International Journal of Mathematical Archive-10(8), 2019, 18-24
Available online through www.ijma.info ISSN 2229 – 5046

# MULTIPLICATIVE KULLI-BASAVA AND MULTIPLICATIVE HYPER KULLI-BASAVA INDICES OF SOME GRAPHS

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*(Received On: 13-07-19; Revised & Accepted On: 13-08-19)*

***ABSTRACT***

*In this paper, we introduce the multiplicative Kulli-Basava indices and multiplicative hyper Kulli-Basava indices of a graph. Also we define the general multiplicative Kulli-Basava indices of a graph. We determine these indices.*

***Keywords:*** *Multiplicative Kulli-Basava indices, multiplicative hyper Kulli-Basava indices.*

***Mathematics Subject Classification:*** *05C05, 05C07, 05C12.*

## 1. INTRODUCTION:

A topological index is a numerical parameter mathematically derived from the graph structure. Numerous topological indices or graph indices have been considered in Mathematical Chemistry.

Let $G$ be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex $v$ is the number of edges incident to $v$. The degree of an edge $e = uv$ in $G$ is defined by $d_G(e) = d_G(u)+ d_G(v) – 2$. The open neighborhood $N_G(v)$ of a vertex $v$ is the set of all vertices adjacent to $v$. The edge neighborhood of a vertex $v$ is the set of all edges incident to $v$ and it is denoted by $N_{e(v)}$. Let $S_{e(v)}$ denote the sum of the degrees of all edges incident to a vertex $v$. For undefined term and notation, we refer [1].

The first and second Kulli-Basava indices were proposed in [2], defined as

$$KB_1(G)=\sum_{uv\in E(G)}\left[S_e(u)+S_e(v)\right] \qquad KB_2(G)=\sum_{uv\in E(G)}S_e(u)S_e(v).$$

In [3], Kulli introduced the first and second hyper Kulli-Basava indices, defined as

$$HKB_1(G)=\sum_{uv\in E(G)}\left[S_e(u)+S_e(v)\right]^2, \qquad HKB_2(G)=\sum_{uv\in E(G)}\left[S_e(u)S_e(v)\right]^2.$$

Recently, some variants of Kulli-Basava indices were introduced and studied such as square Kulli-Basava index [4], connectivity Kulli-Basava indices [5].

We introduce the first and second multiplicative Kulli-Basava indices, defined as

$$KB_1II(G)=\prod_{uv\in E(G)}\left[S_e(u)+S_e(v)\right], \qquad KB_2II(G)=\prod_{uv\in E(G)}S_e(u)S_e(v).$$

Also we propose the first and second multiplicative hyper Kulli-Basava indices, and they are defined as

$$HKB_1II(G)=\prod_{uv\in E(G)}\left[S_e(u)+S_e(v)\right]^2, \qquad HKB_2II(G)=\prod_{uv\in E(G)}\left[S_e(u)S_e(v)\right]^2.$$

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We now introduce the general first and second multiplicative Kulli-Basava indices of a graph G, defined as

$$KB_1^a II(G) = \prod_{uv \in E(G)} \left[S_e(u) + S_e(v)\right]^a, \tag{1}$$

$$KB_2^a II(G) = \prod_{uv \in E(G)} \left[S_e(u) S_e(v)\right]^a, \tag{2}$$

Recently, some new multiplicative indices, were studied see [6, 7, 8, 9, 10].

In this study, the first and second multiplicative Kulli-BAsava Indices, first and second multiplicative hyper. Kulli-Basava indices, general first and second multiplicative Kulli-Basava indices of regular, complete, cycle, wheel, gear and helm graphs are computed.

## 2. RESULTS FOR REGULAR GRAPHS

A graph $G$ is an $r$-regular graph if the degree of each vertex of $G$ is $r$.

**Theorem 1:** Let $G$ be an $r$-regular graph with $n$ vertices and $m$ edges. Then the general first multiplicative Kulli-Basava index of $G$ is

$$KB_1^a II(G) = \left[4r(r-1)\right]^{\frac{anr}{2}}.$$

**Proof:** Let $G$ be an $r$-regular graph with $n$ vertices and $m$ edges. Then $m = \frac{nr}{2}$, $S_e(u) = 2r(r-1)$ for each vertex u of G. Thus

$$KB_1^a II(G) = \prod_{uv \in E(G)} \left[S_e(u) + S_e(v)\right]^a = \left[2r(r-1) + 2r(r-1)\right]^{am}$$

$$= \left[4r(r-1)\right]^{am} = \left[4r(r-1)\right]^{\frac{anr}{2}}.$$

**Corollary 1.1:** If $C_n$ is a cycle with $n$ vertices, then

(i) $KB_1 II(C_n) = 8^n.$ (ii) $HKB_1 II(C_n) = 8^{2n}.$

**Corollary 1.2:** If $K_n$ is a complete graph with $n$ vertices, then

(i) $KB_1 II(K_n) = \left[4(n-1)(n-2)\right]^{\frac{n(n-1)}{2}}.$

(ii) $HKB_1 II(K_n) = \left[4(n-1)(n-2)\right]^{n(n-1)}.$

**Corollary 1.3:** If $G$ is an $r$-regular graph with $n$ vertices, then

(i) $KB_1 II(G) = \left[4r(r-1)\right]^{\frac{nr}{2}}.$ (ii) $HKB_1 II(G) = \left[4r(r-1)\right]^{nr}.$

**Theorem 2:** The general second multiplicative Kulli-Basava index of an $r$-regular graph $G$ is

$$KB_2^a II(G) = \left[2r(r-1)\right]^{anr}.$$

**Proof:** Let $G$ be an $r$-regular graph with $n$ vertices.

Then $|E(G)| = \frac{nr}{2}$ and $S_e(u) = 2r(r-1)$ for any vertex $u$ in $G$. Thus

$$KB_2^a II(G) = \prod_{uv \in E(G)} \left[S_e(u) + S_e(v)\right]^a$$

$$= \left[\left(2r(r-1) \times 2r(r-1)\right)^a\right]^m$$

$$= \left[4r^2(r-1)^2\right]^{am} = \left[2r(r-1)\right]^{anr}.$$

 

**Corollary 2.1:** If $G$ is an $r$-regular graph with $n$ vertices, then

(i) $KB_2II(G)=[2r(r-1)]^{nr}$ (ii) $HKB_2II(G)=[2r(r-1)]^{2nr}$

**Corollary 2.2:** If $C_n$ is a cycle $n$ vertices, then

(i) $KB_2II(C_n)=4^{2n}$. (ii) $HKB_2II(C_n)=4^{4n}$.

**Corollary 2.3:** Let $K_n$ be a complete graph with $n$ vertices. Then

(i) $KB_2II(K_n)=[2(n-1)(n-2)]^{n(n-1)}$

(ii) $HKB_2II(K_n)=[2(n-1)(n-2)]^{2n(n-1)}$.

## 3. RESULTS FOR WHEEL GRAPHS

A wheel $W_n$ is the join of $C_n$ and $K_1$. We see that $W_n$ has $n$+1 vertices and 2$n$ edges. The vertices of $C_n$ are called rim vertices and the vertex $K_1$ is called apex. A graph $W_n$ is shown in Figure 1.

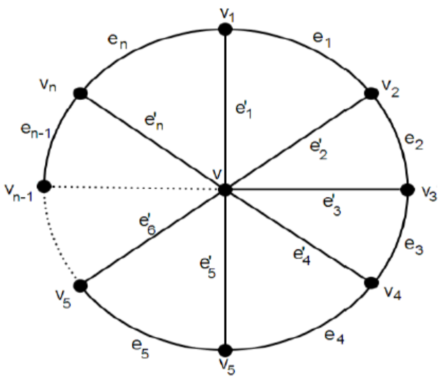


**Figure-1:** Wheel $W_n$

**Lemma 3:** If $W_n$ is a wheel with $n$+1 vertices and 2$n$ edges, then $W_n$ has two types of edges as given below:

$E_1 = \{uv \in E(W_n) \mid S_e(u) = n(n+1), S_e(v) = n+9\}$, $|E_1| = n$.

$E_2 = \{uv \in E(W_n) \mid S_e(u) = S_e(v) = n+9\}$, $|E_2| = n$.

**Theorem 4:** Let $W_n$ be a wheel with $n$+1 vertices and 2$n$ edges. The general first multiplicative Kulli-Basava index of $W_n$ is

$$KB_1^aII(W_n)=(n^2+2n+9)^{an}\times(2n+18)^{an}.$$

**Proof:** By using equation (1) and Lemma 3, we deduce

$$KB_1^aII(W_n)=\prod_{uv\in E(W_n)}\left[S_e(u)+S_e(v)\right]^a$$

$$=\left[(n(n+1)+(n+9))^a\right]^n\times\left[((n+9)+(n+9))^a\right]^n$$

$$=[n^2+2n+9]^{an}\times[2n+18]^{an}.$$

**Corollary 4.1:** The first multiplicative Kulli-Basava index of $W_n$ is

$$KB_1II(W_n)=(n^2+2n+9)^n\times(2n+18)^n.$$

**Proof:** Put $a$ = 1 in equation (3), we get the desired result.

**Corollary 4.2:** The first multiplicative hyper Kulli-Basava index of $W_n$ is

$$HKB_1II(W_n)=(n^2+2n+9)^{2n}\times(2n+18)^{2n}.$$

**Proof:** Put $a$ = 2, in equation (3), we obtain the desired result.

**Theorem 5:** Let $W_n$ be a wheel with $n$+1 vertices and 2$n$ edges. The general second multiplicative Kulli-Basava index of $W_n$ is

$$KB_1^a II(W_n) = [n(n+1)]^{an} \times (n+9)^{3an}. \tag{4}$$

**Proof:** From equation (2) and Lemma 3, we derive

$$KB_2^a II(W_n) = \prod_{uv\in E(W_n)} \left[S_e(u)S_e(v)\right]^a$$

$$= \left[(n(n+1)(n+9))^a\right]^n \times \left[((n+9)(n+9))^a\right]^n$$

$$= [n(n+1)]^{an} \times [n+9]^{3an}.$$

**Corollary 5.1:** The second multiplicative Kulli-Basava index of $W_n$ is

$$KB_1 II(W_n) = [n(n+1)]^n \times (n+9)^{3n}.$$

**Proof:** Put $a$ = 1 is equation (4), we get the desired result.

**Corollary 5.2:** The second multiplicative hyper Kulli-Basava index of $W_n$ is

$$HKB_2 II(W_n) = [n(n+1)]^{2n} \times (n+9)^{6n}.$$

**Proof:** Put $a$ = 2 in equation (4), we obtain the desired result.

## 4. RESULTS FOR GEAR GRAPHS

A graph is a gear graph obtained from $W_n$ by adding a vertex between each pair of adjacent rim vertices and it is denoted by $G_n$. Clearly $G_n$ has 2$n$+1 vertices and 3$n$ edges. A gear graph $G_n$ is presented in Figure 2.

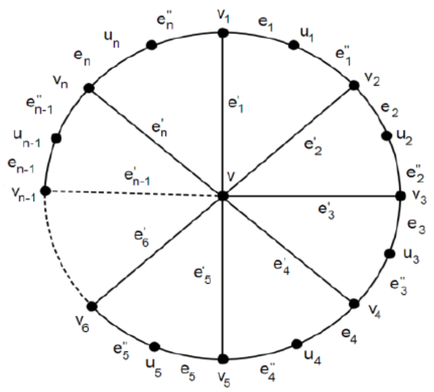

**Figure-2:** Gear graph $G_n$

**Lemma 6:** Let $G_n$ be a gear graph with 2$n$ +1 vertices and 3$n$ edges. Then $G_n$ has two types of edges as follows:

$E_1 = \{uv \in E(G_n) \mid S_e(u) = n(n+1), S_e(v) = n+7\}$, $|E_1| = n$.

$E_2 = \{uv \in E(G_n) \mid S_e(u) = n+7, S_e(v) = 6\}$, $|E_2| = 2n$.

**Theorem 7:** Let $G_n$ be a gear graph with 2$n$+1 vertices and 3$n$ edges. The general first multiplicative Kulli-Basava index of $G_n$ is given by

$$KB_1^a II(G_n) = (n^2+2n+7)^{an} \times (n+13)^{2an}. \tag{5}$$

**Proof:** By using equation (1) and Lemma 6, we obtain

$$KB_2^a II(G_n) = \prod_{uv\in E(G_n)} \left[S_e(u)+S_e(v)\right]^a$$

$$= [n(n+1)+n+7]^n \times (n+7+6)^{a2n}$$

$$= (n^2+2n+7)^{an} \times (n+13)^{2an}.$$

**Corollary 7.1:** The first multiplicative Kulli-Basava index of $G_n$ is

$$KB_1II(G_n) = (n^2 + 2n + 7)^n \times (n+13)^{2n}.$$

**Proof:** Put $a = 1$ is equation (5), we get the desired result.

**Corollary 7.2:** The first multiplicative hyper Kulli-Basava index of $G_n$ is

$$HKB_1II(G_n) = (n^2 + 2n + 7)^{2n} \times (n+13)^{4n}.$$

**Proof:** Put $a = 2$ in equation (5), we obtain the desired result.

**Theorem 8:** Let $G_n$ be a gear graph with $2n$+1 vertices and $3n$ edges. The general second multiplicative Kulli-Basava index of $G_n$ is given by

$$KB_2^aII(G_n) = 6^{2an}\left[n(n+1)\right]^{an} \times (n+7)^{3an}. \tag{6}$$

**Proof:** From equation (2) and by using Lemma 6, we have

$$KB_2^aII(G_n) = \prod_{uv \in E(G_n)} \left[S_e(u)S_e(v)\right]^a$$

$$= \left[n(n+1) + n + 7\right]^{an} \times \left[6(n+7)\right]^{2an}$$

$$= 6^{2an}\left[n(n+1)\right]^{an} \times (n+7)^{3an}.$$

**Corollary 8.1:** The second multiplicative Kulli-Basava index of $G_n$ is

$$KB_2II(G_n) = 6^{2n}\left[n(n+1)\right]^{n} \times (n+7)^{3n}.$$

**Proof:** Put $a = 1$ in equation (6), we get the desired result.

**Corollary 8.2:** The second multiplicative hyper Kulli-Basava index of $G_n$ is

$$HKB_2II(G_n) = 6^{4n}\left[n(n+1)\right]^{2n} \times (n+7)^{6n}.$$

**Proof:** Put $a = 2$ in equation (6), we obtain the required result.

## 5. RESULTS FOR HELM GRAPHS

A helm graph is a graph obtained from $W_n$ by attaching an end edge to each rim vertex and it is denoted by $H_n$. Clearly $H_n$ has $2n$+1 vertices and $3n$ edges. A graph $H_n$ is shown in Figure 3.

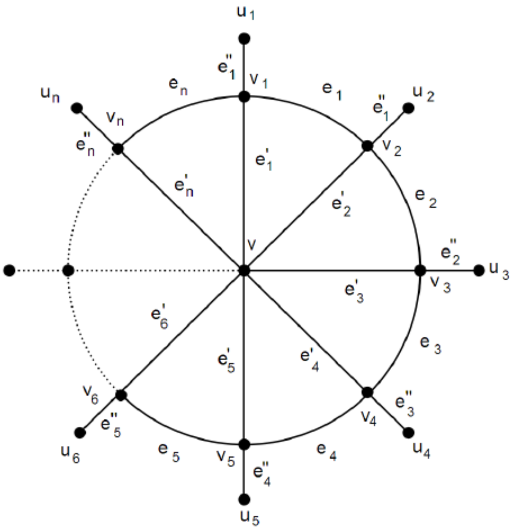


**Figure-3:** Helm graph $H_n$

**Lemma 9:** Let $H_n$ be a helm graph with $2n$+1 vertices and $3n$ edges. Then $H_n$ has three types of edges as follows:

$E_1 = \{uv \in E(H_n) \mid S_e(u) = n(n+2), S_e(v) = n+17\}$, $|E_1| = n$.

$E_2 = \{uv \in E(H_n) \mid S_e(u) = S_e(v) = n+17\}$, $|E_2| = n$.

$E_3 = \{uv \in E(H_n) \mid S_e(u) = n+17, S_e(v) = 3\}$, $|E_3| = n$.

**Theorem 10:** Let $H_n$ be a helm graph with $2n+1$ vertices and $3n$ edges. The general first multiplicative Kulli-Basava index of $H_n$ is

$$KB_2^a II(H_n) = (n^2+3n+17)^{an}(2n+34)^{an}(n+20)^{an}. \tag{7}$$

**Proof:** By using equation (1) and Lemma 9, we deduce

$$KB_1^a II(H_n) = \prod_{uv\in E(H_n)}\left[S_e(u)+S_e(v)\right]^a$$
$$= [n(n+2)+n+17]^{an} \times [n+17+n+17]^{an} \times [n+17+3]^{an}$$
$$= (n^2+3n+17)^{an} \times (2n+34)^{n} \times (n+20)^{an}.$$

**Corollary 10.1:** The first multiplicative Kulli-Basava index of $H_n$ is

$$KB_1 II(H_n) = (n^2+3n+17)^{n} \times (2n+34)^{n} \times (n+20)^{n}.$$

**Proof:** Put $a$ = 1 in equation (7), we obtain the desired result.

**Corollary 10.2:** The first multiplicative hyper Kulli-Basava index of $H_n$ is

$$HKB_1 II(H_n) = (n^2+3n+7)^{2n} \times (2n+34)^{2n} \times (n+20)^{2n}.$$

**Proof:** Put $a$ = 2 in equation (7) we get the desired result.

**Theorem 11:** Let $H_n$ be a helm graph with $2n+1$ vertices and $3n$ edges. The general second multiplicative Kulli-Basava index of $H_n$ is

$$KB_2^a II(H_n) = [3n(n+2)]^{an}(n+17)^{4an}. \tag{8}$$

**Proof:** From equation (2) and by using Lemma 9, we derive

$$KB_2^a II(H_n) = \prod_{uv\in E(H_n)}\left[S_e(u)S_e(v)\right]^a$$
$$= [n(n+2)(n+17)]^{an} \times [(n+17)(n+17)]^{an} \times [3(n+17)]^{an}$$
$$= [3n(n+2)]^{an} \times (n+17)^{4an}.$$

**Corollary 11.1:** The second multiplicative Kulli-Basava index of $H_n$ is

$$KB_2 II(H_n) = [3n(n+2)]^{n}(n+17)^{4n}.$$

**Proof:** Put $a$ = 1 in equation (8), we get the desired result.

**Corollary 11.2:** The second multiplicative hyper Kulli-Basava index of $H_n$ is

$$KB_2 II(H_n) = [3n(n+2)]^{2n}(n+17)^{8n}.$$

**Proof:** Put $a$ = 2 in equation (8), we obtain the required result.

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**Source of support: Nil, Conflict of interest: None Declared.**