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A SPECIAL MODEL GIVEN BY ORDERED PAIRS OF REAL NUMBERS AND AN APPLICATION FOR DESCRIPTION OF PARALLEL UNIVERSES

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ABSTRACT

In [1] we decribe the enlargement of the our universe by compressed numbers. Present paper describes the parallel universes by exploded numbers. Introduction shows the most important concepts of the theory of exploded and compressed numbers. Part 1 summarizes the algebra of exploded numbers detailed in [2]. Part 2 shows some applications concerning the box – model of multiverse which is built by parallel universes.

INTRODUCTION

Let x be an arbitrary real number. The ordered pair

(0.1) $$\left((sgn\ x)\cdot\tanh^{-1}\{|x|\}\ ,(sgn\ x)\cdot[|x|]\right)$$

is called exploded x (or exploded of x) and denoted by $\breve{x}$. (Here, $sgn\ x=\begin{cases}1,\ if\ x>0\\ 0,\ if\ x=0\\ -1,\ if\ x<0\end{cases}$,

$\tanh^{-1}x=\frac{1}{2}ln\frac{1+x}{1-x}\ ,-1<x<1\ ;|x|=\begin{cases}x,\ if\ x>0\\ 0,\ if\ x=0\\ -x,\ if\ x<0\end{cases}$; $[x]$ is the greatest integer number, which is less than or equal to x and $\{x\}=x-[x]$.) Moreover, we mention $\breve{x}$ as an exploded number. The identity

(0.2) $$(sgn\ x)\cdot(tanh^{-1}\{|x|\})=tanh^{-1}(x-(sgn\ x)[|x|]),\ \ x\in\mathbb{R}.$$

yields that for any pair $x\ ,y\ \in\mathbb{R}$, $\breve{x}=\breve{y}$ if and only if $x=y$. We give an ordering for exploded numbers by

Definition 0.3: (Ordering of exploded numbers.)
For any pair $x\ ,y\ \in\mathbb{R}$ we say, that $\breve{x}<\breve{y}$ if

$$(sgnx)\cdot[|x|]<(sgnx)\cdot[|y|]$$
or
$$\text{if } (sgnx)\cdot[|x|]=(sgnx)\cdot[|y|] \text{ then } (sgn\ x)\cdot\tanh^{-1}\{|x|\}<(sgn\ x)\cdot\tanh^{-1}\{|y|\}$$

Theorem 0.4: (Theorem of ordering. See [2], Theorem 3.2.10.) For any pair $x\ ,y\ \in\mathbb{R}$, $\breve{x}<\breve{y}$ if and only if $x<y$.

Consequently, the ***transitivity and trichotomy properties remain valid for extended ordering, too***.

We say that the points $u=(\xi,\eta)\in\mathbb{R}^2$ form a „flag” (in the rectangular Descartes coordinate – system of $\mathbb{R}^2$) if

(0.5) $$\eta\in\mathbb{Z}$$

and

(0.6) $$\xi\cdot\eta\geq 0.$$

By (0.1) it is clear that the exploded numbers are situated on the „flag”. On the other hand, we have the following theorem.

Theorem 0.7: (Theorem of completeness. See [2], Theorem 3.2.8.) If $u=(\xi,\eta)\in\mathbb{R}^2$ is situated on the „flag”, then

$$\left(\widetilde{\eta+\tanh\xi}\right)=u\qquad ,\qquad \left(\tanh\xi=\frac{e^{\xi}-e^{-\xi}}{e^{\xi}+e^{-\xi}}\ ,-\infty<\xi<\infty\right)$$

By Theorem 0.7 we can say that the „flag” is a geometrical representation of the set of exploded numbers

$$\breve{\mathbb{R}}=\{u=(\xi,\eta)\in\mathbb{R}^2|(\xi,\eta)=\breve{x},\qquad x\in\mathbb{R}.\}$$

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Moreover, we may give the concept of compressed of $u \in \breve{\mathbb{R}}$, as follows

(0.8) $$\underline{u} = \eta + \tanh \xi \quad , \quad u = (\xi, \eta) \in \breve{\mathbb{R}}.$$

Hence, we have that $\underline{u} \in \mathbb{R}$.

Definition 0.9: If u is a real number then it is identified with the pair $(u, 0)$, that is $u = (u, 0)$.

Theorem 0.10: (Theorem of expansion.) The set of real number $\mathbb{R}$ is a real subset of the set of exploded numbers $\breve{\mathbb{R}}$. Moreover, the inversion formulas

(0.11) $\underline{(\breve{x})} = x \quad , \quad x \in \mathbb{R}$ (see [1], (0.3))

and

(0.12) $\breve{(\underline{u})} = u \quad , \quad u \in \breve{\mathbb{R}}$ (see [1], (0.4)),

are valid.

Proof: Let x be an arbitrary real number. Considering the real number $\tanh x$ and having that $0 \le |\tanh x| < 1$. By (0.1) we can write

$$\widetilde{(\tanh x)} = \left((sgn\ (\tanh x)) \cdot \tanh^{-1}\{|\tanh x|\}\ , 0 \right) =$$
$$= \left((sgn\ (\tanh x)) \cdot \tanh^{-1}|\tanh x|\ , 0 \right) =$$
$$= \left(\tanh^{-1} \left((sgn\ (\tanh x)) \cdot |\tanh x| \right) , 0 \right) =$$
$$= (\tanh^{-1}(\tanh x)\ , 0) = (x\ , 0).$$

Hence, the Definition 0.9 gives $\widetilde{(\tanh x)} = x$. So, $\mathbb{R} \subseteq \breve{\mathbb{R}}$. As $\breve{1} = (0\ , 1) \notin \mathbb{R}$ but $\breve{1} \in \breve{\mathbb{R}}$ we have that $\mathbb{R}$ is a real subset of $\breve{\mathbb{R}}$. Using (0.8), by (0.1) we can write

$$\underline{(\breve{x})} = (sgn\ x) \cdot [|x|] + \tanh\left((sgn\ x) \cdot \tanh^{-1}\{|x|\}\right) =$$
$$= (sgn\ x) \cdot [|x|] + \tanh\left(\tanh^{-1}\left((sgn\ x) \cdot \{|x|\}\right)\right) =$$
$$= (sgn\ x\ ) \cdot ([|x|] + \{|x|\}) = (sgn\ x) \cdot |x| = x.$$

Denoting $\breve{x} = u$, we have that $\underline{(\breve{x})} = \underline{u}$. Hence, by (0.11) $x = \underline{u}$ is obtained. Using again, that $\breve{x} = u$

we have (0.12).

With respect to the Definition 0.9 for the arbitrary real number x the compression formula (0.8) says that $\underline{x} = \tanh x$. For this reason, the set

$$\underline{\mathbb{R}} = \{\xi \in \mathbb{R} | \xi = \underline{x}, \quad x \in \mathbb{R}.\}$$

may considered as compressed number line. Clearly, $(-1) \notin \underline{\mathbb{R}}$ and $1 \notin \underline{\mathbb{R}}$, so $\underline{\mathbb{R}} =]-1,1[$. If x is running the open interval $]-1,1[$ then $\breve{x}$ is running the real number line. If $x \in \mathbb{R} \setminus \underline{\mathbb{R}}$ then $\breve{x}$ is outside the number line. In this case the number line is considered as an one dimensional space and $\breve{x}$ is called ***invisible exploded number***. If $x \in \underline{\mathbb{R}}$ we can say that the real number $\breve{x}$ is a ***visible exploded number.*** The greatest invisible exploded number which is smaller than all of real numbers is $\widetilde{(-1)}$ is called ***negative discriminator.*** The smallest invisible exploded number which is greater than all of real numbers is $\breve{1}$ called ***positive discriminator***.

Theorem 0.13: (The monotonity property of explosion.) Let $x \neq 0$ an arbitrary real number.

a/ If

(0.14) $0 < x$

then

(0.15) $x < \breve{x}.$

b/ If

(0.16) $x < 0$

then

(0.17) $\breve{x} < x.$

Proof.

Ad a/

Having that $x = (x, 0)$, by (0.14) and (0.1) we have $\breve{x} = (\tanh^{-1}\{x\}\ , [x])$. Starting from the condition (0.14) the Definition 0.3 yields that $0 < \breve{x}$. If $\breve{x} < \breve{1}$, then $\breve{x} = \tanh^{-1} x$. As the function $\tanh^{-1}$ is strictly convex on the interval $[0,1[$, the inequality (0.15) is obtained. If $\breve{1} \le \breve{x}$, then the definition of positive discriminator gives (0.15).

Ad b/

Having that $x = (x, 0)$, by (0.16) and (0.1) we have

$$\breve{x} = (-\tanh^{-1}\{(-x)\}\ , -[(-x)]) = (\tanh^{-1}(\{x\} - 1)\ , [x] + 1).$$

Starting from the condition (0.16) the Definition 0.3 yields that $\breve{x} < 0$. If $\widetilde{(-1)} < \breve{x}$ then $-1 < x$, so, $[x] = -1$ and $\{x\} = x + 1$ so, $\breve{x} = \tanh^{-1} x$. As the function $\tanh^{-1}$ is strictly concave on the interval $]-1,0]$, the inequality (0.17) is obtained. If $\breve{x} \le \widetilde{(-1)}$, then the definition of negative discriminator gives (0.17).

Theorem 0.18: (The monotonity property of compression.) Let $u \neq 0$ an arbitrary exploded number.

a/ If

(0.19) $$0 < u$$

then

(0.20) $$0 < \underline{u} < u$$

b/ If

(0.21) $$u < 0$$

then

(0.22) $$u < \underline{u} < 0.$$

Proof:

Ad a/

By (0.12) and (0.19) we have that $\breve{(\underline{0})} < \breve{(\underline{u})}$. Applying Theorem 0.4, we get the inequaity $\underline{0} < \underline{u}$. As $\underline{0} = 0$, the left hand side of (0.20) is obtained. Having this and applying the part a/ of Theorem 0.13, we get that $\underline{u} < \breve{(\underline{u})}$. Using the inversion formula (0.12) again, we have the right hand side of (0.20).

Ad b/

By (0.12) and (0.21) we have that $\breve{(\underline{u})} < \breve{(\underline{0})}$. Applying Theorem 0.4 we get the inequaity $\underline{u} < \underline{0}$. As $\underline{0} = 0$, the right hand side of (0.22) is obtained. Having this and applying the part b/ of Theorem 0.13, we get that $\breve{(\underline{u})} < \underline{u}$. Using the inversion formula (0.12) again, we have the left hand side of (0.22).

1. SUPER - OPERATIONS IN THE SET OF EXPLODED NUMBERS

Definition 1.1: (The concepts of super – addition and super - multiplication) Let *x* and *y* be arbitrary real numbers. The ***super – sum*** and ***super – product*** of $\breve{x}$ and $\breve{y}$ are

(1.2) $$\breve{x} \overline{\oplus} \breve{y} = \widetilde{(x + y)} \quad , \quad x, y \in \mathbb{R}.$$

and

(1.3) $$\breve{x} \overline{\odot} \breve{y} = \widetilde{(x \cdot y)} \quad , \quad x, y \in \mathbb{R},$$

respectively.

Remark: Some equivalent formulations are

(1.4) $$\breve{x} \overline{\oplus} \breve{y} = \left(\left(sgn\ (x + y) \right) \cdot \tanh^{-1}\{|x + y|\}, \left(sgn\ (x + y) \right) \cdot [|x + y|] \right) \quad , x, y \in \mathbb{R}.$$

(1.5) $$\breve{x} \overline{\odot} \breve{y} = \left(\left(sgn\ (x \cdot y) \right) \cdot \tanh^{-1}\{|x \cdot y|\}, \left(sgn\ (x \cdot y) \right) \cdot [|x \cdot y|] \right) \quad , x, y \in \mathbb{R}.$$

(1.6) $$u \overline{\oplus} v = \widetilde{\left(\underline{u} + \underline{v} \right)} \quad , u, v \in \breve{\mathbb{R}}.$$

(1.7) $$u \overline{\odot} v = \widetilde{\left(\underline{u} \cdot \underline{v} \right)} \quad , u, v \in \breve{\mathbb{R}}.$$

By (1.2) and (1.3) the mutually and unamgiguous map

(1.8) $$x \leftrightarrow \breve{x}$$

is an isomorphism between the fields $(\mathbb{R}, +, \cdot)$ and $\left(\breve{\mathbb{R}}, \overline{\oplus}, \overline{\odot} \right)$.

Theorem 1.9: (The monotonity of super – addition.) Let $\breve{x}$, $\breve{y}$ and $\breve{z}$ be arbitrary exploded numbers. If

(1.10) $$\breve{x} < \breve{y}$$

then

(1.11) $$\breve{x} \overline{\oplus} \breve{z} < \breve{y} \overline{\oplus} \breve{z}$$

holds.

Proof: By (1.10), Theorem 0.4 yields that $x < y$. Hence, $x + z < y + z$. Considering (1.2), by Theorem 0.4, we have (1.11)

Theorem 1.12: (The monotonity of super – multiplication.) Let $\breve{x}$, $\breve{y}$ be arbitrary and $\breve{z}$ be an arbitrary positive exploded numbers. If (1.10) is valid then

(1.13) $$\breve{x} \overline{\odot} \breve{z} < \breve{y} \overline{\odot} \breve{z}$$

holds.

Proof: By (1.10) and $\breve{z} > 0$, Theorem 0.4 yields that $x < y$ and $z > 0$. Hence, $x \cdot z < y \cdot z$.

Considering (1.3), by Theorem 0.4, we have (1.13)

Theorem 1.14: If $\breve{x}$, $\breve{y}$ are arbitrary and $\breve{z}$ is an arbitrary negative exploded numbers, then

(1.15) $$\breve{x}\overline{\odot}\breve{z} > \breve{y}\overline{\odot}\breve{z}$$

holds.

Proof: By (1.10) and $\breve{z} < 0$, Theorem 0.4 yields that $x < y \; and \; z < 0$. Hence, $x \cdot z > y \cdot z$.

Considering (1.3), by Theorem 0.4, we have (1.15)

By (1.2) , (1.3) and Theorems 1.9, 1.12 and 1.14 we see that

$(\mathbb{R}\,,+\,,\,\cdot\,,<)$ **and** $\left(\breve{\mathbb{R}}\;,\;\overline{\oplus}\,,\overline{\odot}\,,<\right)$

are isomorphic ordered fields.

Further important properties of the ordered field $\left(\breve{\mathbb{R}}\,,\;\overline{\oplus}\,,\overline{\odot}\,,<\right)$:

1.16 . (Associative laws.) $\left(\breve{x}\overline{\oplus}\breve{y}\right)\overline{\oplus}\breve{z} = \breve{x}\overline{\oplus}\left(\breve{y}\overline{\oplus}\breve{z}\right)$ and $\left(\breve{x}\overline{\odot}\breve{y}\right)\overline{\odot}\breve{z} = \breve{x}\overline{\odot}\left(\breve{y}\overline{\odot}\breve{z}\right)$.

1.17 . (Commutative laws.) $\breve{x}\overline{\oplus}\breve{y} = \breve{y}\overline{\oplus}\breve{x}$ and $\breve{x}\overline{\odot}\breve{y} = \breve{y}\overline{\odot}\breve{x}$.

1.18 . (Distributive law.) $\left(\breve{x}\overline{\oplus}\breve{y}\right)\overline{\odot}\breve{z} = \left(\breve{x}\overline{\odot}\breve{z}\right)\overline{\oplus}\left(\breve{y}\overline{\odot}\breve{z}\right)$.

1.19 . (Additive unit element.) $\breve{x}\overline{\oplus}0 = \breve{x}$, uniqueness of additive unit element can be proved.

1.20 . (Additive inverse element.) $\breve{x}\overline{\oplus}\widetilde{(-x)} = 0$, uniqueness of additive inverse element can be proved.

1.21 . (Multiplicative unit element.) $\breve{x}\overline{\odot}\breve{1} = \breve{x}$, uniqueness of multiplicative unit element can be proved.

1.22 . (Multiplicative iverse element.) If $x \neq 0 \; then \; \breve{x}\overline{\odot}\widetilde{\left(\frac{1}{x}\right)} = \breve{1}$, uniqueness of multiplicative inverse element can be proved.

The extension of the sign „minus". By the uniqueness of additive inverse element we use

(1.23) $$-\breve{x} =^{def} \widetilde{(-x)} \qquad , x \in \mathbb{R}.$$

Clearly, if $x \in \underline{\mathbb{R}}$ then $\breve{x} \in \mathbb{R} \;\; and \quad \widetilde{(-x)} = \tanh^{-1}(-x) = -\tanh^{-1} x = -\breve{x}$.

Other equivalent formulations are

(1.24) $$-\breve{x} = \left(-(sgn\, x)\cdot \tanh^{-1}\{|x|\}\,, -(sgn\, x)\cdot[|x|]\right) \quad , x \in \mathbb{R}.$$

(1.25) $$-\breve{x} = \widetilde{(-1)}\overline{\odot}\breve{x} = \left(-\breve{1}\right)\overline{\odot}\breve{x} \;\; , x \in \mathbb{R}.$$

(1.26) $$-u = \widetilde{\left(-\underline{u}\right)} \qquad , u \in \breve{\mathbb{R}}.$$

Hence, by the inversion formula (0.11) yields

(1.27) $$\underline{(-u)} = -\underline{u} \qquad , u \in \breve{\mathbb{R}}.$$

(1.28) $$-u = \widetilde{(-1)}\overline{\odot}u = \left(-\breve{1}\right)\overline{\odot}u \;\; , u \in \breve{\mathbb{R}}.$$

Hence, by the associative law and (1.3) with the multiplicative unit element property, we have

(1.29) $$-(-u) = \widetilde{(-1)}\overline{\odot}\left(\widetilde{(-1)}\overline{\odot}u\right) = \left(\widetilde{(-1)}\overline{\odot}\widetilde{(-1)}\right)\overline{\odot}u = u \quad , \; u \in \breve{\mathbb{R}}.$$

The extension of „absolute value".

(1.30) $$|u| =^{def} \widetilde{\left(|\underline{u}|\right)} \qquad , u \in \breve{\mathbb{R}}.$$

Clearly, if $u \in \mathbb{R}$ then $|\underline{u}| = |\tanh u| = \tanh|u|$ and $\widetilde{\left(|\underline{u}|\right)} = \tanh^{-1}(\tanh|u|) = |u|$.

Equivalent formulation is

(1.31) $$|u| = \begin{cases} u \;\; if \; u > 0 \\ 0 \;\; if \;\; u = 0 \\ -u \;\; if \;\; u < 0 \end{cases} , \qquad , u \in \breve{\mathbb{R}}. \quad \text{(See (1.26).)}$$

Hence, by (1.29) and (1.31) the equality

(1.32) $$|-u| = |u| \qquad , \; u \in \breve{\mathbb{R}},$$

is obtained.

Theorem 1.33: If u and v are arbitrary exploded numbers, then we have

(1.34) $$\left|u\overline{\odot}v\right| = |u|\overline{\odot}|v|.$$

Proof: By (0.11), (1.7) and (1.30) we can write

$$|u\overline{\odot}v| = \left(\widetilde{|\underline{u\overline{\odot}v}|}\right) = \left(\widetilde{|\underline{u}\cdot\underline{v}|}\right) = \left(\widetilde{|\underline{u}|\cdot|\underline{v}|}\right) = \left(\widetilde{|\underline{u}|}\right)\overline{\odot}\left(\widetilde{|\underline{v}|}\right) = |u|\overline{\odot}|v|.$$

Theorem 1.35: (Triangle inequality for exploded numbers.) If u and v are arbitrary exploded numbers, then we have

(1.36) $$|u\overline{\oplus}v| \leq |u|\overline{\oplus}|v|.$$

Proof: By Theorem 0.4, (0.11), (1.6) and (1.30) we can write

$$|u\overline{\oplus}v| = \left(\widetilde{|\underline{u\overline{\oplus}v}|}\right) = \left(\widetilde{|\underline{u}+\underline{v}|}\right) \leq \left(\widetilde{|\underline{u}|+|\underline{v}|}\right) = \left(\widetilde{|\underline{u}|}\right)\overline{\oplus}\left(\widetilde{|\underline{v}|}\right) = |u|\overline{\oplus}|v|.$$

Definition 1.37: Let $\mathbb{S}$ be a set of exploded numbers.

- We say that the set $\mathbb{S}$ is bounded from above if there exists $u_{upper} \in \breve{\mathbb{R}}$ such that for any $u \in \mathbb{S}$ the inequality $u \leq u_{upper}$. holds. The u_{upper} is called an upper bound of $\mathbb{S}$.
- We say that the set $\mathbb{S}$ is bounded from below if there exists $u_{lower} \in \breve{\mathbb{R}}$ such that for any $u \in \mathbb{S}$ the inequality $u \geq u_{lower}$ holds. The u_{lower} is called a lower bound of $\mathbb{S}$.
- We say that the set $\mathbb{S}$ is bounded if for any $u \in \mathbb{S}$, $u_{lower} \leq u \leq u_{upper}$.

Definition 1.38: Let $\mathbb{S}$ be a set of exploded numbers.

- We say that, that the exploded number $sup_{u\in\mathbb{S}}\, u$ is the upper limit of the set $\mathbb{S}$, if it is an upper bound of $\mathbb{S}$ but for any $v(\in \breve{\mathbb{R}})$ which is less than $sup_{u\in\mathbb{S}}\, u$ and there exists $u_0 \in \mathbb{S}$, such that $v < u_0$. (The $sup_{u\in\mathbb{S}}\, u$ is mentioned as the least upper bound, too.)
- We say that, that the exploded number $inf_{u\in\mathbb{S}}\, u$ is the lower limit of the set $\mathbb{S}$, if it is a lower bound of $\mathbb{S}$ but for any $v(\in \breve{\mathbb{R}})$ which is greater than $inf_{u\in\mathbb{S}}\, u$ and there exists $u_0 \in \mathbb{S}$, such that $u_0 < v$. (The $inf_{u\in\mathbb{S}}\, u$ is mentioned as the greaeast lower bound, too.)

Definition 1.39: Let $\mathbb{S}$ be a set of exploded numbers. The set

$$\underline{\mathbb{S}} = \{x = \underline{u} | u \in \mathbb{S}\}$$

is called the compressed of set $\mathbb{S}$. Clearly, $\underline{\mathbb{S}} \subseteq \mathbb{R}$. Moreover, $\underline{(\breve{\mathbb{R}})} = \mathbb{R}$.

Theorem 1.40. (The upper limit property of the set of exploded numbers.) Let $\mathbb{S}$ be a set of exploded numbers. If it the $\mathbb{S}(\subset \breve{\mathbb{R}})$ is non empty and bounded from above, then there exists its upper limit.

Proof: Let us consider the upper bound u_{upper} of $\mathbb{S}$ and the set $\underline{\mathbb{S}}$. (See Definition 1.39.) Now, through the inversion formulas, Theorem 0.4 says that for any $x \in \underline{\mathbb{S}}$, the inequality $x \leq \underline{u_{upper}}$ is valid. By the completeness axiom $\underline{\mathbb{S}}$ has its least upper bound $sup_{x\in\underline{\mathbb{S}}}\, x$. Hence for any $x \in \underline{\mathbb{S}}$, the inequalty $x \leq sup_{x\in\underline{\mathbb{S}}}\, x$ holds. Applying Theorem 0.4 again, we have that for any $u \in \mathbb{S}$ the inequality

$$u \leq \left(\widetilde{sup_{x\in\underline{\mathbb{S}}}\, x}\right)$$

holds. We state, that $\left(\widetilde{sup_{x\in\underline{\mathbb{S}}}\, x}\right)$ is the least upper bound of $\mathbb{S}$. If our statement is not true then there exists $v(\in \breve{\mathbb{R}})$ which is less then $\left(\widetilde{sup_{x\in\underline{\mathbb{S}}}\, x}\right)$ and an upper boumd of $\mathbb{S}$. So, for any $u \in \mathbb{S}$ the inequality $u \leq v < \left(\widetilde{sup_{x\in\underline{\mathbb{S}}}\, x}\right)$ is valid.

By Theorem 0.4, for any $x \in \underline{\mathbb{S}}$ the inequality

$$x \leq \underline{v} < sup_{x\in\underline{\mathbb{S}}}\, x.$$

This is a contradiction, because $sup_{x\in\underline{\mathbb{S}}}\, x$ is the least upper bound of the set $\underline{\mathbb{S}}$. So,

$$\left(\widetilde{sup_{x\in\underline{\mathbb{S}}}\, x}\right) = sup_{u\in\mathbb{S}}\, u.$$

In a similar way, we are able to prove

Theorem 1.41: (The lower limit property of the set of exploded numbers.) Let $\mathbb{S}$ be a set of exploded numbers. If it the $\mathbb{S}(\subset \breve{\mathbb{R}})$ is non empty and bounded from below, then there exists its lower limit.

Definition 1.42: (The concepts of super – subtraction and super - division.)

(1.43) $$\breve{x}\,\overline{\ominus}\,\breve{y} = (\widetilde{x-y}) \quad , \ x, y \in \mathbb{R}.$$

and

(1.44) $$\breve{x}\,\overline{\oslash}\,\breve{y} = \left(\widetilde{\frac{x}{y}}\right) \quad , \ x, y(\neq 0) \in \mathbb{R},$$

Equivalent formulations are

(1.45) $$u\,\overline{\ominus}\,v = \left(\widetilde{\underline{u}-\underline{v}}\right) \quad , u, v \in \breve{\mathbb{R}}.$$

(1.46) $$u \overline{\oslash} v = \breve{\left(\frac{\underline{u}}{\underline{v}}\right)} \qquad , u, v(\neq 0) \in \breve{\mathbb{R}}.$$

For real numbers the super – operations are able to express tradinional addition and subtraction.

Theorem 1.47: If x and y are arbitrary real numbers, then

$$x + y = (x \overline{\oplus} y) \overline{\oslash} \left(\breve{1} \overline{\oplus} (x \overline{\odot} y)\right)$$

and

$$x - y = (x \overline{\ominus} y) \overline{\oslash} \left(\breve{1} \overline{\ominus} (x \overline{\odot} y)\right)$$

are valid.

Proof: If x and y are real numbers, then $\underline{x}$, $\underline{y}$ *and* $\frac{\underline{x}+\underline{y}}{1+\underline{x}\cdot\underline{y}} \in \underline{\mathbb{R}}$. Using (1.6), (1.7), (1.2) and (1.44) we can write

$$(x \overline{\oplus} y) \overline{\oslash} \left(\breve{1} \overline{\oplus} (x \overline{\odot} y)\right) = \left(\breve{\underline{x} + \underline{y}}\right) \overline{\oslash} \left(\breve{1} \overline{\oplus} \left(\breve{\underline{x} \cdot \underline{y}}\right)\right) = \left(\breve{\underline{x} + \underline{y}}\right) \overline{\oslash} \left(\breve{1 + \underline{x} \cdot \underline{y}}\right) = \breve{\left(\frac{\underline{x} + \underline{y}}{1 + \underline{x} \cdot \underline{y}}\right)}.$$

Using (0.1) and (0.8) with Definition 9,

$$\breve{\left(\frac{\underline{x} + \underline{y}}{1 + \underline{x} \cdot \underline{y}}\right)} = \tanh^{-1} \frac{\tanh x + \tanh y}{1 + \tanh x \cdot \tanh y} = x + y$$

is obtained. The statement concerning subtraction we ara able to prove in a similar way.

2. A MATHEMATICAL MODEL FOR MULTIVERSE AND PARALEL UNIVERSES

In this part the space $\mathbb{R}^3$ will be mentioned as „our universe", too. Using the explosion of numbers we are able to explode the universe $\mathbb{R}^3$. So, the multiverse

(2.1) $$\breve{\mathbb{R}^3} = \left\{\breve{P} = (\breve{x}, \breve{y}, \breve{z}) \middle| P = (x, y, z) \in \mathbb{R}^3\right\}$$

is obtained. In general, point $\breve{P} = (\breve{x}, \breve{y}, \breve{z})$ is invisible in our universe $\mathbb{R}^3$. Having the open cube

$$\underline{\mathbb{R}^3} = \left\{(\xi, \eta, \zeta) \in \mathbb{R}^3 \middle| \begin{matrix} -1 < \xi < 1 \\ -1 < \eta < 1 \\ -1 < \zeta < 1 \end{matrix}\right\}$$

Definition (0.1) yields that condition $\breve{P} = (\breve{x}, \breve{y}, \breve{z}) \in \mathbb{R}^3$ is fulfilled if and only if $P = (x, y, z) \in \underline{\mathbb{R}^3}$.

For example we show the case of plane $\mathbb{S} = \{P = (x, y, z) | z = x + y \quad , x, y \in \mathbb{R}\}$

The exploded of plane $\mathbb{S}$ is the super-plane $\breve{\mathbb{S}} = \left\{\breve{P} = (\breve{x}, \breve{y}, \breve{z}) \middle| P = (x, y, z) \in \mathbb{S}\right\}$. Its visible part is the exploded of $\mathbb{S} \cap \underline{\mathbb{R}^3}$

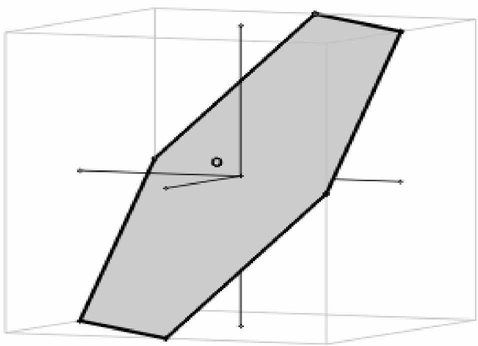


Fig.-2.2

Exploding the open hexagon presented by Fig. 2.2 , we have the visible part of the super – plane $\breve{\mathbb{S}}$

$$\breve{\mathbb{S}} \cap \mathbb{R}^3 = \left\{\breve{P} = (\breve{x}, \breve{y}, \breve{z}) \middle| P = (x, y, z) \in (\mathbb{S} \cap \underline{\mathbb{R}^3})\right\}$$

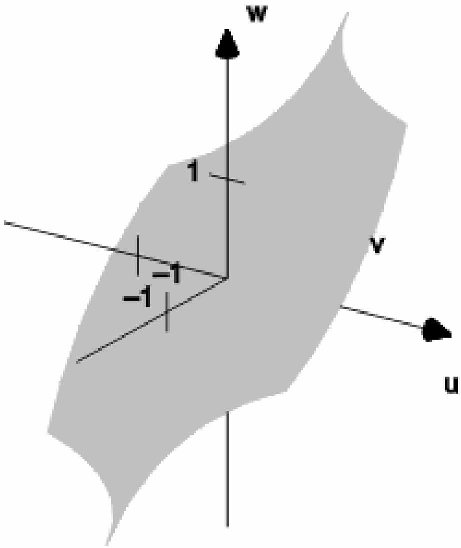


Fig.-2.3

is obtained. For the sake of visibility of more parts of super-plane $\breve{\mathbb{S}}$ we introduce the box – model of multiverse $\widetilde{\mathbb{R}^3}$. First we devide the universe $\mathbb{R}^3$ by cube-compositions

(2.4) $C^{box}_{(p,q,r)} = \{(x,y,z) \in \mathbb{R}^3 | \, p \le |x| < p+1 \,; q \le |y| < q+1 \,; r \le |z| < r+1 \,; p,q,r \in \mathbb{N}_0^+\}$

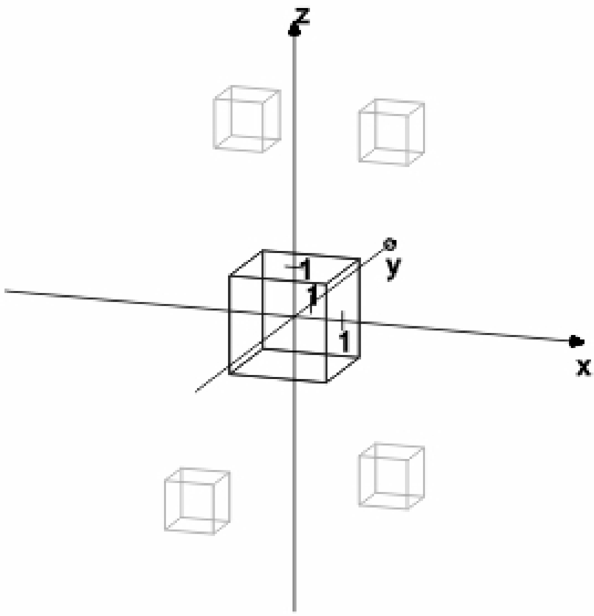


Fig.-2.5

Clearly, $\qquad \bigcup_{p,q,r \in \mathbb{N}_0^+} C^{box}_{(p,q,r)} = \mathbb{R}^3$ and $C^{box}_{(0,0,0)} = \underline{\mathbb{R}^3}$.

Second, we define the box-exploded of the point $P = (x,y,z) \in \mathbb{R}^3$

(2.6) $\quad \breve{P}^{box} = \left((sgn\, x) \cdot \tanh^{-1}\{|x|\} \,,\, (sgn\, y) \cdot \tanh^{-1}\{|y|\} \,,\, (sgn\, z) \cdot \tanh^{-1}\{|z|\} \,,\, d_{(x,y,z)}\right)$

where

(2.7) $$d_{(x,y,z)} = (sgn\, x) \cdot [|x|] + (sgn\, y) \cdot [|y|] \cdot \sqrt{2} + (sgn\, z) \cdot [|z|]\sqrt{3} \,.$$

Moreover, for any set $\mathbb{S} \subseteq \mathbb{R}^3$

(2.8) $$\breve{\mathbb{S}}^{box} = \{\breve{P}^{box} \,|\, P = (x,y,z) \in \mathbb{S}\}.$$

Especially,

(2.9) $$\widetilde{\mathbb{R}^3}^{box} = \{\breve{P}^{box} \,|\, P = (x,y,z) \in \mathbb{R}^3\}.$$

Hence, by (2.6) and (2.9) we can see that $\widetilde{\mathbb{R}^3}^{box} \subset \mathbb{R}^4$.

Using (2.4) – (2.6) we explode the boxes $C^{box}_{(p,q,r)}$ and get„ parallel" three - dimensional spaces

(2.10) $\left(\widetilde{C^{box}_{(p,q,r)}}\right)^{box} = \left\{(u,v,w,d) \in \widetilde{\mathbb{R}^3}^{box} \,\middle|\, \begin{smallmatrix} -\infty<u<\infty \\ -\infty<v<\infty \\ -\infty<w<\infty \\ d=\pm p\pm q\sqrt{2}\pm r\sqrt{3} \end{smallmatrix}\ p,q,r \in \mathbb{N}_0^+\right\}$. (See, [1], (6.2.12)-(6.2.19).)

Having that $C^{box}_{(0,0,0)} = \underline{\mathbb{R}^3}$, by (2.6) , (2.7) and (2.8) is $\left(\widetilde{C^{box}_{(0,0,0)}}\right)^{box} = \left\{(u,v,w,d) \in \widetilde{\mathbb{R}^3}^{box} \,\middle|\, \begin{smallmatrix} -\infty<u<\infty \\ -\infty<v<\infty \\ -\infty<w<\infty \\ d=0 \end{smallmatrix}\right\}$,

we obtain that our universe in the box- model is represented by $\left(\widetilde{C^{box}_{(0,0,0)}}\right)^{box}$.

Plane $\mathbb{S}$ turns out from box $C^{box}_{(0,0,0)}$ and a part is situated in the „Right – Back- Upper" part of the box

$$C^{box}_{(0,0,1)} = \{(x,y,z) \in \mathbb{R}^3 | -1 < x < 1 ; -1 < y < 1 ; 1 \le |z| < 2\}$$

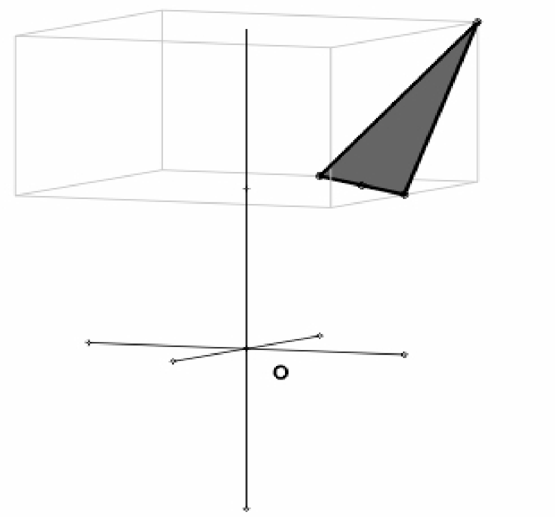


Fig.-2.11

Super – plane $\breve{\mathbb{S}}$ has the equation

(2.12) $w = u \overline{\oplus} v \quad , \ u,v \in \breve{\mathbb{R}}.$

and $\breve{\mathbb{S}}^{box} \subset \widetilde{\mathbb{R}^3}^{box}$. Moreover, the exploded of the plane-part which is in the Fig. 2.11 will be situated in the three - dimensional space

$$\left(\widetilde{C^{box}_{(0,0,1)}}\right)^{box} = \left\{(u,v,w,d) \in \widetilde{\mathbb{R}^3}^{box} \,\middle|\, \begin{smallmatrix} -\infty<u<\infty \\ -\infty<v<\infty \\ -\infty<w<\infty \\ d=\pm\sqrt{3} \end{smallmatrix}\right\}.$$

Writing that

$$u = \tanh^{-1} x \ , \ 0 \le x < 1; v = \tanh^{-1} y \ , \ 0 \le y < 1 ; w = \tanh^{-1}(z-1), \ 1 \le z < 2;$$

by (2.12) the equation

(2.13) $w = \tanh^{-1}(\tanh u + \tanh v - 1) \qquad ,1 \le \tanh u + \tanh v < 2 \ , \ u,v \in \mathbb{R}_0^+$

is obtained. This is the equation of continuation (in $\left(\widetilde{C^{box}_{(0,0,1)}}\right)^{box}$) of $\breve{\mathbb{S}} \cap \mathbb{R}^3$ introduced on Fig. 2.3.

$$\breve{\mathbb{S}}^{box} \cap \left(\widetilde{C^{box}_{(0,0,1)}}\right)^{box}$$

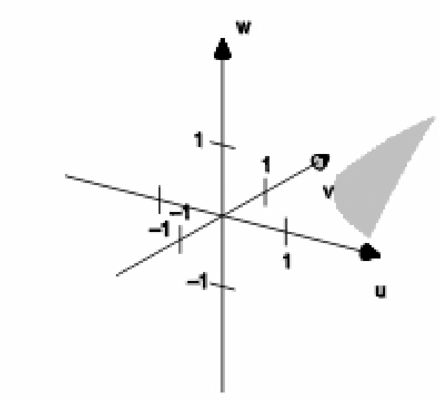


Fig.-2.14

By (2.13), in the highness" $w = \breve{1}$ the level – curve of super-plane $\breve{\mathbb{S}}$ has the equation

$$\tanh u + \tanh v = 1 \quad , u, v \in \mathbb{R}_0^+.$$

This level – curve is situated on the border of our universe, so, it is invisible for us. On the other side", it is visible in the neighbouring" universe $\left(\widetilde{C_{(0,0,1)}^{box}}\right)^{box}$. (See Fig. 2.14.)

Let us imagine that a surveyor starts from the origo of our three - dimensional space $\mathbb{R}^3$ and moves on the super – line $\breve{\mathbb{L}} = \{\breve{P} = (\breve{x}, \breve{y}, \breve{z}) | P = (x, y, z) \in \mathbb{L}\}$,where

(2.15) $$\mathbb{L} = \left\{ P = (x, y, z) \in \mathbb{R}^3 | \begin{smallmatrix} x = \frac{1}{\sqrt{6}} \cdot t \\ y = \frac{1}{\sqrt{6}} \cdot t \\ z = \frac{2}{\sqrt{6}} \cdot t \end{smallmatrix} \quad , t \in \mathbb{R} \right\}.$$

The line $\mathbb{L}$ bores through infinite many box – compositions given under (2.4).

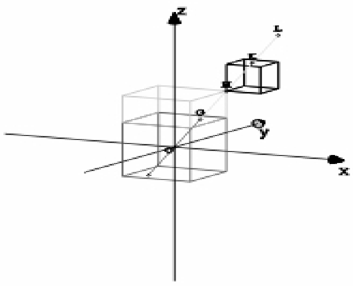


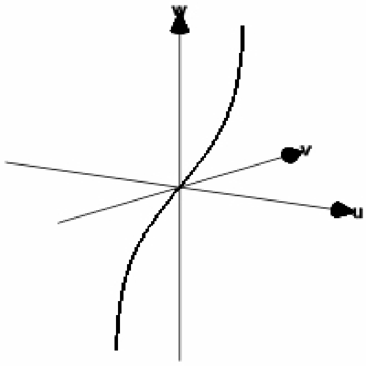
Fig.-2.16

By (2.4) and (2.15) we can see that $\mathbb{L} \cap C_{(0,0,0)}^{box} = \left\{ P = (x, y, z) \in \mathbb{R}^3 | \begin{smallmatrix} x = \frac{1}{\sqrt{6}} \cdot t \\ y = \frac{1}{\sqrt{6}} \cdot t \\ z = \frac{2}{\sqrt{6}} \cdot t \end{smallmatrix} \quad , -\frac{\sqrt{6}}{2} < t < \frac{\sqrt{6}}{2} \right\}.$

Hence, $\breve{\mathbb{L}} \cap \left(\widetilde{C_{(0,0,0)}^{box}}\right)^{box} = \left\{ (u, v, w) \in \mathbb{R}^3 | \begin{smallmatrix} u = \tanh^{-1} \frac{t}{\sqrt{6}} \\ v = \tanh^{-1} \frac{t}{\sqrt{6}} \\ w = \tanh^{-1} \frac{2t}{\sqrt{6}} \end{smallmatrix} \quad , -\frac{\sqrt{6}}{2} < t < \frac{\sqrt{6}}{2} \right\}$ (See the next figure.)

It is easy to see that $\lim_{t \to \frac{\sqrt{6}}{2}} u(t) = \tanh^{-1} \frac{1}{2}$, $\lim_{t \to \frac{\sqrt{6}}{2}} v(t) = \tanh^{-1} \frac{1}{2}$ *and* $\lim_{t \to \frac{\sqrt{6}}{2}} w(t) = \infty$,

and $\breve{G} = \left(\widetilde{\left(\frac{1}{2}\right)}, \widetilde{\left(\frac{1}{2}\right)}, \breve{1}\right) \notin \mathbb{R}^3$. (See Fig. 2.16.)



At the point $\breve{G}$ the surveyor turns into the three - dimensional space $\left(\widetilde{C_{(0,0,1)}^{box}}\right)^{box}$ and moves towards the point $\breve{H} = (\breve{1}, \breve{1}, \breve{2}) \notin \left(\widetilde{C_{(0,0,1)}^{box}}\right)^{box}$. (See Fig. 2.16.) on the way

$$\breve{\mathbb{L}} \cap \left(\widetilde{C_{(0,0,1)}^{box}}\right)^{box} = \left\{ (u, v, w) \in \mathbb{R}^3 | \begin{matrix} u = \tanh^{-1} \frac{t}{\sqrt{6}} \\ v = \tanh^{-1} \frac{t}{\sqrt{6}} \\ w = \tanh^{-1} \left(\frac{2t}{\sqrt{6}} - 1 \right) \end{matrix} \quad , \frac{\sqrt{6}}{2} \le t < \sqrt{6} \right\}.$$

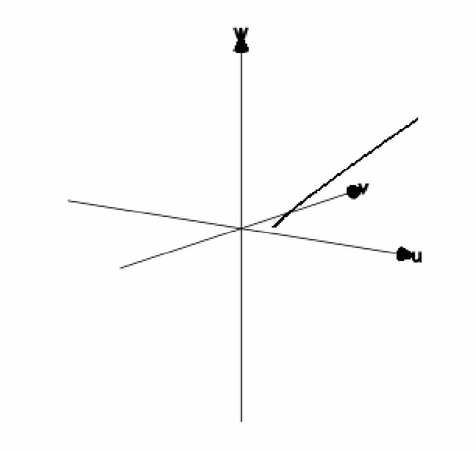


Fig.-2.17

As

$$\lim_{t\to\sqrt{6}} u(t)=\infty\,,\ \lim_{t\to\sqrt{6}} v(t)=\infty\ \ and\ \ \lim_{t\to\sqrt{6}} w(t)=\infty\,,$$

and the box – explosion formula (2.6) with (2.7) shows that $\breve{H}^{box}=\left(0\,,0\,,0\,,1+\sqrt{2}+\ 2\sqrt{3}\right)$ is the origo of the three - dimensional space

$$\left(\widetilde{C^{box}_{(1,1,2)}}\right)^{box}=\left\{(u,v,w,d)\in\widetilde{\mathbb{R}^3}^{box}\ \middle|\ \begin{matrix}-\infty<u<\infty\\ -\infty<v<\infty\\ -\infty<w<\infty\\ d=\pm1+\pm\sqrt{2}\pm2\sqrt{3}\end{matrix}\right\}$$

where the surveyor arrives. (This point is already invisible in the universe $\left(\widetilde{C^{box}_{(0,0,1)}}\right)$.) In this newer universe we have

$$\breve{\mathbb{L}}\cap\left(\widetilde{C^{box}_{(1,1,,2)}}\right)^{box}=\left\{(u,v,w)\in\mathbb{R}^3\middle|\ \begin{matrix}u=\tanh^{-1}\left(\frac{t}{\sqrt{6}}-1\right)\\ v=\tanh^{-1}\left(\frac{t}{\sqrt{6}}-1\right)\\ w=\tanh^{-1}\left(\frac{2t}{\sqrt{6}}-2\right)\end{matrix}\ ,\sqrt{6}\le t<\frac{3\sqrt{6}}{2}\right\}$$

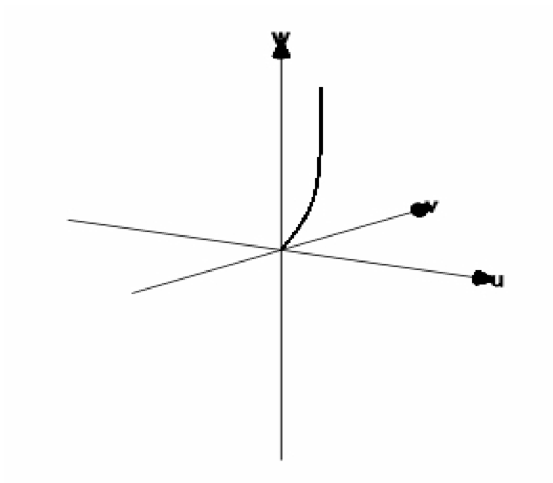


Fig.-2.18

and the surveyor tends to the point $\breve{K}=\left(\widetilde{\left(\frac{3}{2}\right)},\widetilde{\left(\frac{3}{2}\right)},\breve{3}\right)$ assuming that it has sufficient capacity to move.

The Fig. 2.16 , 2.17 and 2.18 show that the further way in the multiverse is periodical.

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