CHARACTERS OF NAGENDRAM Γ-SEMI SUB NEAR-FIELD SPACE
OF A Γ-NEAR-FIELD SPACE OVER NEAR-FIELD

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ABSTRACT

In this manuscript we obtain the notion of characters of Nagendram Γ-semi sub near-field space of a Γ-near-field space over near-field almost with few of their characterizations. We also present the interesting relations on orthogonality characters of Nagendram Γ-semi sub near-field space of a Γ-near-field space over near-field.

Keywords: characters of Nagendram Γ-semi sub near-field space, Γ-near-field space; Γ-Semi sub near-field space of Γ-near-field space, Nagendram Γ-semi sub near-field space, Nagendram Γ-semi near-field space, closed, compact, connected Nagendram Γ-semi sub near-field spaces of a Γ-near-field space over near-field, orthogonality characters of Nagendram Γ-semi sub near-field space.


SECTION 1: INTRODUCTION AND PRELIMINARIES

In this paper author introduced characters of Nagendram Γ-semi sub near-field space of a Γ-near-field space over near-field and discussed about orthogonality characters of Nagendram Γ-semi sub near-field space of a Γ-near-field space over near-field.

Definition 1.1: Characters of Nagendram Γ-semi sub near-field space of a Γ-near-field space over near-field. Let N be a Nagendram Γ-semi sub near-field space K of a Γ-near-field space over near-field and ρ : N → NL(V) a complex representation. The character of the representation is defined as the function χρ = χV : N → C and χV (g) = tv (ρ(g)).

Note 1.2: If S, T are complex matrices such that tr(S T) = tr (TS) then tr (STS⁻¹) = tr (S). So tr is independent of the chosen basis. Also, if M : V → V is linear, {v₁, v₂, ...., vₙ} is a basis of V, v₁*, v₂*,....., vₙ* the corresponding dual basis of V*. Then tr (M) = ∑ᵢ vᵢ* (M(vᵢ)).

If V is a representation of N, then V* = Hom (V, C) is the dual representation of N. If N is compact, we may choose a N-invariant. Hermitian inner product ⟨,⟩ on V. This gives a N-equivalent complex anti-linear map V → V* v → ⟨v,·⟩. This gives an isomorphism V* ≅ V where V is the complex Nagendram Γ-semi sub near-field space of a Γ-near-field space over near-field with the same addition as V and scalar multiplication is given by λ . V = λV and λ ∈ C, v ∈ V.

SECTION 2: CHARACTERS OF NAGENDRAM GAMMA SEMI SUB NEAR-FIELD SPACES OF A GAMMA NEAR-FIELD SPACE OVER A NEAR-FIELD.

In this section, author present propositions on characters of Nagendram Gamma semi sub near-field spaces of a Gamma near-field space over near-field.
Proposition 2.1: Let N be a Nagendram $\Gamma$-semi sub near-field space of a $\Gamma$-near-field space over near-field. Then

(a) a character of a representation of N is a $C^\infty$ function on N,
(b) if $V$ and $W$ are isomorphic representations of N, then $\chi_V = \chi_W$
(c) $\chi_V (ghg^{-1}) = \chi_V (h)$, for all $g, h \in N$,
(d) $\chi_V \otimes W = \chi_V \chi_W$,
(e) $\chi_V \odot W = \chi_V \chi_W$,
(f) $\chi_V \cdot (\rho(g)) = \chi_V (\rho(g)) = \chi_V (\rho(g^{-1}))$,
(g) $\chi_V (1) = \dim_{C}(V)$.

Proof: Given N be a Nagendram $\Gamma$-semi sub near-field space of a $\Gamma$-near-field space over near-field.

(a): By definition of character of Nagendram $\Gamma$-semi sub near-field space of a $\Gamma$-near-field space over near-field and $\rho : N \rightarrow NL(V)$ a complex representation. The character of the representation is defined as the function $\chi_{\rho} = \chi_{\rho} : N \rightarrow C$ and $\chi_V (g) = \text{tr} (\rho(g))$ is a complex representation on from $C^\infty$ on N. hence it is obvious that a character of a representation of N is a $C^\infty$ function on N. Hence Proved (a).

(b): If $\rho_1, \rho_2 : N \rightarrow NL(n, C)$ are two representations and

\[
\begin{array}{ccc}
\rho_1(g) & M & \rho_2(g) \\
\downarrow & M & \downarrow \\
C^n & \rightarrow & C^n \\
\end{array}
\]

Commutes then $\text{tr} (\rho_2(g)) = \text{tr} (\rho_1(g))$. Proved (b).

(c) $\text{Tr} (\rho(ghg^{-1})) = \text{tr} (\rho_1(g) (\rho(h)(\rho(g^{-1}))) = \text{tr} (\rho(g))$.

(d) and (e) recall from linear algebra that if $S : V \rightarrow W$ and $T : V \rightarrow V$ are linear, then $\text{tr} (\rho \otimes T) = \text{tr} (S) + \text{tr} (T)$ and $\text{tr} (S \otimes T) = \text{tr} (S) \cdot \text{tr} (T)$.

(f) If $\rho : N \rightarrow NL(V)$ is a representation, $(v_1, v_2, \ldots, v_n)$ is a basis for $V$ and $v_1^*, v_2^*, \ldots, v_n^*$ is the associated dual basis, then

$\chi_{\rho} (g) = \text{tr} (\rho^* (g) v_i^*) = \sum_i v_i^* (\rho(g^{-1}) v_i^* ) = \chi_{\rho} (g^{-1})$.

If $(, )$ is an invariant Hermitian inner product, and \{ $v_i$ \} is an orthogonal basis, then

$\text{tr} \rho^*(g) = \sum_i (v_i, v_i^*) \rho(g^{-1}) v_i$

(g) $\chi_V (1) = \text{tr} (\text{id}) = \dim_C V$.

This completes the proof of the proposition.

Proposition 2.2: Let $\rho : N \rightarrow NL(V)$ be a representation of N and $V^g = \{ g \in V : g.v = v \}$.

Then $\int_N \chi_V (g) dg = \dim_{C} V$.

Proof:

Consider $Q : V \rightarrow V$ given by $Q(V) = \int_N \rho(g) v dg$. We claim that Q is a linear N-equivalent map such that $Q(V) \subseteq V^g$ and $Q|V^g = \text{id}_{V^g}$.
It is clear that $Q$ is linear. Now,
\[
Q(\rho(a) v) = \int_N \rho(g) \rho(a) v \, dg = \int_N \rho(\rho(a) v) \, dg = \int_N \rho(g) v \, dg = Q(v) = \int_N \rho(g) v \, dg
\]

and so $Q(v) \subseteq V^N$ and $Q(\rho(a) . v) = Q(v)$ for all $g \in N$ and $v \in V$. Also if $v \in V^N$ we have
\[
Q(v) = \int_N \rho(g) c \, dg = \int_N v \, dg = v. \quad \text{since} \quad \int_N dg = 1.
\]

This claim implies that $\text{tr}(Q) = \dim V^N$. On the other hand,
\[
\text{Tr}(Q) = \sum_{ii} (\chi_L, \chi_M) = \int_N \chi_L(g) \chi_M(g) \, dg = \dim C(L, M).
\]

In particular, if $L, M$ are irreducible, then $(\chi_L, \chi_M) = \begin{cases} 0 & L \cong M \\ 1 & L \text{ not } \cong M \end{cases}$

**Proof:**

\[
\int_N (\chi_L, \chi_M) (g) \, dg = \int_N \chi_{\text{Hom}}(L, M) (g) \, dg = \text{Hom}(L, M)^N = \text{Hom}_N(L, M).
\]

This completes the proof of the theorem.

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