  | 1  | 0  | 1  | 2  | 2  | 1  | 2  | 1  | 2  | 1  | 2  | 1  | 2  | 1  | 2  | 22 |
| 𝑣₃  | 1  | 1  | 0  | 2  | 2  | 1  | 2  | 1  | 2  | 1  | 2  | 1  | 2  | 1  | 2  | 22 |
| 𝑣₄  | 2  | 1  | 0  | 3  | 3  | 2  | 3  | 2  | 3  | 3  | 2  | 3  | 3  | 2  | 3  | 36 |
| 𝑣₅  | 1  | 2  | 2  | 3  | 0  | 3  | 2  | 3  | 2  | 3  | 3  | 2  | 3  | 2  | 3  | 36 |
| 𝑣₆  | 2  | 2  | 1  | 3  | 3  | 2  | 3  | 2  | 3  | 3  | 2  | 3  | 3  | 2  | 3  | 36 |
| 𝑣₇  | 1  | 2  | 1  | 3  | 2  | 2  | 0  | 3  | 1  | 2  | 1  | 2  | 1  | 3  | 1  | 2  | 27 |
| 𝑣₈  | 2  | 1  | 2  | 2  | 3  | 3  | 0  | 2  | 3  | 2  | 3  | 2  | 3  | 2  | 3  | 36 |
| 𝑣₉  | 2  | 1  | 2  | 3  | 2  | 2  | 0  | 3  | 1  | 2  | 1  | 2  | 1  | 2  | 1  | 2  | 27 |
| 𝑣₁₀ | 1  | 2  | 2  | 3  | 2  | 3  | 1  | 3  | 3  | 0  | 2  | 3  | 2  | 3  | 2  | 3  | 36 |
| 𝑣₁₁ | 1  | 1  | 2  | 2  | 3  | 1  | 2  | 1  | 2  | 0  | 3  | 1  | 2  | 1  | 3  | 2  | 27 |
| 𝑣₁₂ | 2  | 2  | 1  | 3  | 3  | 2  | 2  | 3  | 2  | 3  | 0  | 2  | 3  | 3  | 2  | 3  | 36 |
| 𝑣₁₃ | 1  | 2  | 1  | 3  | 2  | 1  | 3  | 1  | 2  | 1  | 2  | 0  | 3  | 2  | 2  | 2  | 28 |
| 𝑣₁₄ | 2  | 1  | 2  | 3  | 3  | 2  | 2  | 3  | 2  | 3  | 2  | 3  | 0  | 2  | 3  | 3  | 36 |
| 𝑣₁₅ | 1  | 1  | 2  | 2  | 3  | 1  | 2  | 1  | 2  | 1  | 3  | 2  | 2  | 0  | 3  | 2  | 28 |
| 𝑣₁₆ | 2  | 2  | 1  | 3  | 3  | 2  | 3  | 2  | 3  | 3  | 2  | 3  | 3  | 0  | 3  | 36 |
| ∑ 𝑐ⱼ| 22 | 22 | 22 | 36 | 36 | 36 | 27 | 36 | 27 | 36 | 27 | 36 | 27 | 36 | 28 | 36 | 491 |

Table – 1: distance matrix of fig19 (7 semi regular)