International Journal of Mathematical Archive-10(7), 2019, 43-46
Available online through www.ijma.info ISSN 2229 – 5046

SOME MATHEMATICAL ASPECTS OF BEEHIVE AND HONEYCOMB CONSTRICTION BY BEES

AVHALE.P.S*

**Department of Mathematics,
Shivaji Arts and Science College Kannad, Dist. Aurangabad (M.H), India.**

(Received On: 22-06-19; Revised & Accepted On: 23-07-19)

ABSTRACT

In this article, observation on the building technique of Apis Mellifera describing the construction of beehive and honeycomb is proposed. Honeycomb is compact hexagonal structure for mention the least building material and maximum storage of honey. The construction of hive is represented by a set of dynamical non-linear partial differential equations for the density of bees and quantity of wax distributed in the hive.

Key words and Phrases: *Honeycomb, hive.*

1. INTRODUCTION

We know honey bee is small insect till he make management of hive which is mathematically can be proved as top management. We interested in study of store of honey, population of bees and shape of hive.

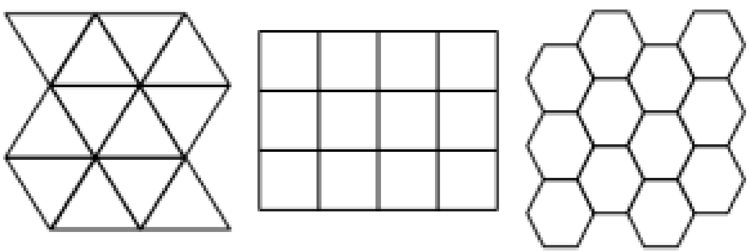
2. HONEYCOMB CONJECTURE

A mathematician would describe a cell as ‘a’ hexagonal prism. We discuss why bee story honey in hexagonal prism. Wax is expensive, bees select the structure which require less wax and store maximum amount of honey. The Bee stores the honey in hexagonal tube. We see the mathematical view of tiny bee.

The first requirements are that there should be no gaps. When multiple copies of the shapes used are stacked adjacent to each other on the plane otherwise space will be west.

We know some shapes are, the circle, the pentagon and polygon with eight or more sides leave the gap when they put together. While square, the equilateral triangle and the regular hexagon leave no gap when they put together.

The second requirement is that the amount of material needed to construct the cell should be the least possible. If the three remaining shapes are drawn on a graph as indicated the peripheral length of each can be calculated easily.



The triangle, square and hexagon have wall lengths respectively of $7 \times 3 = 21$ units $6 \times 4 = 24$ units and $3 \times 6 = 18$ units. Thus by similar calculation it would appear that the hexagon which has the shortest periphery is the most economical to make.

Corresponding Author: Avhale.P.S*
Department of Mathematics,
Shivaji Arts and Science College Kannad, Dist. Aurangabad (M.H), India.

Thus the hexagon shape optimized both space and the use of building materials. One hypothesis be mentions that the six sides of the cell were necessary as they made better allowance for the six legs of the bee.

In 1943 the Hungarian mathematician L Fejes Toth (1915-2005) provided a mathematical proof that the regular hexagon did give the smallest total perimeter for any pattern comprising polygons with straight edges and it wasn't until 1999 that Thomas C Hales (born 1958) of the university of Michign in US

3. HIVE CONSTRUCTION ACTIVITY AND MANAGEMENT OF LONG CHAINS OF PARALLEL BEES.

The competition between two differently oriented Constructional activity in social insects is a highly co-operative phenomenon of great complexity This structure results from the multitude of interactions between the workers as well as between the workers and the building material However, a mathematical modeling of one of the crucial aspects of such a phenomenon, the parallelism of combs Honey-bees, Apis mellifera, are able to construct parallel combs and to maintain and even to restore their parallelism when it is disturbed So, in an empty(circular) beehive a swarm constructs more or less parallel and equidistant combs. The average distance between combs varies from 2'5-5 cm (Darchen, 1959).

The building activities are social phenomena. In order to construct, a minimum number of bees is necessary: at least 100 in the presence of the queen, and about 10 000 in her absence (Darchen, 1959).

The swarm chooses the highest place in a nest cavity, its ceiling, and hangs from there. It forms a drop like cluster in which the bees are in close contact, hanging one to the other or crawling about. At the beginning of the building, workers deposit at random small balls of wax on the ceiling. An arbitrary deposit of a new ball of wax on one side of the depot breaks its central symmetry. This may be understood as a small fluctuation. However, this fluctuation can be amplified by the further deposition of new wax and the oval deposit becomes more and more elongated.

In order to describe the co-operatively effects, the competition of groups of bees has to be taken into account. Darchen (1959) reports the competition of differently oriented groups during the construction. The form of a deposit influences this competition, since the number of workers along the longer side of an oval deposit is larger than the number of those along the shorter side. The larger group acts to extend even more the longer side of deposit. As a result of this competition one group wins (usually the larger), and one orientation of the comb is adopted, Darchen finds the confirmation of this in the formation groups of bees plays an important role in comb orientation

4. MATHEMATICAL DESCRIPTION OF THE MODEL

We consider only the beginning of the co-operative construction in the plane parallel to the ceiling .We consider only two extreme cases: the workers parallel either to $X0Z$ or to $Y0Z$ plane ($\underline{Z}$ axis being perpendicular to our chosen $X-Y$ plane, and pointing downward). By $A_x(x,y)$ we denote the average density of the bees parallel to the $X0Z$ plane. Similarly, the average density of workers in the $Y0Z$ plane is given by $A_y(x,y)$ We may now write partial differential equations which describe how the volume of deposited wax C and the density of oriented bees A_x and A_y change in time:

$$\frac{\partial A_x}{\partial t} = \phi - \pi A_x + \beta\left(A_x^2 A_y - A_x A_y^2\right) + \theta \Delta A_x + \gamma A_x \frac{\partial^2 C}{\partial x} \quad (1)$$

$$\frac{\partial A_y}{\partial t} = \phi - \pi A_y + \beta\left(A_x^2 A_y - A_x A_y^2\right) + \theta \Delta A_y + \gamma A_y \frac{\partial^2 c}{\partial y} \quad (2)$$

$$\frac{\partial C}{\partial t} = \alpha\left(A_x + A_y\right) - \upsilon C + \left(A_x + A_y\right) D \Delta C \quad (3)$$

where ∂_t denotes the partial derivative with respect to t, Δ stands for the two dimensional Laplasian, and $\alpha, \beta, \gamma, \dots$ are various phenomenological parameters.

We discuss on the right of equation (1). The first term on the right of equation(1) is the flux of differently oriented bees active in the construction, which come into the considered volume near the top of the beehive the loss of some bees parallel to the plane xoz due to their orientation change or departure is taken into account by second term. The competition of two oppositely oriented groups of workers as described above is expressed by the next term.

If originates from the local nonlinear coupling between A_x and A_y which is modeled by a gain term $A_y F(A_x)$ which corresponds to the opposite situation.

In such an autocatalytic reaction

$$A_y \underset{F(A_y)}{\overset{F(A_x)}{\rightleftharpoons}} A_x \quad (4)$$

It is assumed that the function $F(A)$ chosen at convince, can be expanded in a power series

$$F(A) = \alpha A + \beta A^2 + \dots \quad (5)$$

5. GROWTH OF HIVE AND POPULATION OF BEES

Workers bees enter the population from eggs laid by the queen and existing population of workers raise a proportion from eggs to adulthood. It takes three weeks for worker bee to develop from eggs to adults but their lifespan as adults is strongly influenced by their behavior role in the colony. The average foraging life of a bee has been estimated as less than seven days because the many risks and severe metabolic costs associated with foraging.

Model

A mathematical model allows us to explore the effects of different factor and forces on the population of the hive in a quantiteve way. Let H be the number of bees working in the hive and F the number of bees who work outside the hive referred to here as foragers. We assume that all adult workers bees can be classed either as hive bees or as foragers and that there is no overlap between these two behavioral classes. Hence the total number of adult worker bees in the colony is $N = H + F$ workers are recruited to the forger class from the hive bee class and die at the rate m. Let t be the time measured in days. Then we can represent this process as a differential equation model.

Rate of change of hive bee numbers

$$\frac{dH}{dt} = E(H,F) - HR(H,F)$$

Eclosion recruitment to forager class

Rate of change forager numbers

$$\frac{dF}{dt} = HR(H,F) - mF$$

Recruitment death.

The function $E(H,F)$ describes the way that eclosion depends on the number of hive bees and foragers. The recruitment rate function $R(H,F)$ models the effect of social inhibition on the recruitment rate.we assume that the maximum rate of eclosion is equivalent to the queen's laying rate land that the ecolosion rate approaches this maximum as N the number of workers in the hive increases . In the absence of other information we use the simplest function that increase from zero for no. workers and tends to L as N becomes very large

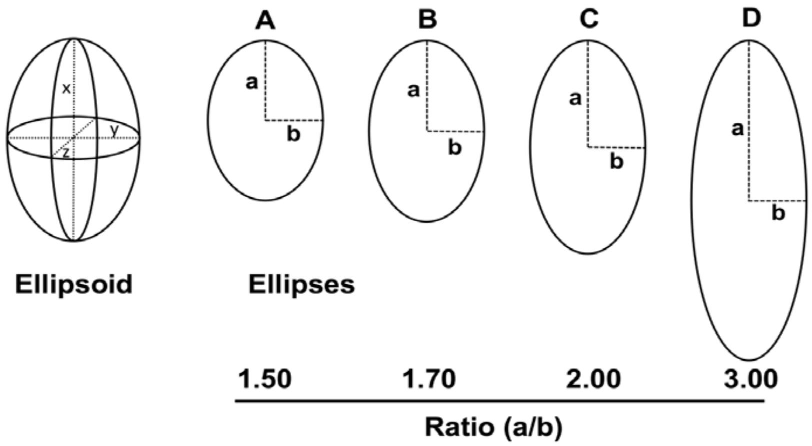
$$E(H,F) = L\left(\frac{N}{W+N}\right) = L\left(\frac{H+F}{W+H+F}\right)$$

Hence W determine the rate at which $E(H,T)$ approaches L as N gets large. [4]

6. GOLDEN RATIO IN THE ELLIPTICAL HIVE

I found that the general two dimensional elliptical form of the newly constructed honeycombs could be drawn into a rectangle of modules having values approaching either 2.00 or 1.62, where the module of the rectangle is the simple division of its long by its short side lengths. it is proposed here that the elliptical form of the early stage honeybee comb is not random, but is following mathematical rules reflecting some geometry intimately related to the golden ratio, also called golden mean or divine proportion. This mathematical presence of the golden ratio might reveal the effect of an inherent law of the Cosmos in the honeybee's world. [5]

In this article, I am presenting circumstantial evidence that the elliptical honeycomb is based on the golden ratio. The golden ratio is an irrational number. It is represented by the Greek letter Φ of ϕ (Phi) and has the value 1.6180339887, approximately. The value of Φ is calculated as $(1+\sqrt{5})$ divided by 2. Throughout history, the golden ratio has been studied not solely by mathematicians and philosophers, but also by biologists, naturalists, artists, architects and musicians, since it was also for them an essential element for the creation and keeping of order, form and beauty. The fascinating presence of the golden ratio in the early honeycomb is an additional stone in the edification of the Cosmos, in which it's ubiquitous presence can only be deciphered but not formally explained.



7. CONCLUSION

Honeybees be make to management according mathematics four sense i.e. in storage of honey in honeycomb which is hexagonal structure, highly co-operative phenomenon of great complexity, maximum rate of eclosion is equivalent to the queen's laying rate ,the fascinating presence of the golden ratio in the early honeycomb.

REFERENCES

1. V Skarka,J.L Deneubouro and M R Belic, *Mathematical Model of Building Behavior Of Apis mellifera*, J theor Biol, 1990.1-16.
2. Tim Raz , *On the Application of the Honeycomb Conjecture to the Bee,s Honeycombe*, 2014.
3. ImranAli and Yu Jing Jim, *Mathematical Model for in-Plane Moduli of Honeycombe Structures*, Research Journal of Applied Science, Engineering and Technology 7(3), 581-592, 2014.
4. David S. Khoury Mary R. Myerscough, Andrew B Barron, *A Quantitive Model of Honey Bee Colony Population Dynamics*, PLOS ONE, April 2011 6(4) e18491.
5. Daniel Favre, *Golden ratio (Sectio Aurea) in the elliptical honeycomb*, journal of nature and Science (JNSCI), 2(1):e173, 2016.

Source of support: Nil, Conflict of interest: None Declared.