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EFFECT OF HEAT SOURCES AND THERMAL RADIATION ON CONVECTIVE HEAT TRANSFER FLOW PAST A VERTICAL WAVY WALL WITH VARIABLE VISCOSITY AND THERMAL CONDUCTIVITY

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ABSTRACT

We investigate effect of variable properties on natural convective heat transfer flow over a vertical wavy surface embedded in a fluid saturated porous medium with thermal radiation and heat sources. The Darcy model is used to study the fluid flow in the saturated porous medium. Moreover, the temperature dependent variables properties are considered. The vertical wavy wall and the governing equations for flow, heat ad mass transfer are transformed to a plane geometry case by employing the Runge-Kutta fourth order with Shooting technique. The non-dimensional velocity, temperature graphs as well as rate of heat transfer coefficients are displayed graphically for different values of variable parameter, variable thermal conductivity parameter, radiation parameter and amplitude of the wavy surface.

Keywords: *Wavy Wall, Thermal Radiation, Chemical reaction, Variable viscosity and thermal conductivity, Heat Sources.*

1. INTRODUCTION

In recent years, energy and material saving considerations have prompted an expansion of the efforts at producing efficient heat exchanger equipment through augmentation of heat transfer. It has been established that channels with diverging – converging geometries augment the transportation of heat transfer and momentum. As the fluid flows through a tortuous path viz., the dilated – constricted geometry, there will be more intimate contact between them. The flow takes place both axially (primary) and transversely (secondary) with the secondary velocity being towards the axis in the fluid bulk rather than confining within a thin layer as in straight channels. Hence it is advantageous to go for converging-diverging geometries for improving the design of heat transfer equipment. Vajravelu and Nayfeh [23] have investigated the influence of the wall waviness on friction and pressure drop of the generated coquette flow. Vajravelu and Sastry [22] have analyzed the free convection heat transfer in a viscous, incompressible fluid confined between long vertical wavy walls is the presence of constant heat source. Later Vajravelu and Debnath [21] have extended this study to convective flow is a vertical wavy channel in four different geometrical configurations. This problem has been extended to the case of wavy walls by Deshikachar *et.al* [5] Rao *et.al* [14] and Sree Ramachandra Murthy [20]. Rees & Pop [18] have consider free convection induced by a vertical wavy surface with uniform heat flux in a porous medium. Shalini and Rathish Kumar [20] have discussed the influence of variable heat flux on natural convection along a corrugated wall in porous media.

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With the fuel emergency extending everywhere throughout the world, outside layer in the geothermal region. Fluid in the geothermal area is an interaction of the geomagnetic field with the fluid in the geothermal region is of awesome interest, in this way prompting enthusiam for the investigation of MHD convective courses through permeble medium. The application of electromagnetic fields in controlling the heat transfer as in aerodynamic heating leads to the study of magneto hydrodynamic heat transfer. This MHD heat transfer has gained significance owing to recent advancement of space technology. The MHD heat transfer can be divided into two sections. One contains problems in which the heating is an incidental by-product of the electro magnetic fields as in MHD generators, pumps etc., and the second consists of problems in which the primary use of electromagnetic fields is to control the heat transfer. With the fuel crisis deepening all over the world, there is a great concern to utilize the enormous power beneath the earth's crust in the geothermal region. Liquid in the geothermal region is an electrically conducting liquid because of high temperature. Hence the study of interaction of the geomagnetic field with the fluid in the geothermal region is of great interest, thus leading to interest in the study of MHD convection flows through porous medium. Bharathi *et.al* [3] have discussed Non Darcy Hydromagnetic Mixed convective Heat and Mass Transfer flow of a viscous fluid in a vertical channel with Heat generating sources. Non-Darcy Hydromagnetic convective Heat and Mass transfer through a porous medium in a cylindrical annulus with Soret effect, radiation and dissipation. Balasubramanyam *et.al* [1] have discussed Non Darcy viscous electrically conductive Heat and Mass transfer flow through a porous medium in a vertical channel in the presence of heat generating sources.

The study of heat generation or absorption effects in moving fluids is important in view of several physical problems such as fluids undergoing exothermic or endothermic chemical reactions. The volumetric heat generation has been assumed to be constant or a function of space variable. For example, a hypothetical core – disruptive accident in a liquid metal fast breeder reactor (LMFBR) could result in the setting of fragmented fuel debris on horizontal surfaces below the core. The porous debris could be saturated sodium coolant and heat generation will result from the radioactive decay of the fuel particulate. Vajravelu and Hadjinicolaou [24] studied the heat transfer characteristics in the laminar boundary layer of a viscous fluid over a stretching sheet with viscous dissipation or frictional heating and internal heat generation. Hossain *et.al* [7] studied the problem of natural convection flow along a vertical wavy surface with uniform surface temperature in the presence of heat generation or absorption. Hady *et.al* [6] studied the problem of free convection flow along a vertical wavy surface embedded in electrically conducting fluid saturated porous media in the presence of internal heat generation or absorption effect.

Most of the researchers have only considered the effect of constant viscosity and thermal conductivity on boundary layer developed by a vertical wavy surface.However,it is known that the fluid viscosity changes with temperature, for example the absolute viscosity of water decreases by 240% when the temperature increases from 10 0C to 50 0C.Variable thermal conductivity takes place I many engineering applications such as Heat transfer in furnace, boilers, porous burners, volumetric solar receivers, fibrous and foam insulations.

In recent years, progress has been considerably made in the study of radiation effect on convective heat and mass transafrer flow due to ts importance in several engineeringt problems,geothermal ,geophysical,technological and industrial areas such as nuclear power plants,various propulsion devices for missiles,satellites,gas turbances,space vehicles and aircraft.In view of the above applications,we considered the combined effect of thermal radiation and variables properties on convective heat and mass transfer flow past a vertical wavy surface in fluid saturated porous medium. Recently Mallikarjuna [12] have discussed the effect of variable viscosity and thermal conductivity on convective heat and mass transfer flow over a vertical wavy suface in a porous medium with variable properties. Bejan and Khair [2] and Lai [10] excellent agreement has been reported in the absence of thermal radiation, heat source and variable properties. Ling and Dybbs [11] has been investigated theoretically a very interesting effect to temperature dependent viscosity on free and mixed convective heat transport in a saturated porous medium. Recently Hossain et al [8], Nasser *et.al* [13] studied the effects of variable properties on natural convection along a vertical wavy surface without porous media.

In this paper, we investigate effect of variable properties on natural convective heat transfer flow over a vertical wavy surface embedded in a fluid saturated porous medium with thermal radiation and heat sources. The Darcy model is used to study the fluid flow in the saturated porous medium. Moreover, the temperature dependent variables properties are considered. The vertical wavy wall and the governing equations for flow, heat ad mass transfer are transformed to a plane geometry case by employing the Runge-Kutta fourth order with Shooting technique. The non-dimensional velocity, temperature graphs as well as rate of heat transfer coefficients are displayed graphically for different values of variable parameter, variable thermal conductivity parameter, radiation parameter and amplitude of the wavy surface.

2. FORMULATION OF THE PROBLEM

We consider a steady, incompressible, two-dimensional laminar natural convective heat and mass transfer flow over a vertical wavy surface embeddede ina saturated porous medum.The porous medium is uniform and local thermal equilibrium with the fluid. The Darcy law is used to describe the fluid saturated porous medium. The fluid is assumed to be gray, absorbing-emitting radiation but non-scattering medium.The wavy surface profile is given by

$$y = \bar{\sigma}(\bar{x}) = \bar{a}Sin\left(\frac{\pi\bar{x}}{l}\right) \tag{1}$$

Where l is the characteristic length of wavy surface and $\bar{a}$ is the amplitude of the wavy surface. The wavy surface is maintained at constant temperature Tw which are higher than the ambient fluid temperature T_∞. We consider the natural convection-radiation flow in the presence of heat sources to be governed by the following equations under Boussinesq approximations;

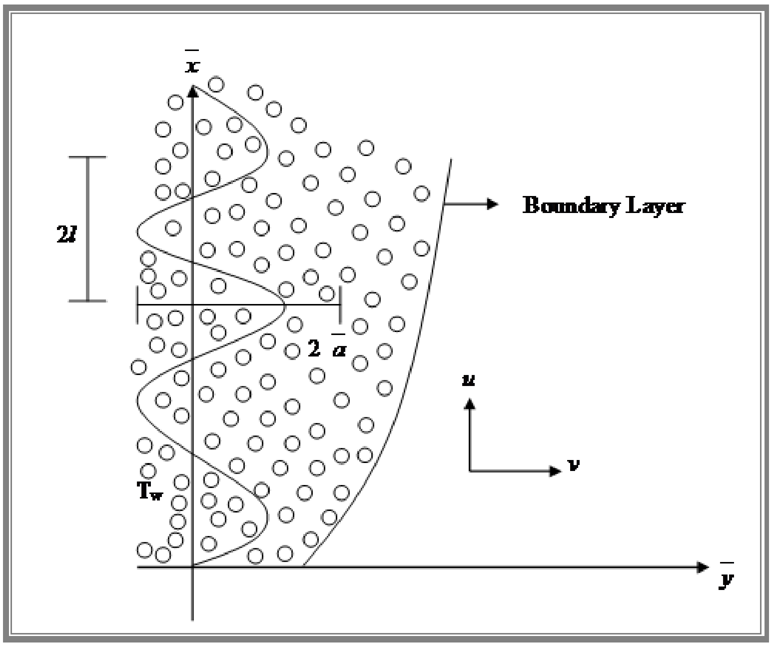

Fig.-1: Physical Configuration and Co- ordinate System

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$\frac{\partial}{\partial y}\left(\frac{\mu}{k}\bar{u}\right) - (\sigma\mu_e^2 H_o^2)u = \frac{\partial}{\partial x}\left(\frac{\mu}{k}\bar{v}\right) + \rho g\left(\beta_o\frac{\partial T}{\partial y}\right)\sqrt{a^2+b^2} \tag{3}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \frac{\partial}{\partial \bar{x}}\left(\alpha\frac{\partial T}{\partial \bar{x}}\right) + \frac{\partial}{\partial \bar{y}}\left(\alpha\frac{\partial T}{\partial \bar{y}}\right) - \nabla.q_r - Q_H(T - T_\infty) \tag{4}$$

The relevant boundary conditions are

$$\vec{u} = 0, \bar{v} = 0, T = T_w \quad at \;\; \bar{y} = \bar{\sigma}(\bar{x}) = \bar{a}\,Sin\left(\frac{\pi\bar{x}}{l}\right)$$

$$\vec{u} = 0, T \to T_\infty \quad as \quad \bar{y} \to \infty \tag{5}$$

where $\bar{u}$ and $\bar{v}$ are the velocity components in the directions of x and y respectively, T is temperature,, ρ is the density of the fluid, μ is the dynamic viscosity of the fluid,k is the permeability of the porous medium, σ is the electrical conductivity, μ_e is the magnetic permeability, Ho is the strength of the magnetic field. β_o are the coefficients of thermal expansion, α is the thermal conductivity, q_r is the radiative heat flux, g is the acceleration due to gravity and Q_H is the strength of the heat source.

By applying Rosseland approximation (Brewester [4]) the radiative heat flux q_r is given by

$$q_r = -\left(\frac{4\sigma^*}{3\beta_R}\right)\frac{\partial}{\partial y}\left[T'^4\right] \tag{6}$$

where σ* is the Stephan – Boltzmann constant and mean absorption coefficient.

Assuming that the difference in temperature within the flow are such that T'^4 can be expressed as a linear combination of the temperature. We expand T'^4 in Taylor's series about Te as follows

$$T'^4 = T_\infty^4 + 4T_0^3(T - T_0) + 6T_0^2(T - T_0) + \ldots\ldots \tag{7}$$

Neglecting higher order terms beyond the first degree in $(T - T_\infty)$ .we have

$$T'^4 \cong -3T_0^4 + 4T_0^3 T \tag{8}$$

Differentiating equation (6) with respect to y and using (7) we get

$$\frac{\partial(q_R)}{\partial y} = -\frac{16\sigma^* T_0^3}{3\beta_R}\frac{\partial^2 T}{\partial y^2} \tag{9}$$

On using equations (9) in the last term of equation (4) we get

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \frac{\partial}{\partial \bar{x}}\left(\alpha\frac{\partial T}{\partial \bar{x}}\right) + \frac{\partial}{\partial \bar{y}}\left(\alpha\frac{\partial T}{\partial \bar{y}}\right) + \frac{16\sigma^* T_\infty^3}{3\beta_R}\left(\frac{\partial^2 T}{\partial \bar{x}^2} + \frac{\partial^2 T}{\partial \bar{y}^2}\right) - Q_H(T - T_\infty) \tag{10}$$

The fluid properties are assumed to be constant except fluid viscosity and thermal conductivity.Terefore we assume that the viscosity of the fluid is to be an inverse function of the temperature and it can be expressed as [Lai and Kulacki [9]].

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}(1 + \delta(T - T_\infty)\ o\ \ \frac{1}{\mu} = b((T - T_\infty) \tag{11}$$

where $b = \frac{\delta}{\mu_\infty}$ and $T = T_\infty - \frac{1}{\delta}$. Both b and Tr are constants and their values depend on the reference state and the thermal property of the fluid i.e. δ . In general b>0 for liquids and b<0 for gases θ_r , which is defined by

$$\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} = -\frac{1}{\delta(T_w - T_\infty)} \tag{12}$$

is constant. The parameter θ_r was first introduced by Ling and Dybbs [11]. It is important to note that for $\delta \to 0 (i.e\ \mu = \mu_\infty = cons\tan t)$ then $\theta_r \to \infty$, the effect of viscosity is negligible. The value of θ_r is determined by the temperature difference $(T_w - T_\infty)$ and viscosity δ of the fluid in consideration.A smaller values of θ_r is implies either the fluid viscosity changes considerably or the temperature difference is high.On the otherhand,for a larger values of θ_r implies either $(T_w - T_\infty)$ or δ is small, and therefore the effects of variable viscosity can be neglected.In either case the influence of variable viscosity plays very important role and the liquid viscosity varies differently with temperature than that of gases.Therefore, θ_r is positive for gases and negative for liquids respectively.

Also, we assume that the fluid thermal conductivity α is to be varying as a linear function of temperature in the form [Seddeek and Salem [17]]

$$\alpha = \alpha_o(1 + E(T - T_\infty))$$

where, α_o is the thermal diffusivity at the wavy surface temperature Tw and E is a constant depending on the nature of the fluid.It is worth mentioning here that E is positive for fluids such as air and E is negative for fluids such as lubrication oils. This can be written in the non-dimensional form [Slattery [19]] as

$$\alpha = \alpha_o(1 + \beta\theta) \tag{13}$$

where $\beta = E(T - T_\infty)$ is the thermal conductivity parameter, the variation of β can be taken in the range $-0.1 \le \beta \le 0$ for lubrication oils, $0 \le \beta \le 0.12$ for water and $0 \le \beta \le 6$ for air.

In view of the continuity equation (2) we define the stream function ψ as

$$\bar{u} = \frac{\partial \bar{\psi}}{\partial \bar{y}}\quad ,\ v = -\frac{\partial \bar{\psi}}{\partial \bar{x}} \tag{14}$$

In order to write the governing equations in the dimensionless form, we introduce the following non-dimensional variables as

$$x=\frac{\overline{x}}{l},\, y=\frac{\overline{y}}{l},\, a=\frac{\overline{a}}{l},\, \sigma=\frac{\overline{\sigma}}{l},\, \psi^{*}=\frac{\overline{\psi}}{l},\, \theta=\frac{T-T_{\infty}}{T_w-T_{\infty}}, \tag{15}$$

The equations (12) - (15).equations (2) - (3) and (10) reduce to

$$\left(\frac{1}{\theta-\theta_r}\right)\left(\frac{\partial\theta}{\partial y}\frac{\partial\psi^{\bullet}}{\partial y}-\frac{\partial\theta}{\partial x}\frac{\partial\psi^{\bullet}}{\partial x}\right)+Ra\left(1-\frac{\theta}{\theta_r}\right)\left(\frac{\partial\theta}{\partial y}\right)-M^2\frac{\partial^2\psi^{\bullet}}{\partial y^2} \tag{16}$$

$$\frac{\partial\theta}{\partial x}\frac{\partial\psi^{\bullet}}{\partial y}-\frac{\partial\theta}{\partial y}\frac{\partial\psi^{\bullet}}{\partial x}=\beta\left(\left(\frac{\partial\theta}{\partial x}\right)^2+\left(\frac{\partial\theta}{\partial y}\right)^2\right)+\left(1+\beta\theta+\frac{4Rd}{3}\right)\left(\frac{\partial^2\theta}{\partial y^2}+\frac{\partial^2\theta}{\partial x^2}\right)-Q\theta \tag{17}$$

where

$Ra=\dfrac{\beta_T g(T_w-T_{\infty})l}{\alpha_o \nu}$ is the Darcy-Rayleigh Number,

$\nu=\dfrac{\mu_{\infty}}{\rho}$ is the kinematic viscosity of the fluid,

$Rd=\dfrac{4\sigma^{*}T_{\infty}^{3}}{k_f\beta_R}$ is the Radiation parameter,

$Q=\dfrac{Q_H l^2}{\alpha_o C_p}$ is heat source parameter,

$M^2=\dfrac{\sigma\mu_e^2 H_0^2 l^2}{\mu}$ .is the magnetic parameter.

The transformed boundary conditions are

$$\begin{aligned}&\psi^{\bullet}=0,\theta=1 \quad at \quad y=aSin(x)\\ &\frac{\partial\psi^{\bullet}}{\partial y}\to 0,\theta\to 0 \quad as \; y\to\infty\end{aligned} \tag{18}$$

We can transform the effect of wavy surface from the boundary conditions into the governing equations by using suitable coordinate transformation with boundary layer scaling,for the case of free convection .The Cartesian coordinates(x, y) are transformed into the new variables(ξ,η).

We incorporate the effect of effect of wavy surface and the usual boundary layer scaling into the governing equations(16) & (17) for free convection ,using the transformations and $Ra\to\infty$ (i.e boundary layer approximation),

$$x=\xi,\overline{\eta}=\frac{y-aSin(x)}{\xi^{1/2}Ra^{-1/2}},\psi^{*}=Ra^{1/2}\psi$$

These transformations are similar to those presented in, for instance, Rees and Pop [17]. We obtain the following boundary layer equations:

$$\begin{aligned}&\left(\frac{1}{\theta-\theta_r}\right)(1+a^2Cos^2\xi)\frac{\partial\theta}{\partial\eta}\frac{\partial\psi}{\partial\eta}+(1+a^2Cos^2\xi)\frac{\partial^2\psi}{\partial\eta^2}=\\ &\qquad Ra\,\xi^{1/2}\left(1-\frac{\theta}{\theta_r}\right)\left(\frac{\partial\theta}{\partial\eta}\right)-M^2\frac{\partial^2\psi}{\partial\eta^2}\end{aligned} \tag{19}$$

$$\xi^{1/2}\left(\frac{\partial\theta}{\partial\xi}\frac{\partial\psi}{\partial\eta}-\frac{\partial\theta}{\partial\eta}\frac{\partial\psi}{\partial\xi}\right)=\left(1+a^2Cos^2(\xi)\beta\left(\left(\frac{\partial\theta}{\partial\eta}\right)^2\right)+\left(1+\beta\theta+\frac{4Rd}{3}\right)\frac{\partial^2\theta}{\partial\eta^2}\right)-Q\theta \tag{20}$$

3. SOULUTION METHODOLOGY

We now introduce the following similarity variables as

$$\eta = \frac{\overline{\eta}}{(1+a^2 Cos^2(\xi))},\ \psi = \xi^{1/2} f(\eta),\ \theta = \theta(\eta)\ \ and\ \ \phi = \phi(\eta)$$

In equations (19) & (20), we obtain a system of ordinary differential equations as follows:

$$f'' + \left(\frac{1}{\theta - \theta_r}\right)\theta' f' - \frac{M^2}{(1+a^2 Cos^2 \xi)} f'' = Ra\left(1 - \frac{\theta}{\theta_r}\right)(\theta')\frac{1}{2} \tag{21}$$

$$\beta(\theta')^2 + \left(1 + \beta\theta + \frac{4Rd}{3}\right)\theta'' + \frac{1}{2} f\theta' - Q\left(1 + a^2 Cos^2(\xi)\right)\theta \tag{22}$$

where prime denotes differentiation with respect to η.

The corresponding boundary conditions are

$f = 0$, $\theta=1$ at $\eta = 0$ (23)

$f' \to 0$, $\theta \to 1$ as $\eta \to \infty$

In equation (22) the radiation parameter $Rd = \dfrac{4\sigma^* T_\infty^3}{k_f \beta_R}$ means that the rate of thermal radiation contribution relative to the thermal conditions. As $Rd \to \infty$, influence of thermal radiation is high in the boundary layer regime. For $Rd \to 0$, the term 4Rd/3 tends to zero. For Rd=1, thermal radiation and thermal conduction will give equal contribution.

The main results of practical interest in many applications are heat transfer coefficient at the surface.The heat transfer coefficient are expressed in terms of Nusselt number Nux.
Nusselt number Nux is given by

$$Nux = \frac{x q_w}{\alpha_o (T_w - T_\infty)} \tag{24}$$

where q_w is the heat flux on the wavy surface, and is defined by

$q_w = -\alpha_0 \overline{n}.\nabla T\ \ and\ \ \overline{n} = \left(-\dfrac{aCos(\xi)}{\sqrt{(1+a^2 Cos^2(\xi))}}, \dfrac{1}{\sqrt{(1+a^2 Cos^2(\xi))}}\right)$ is the unit normal vector to the wavy surface, α_0 is the effective porous medium thermal conductivity.Therefore

$$Nu_\xi = -\frac{\theta'(0) Ra_\xi^{1/2}}{\sqrt{(1+a^2 Cos^2(\xi))}} \tag{25}$$

4. RESULTS AND DISCUSSION

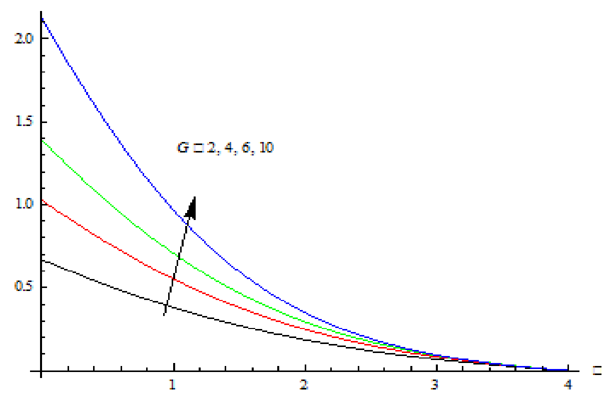

Fig.-2a: Variation of velocity (u) with G
M=0.5, Q=0.5,β=0.2,Rd=0.5,a=0.1,ξ=π/4,θr=-2

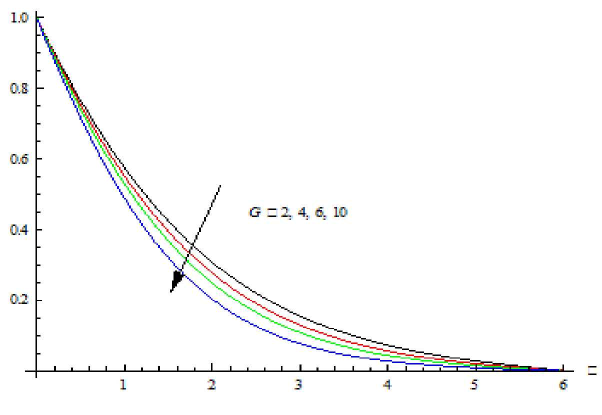

Fig.-2b: Variation of temperature (θ) with G
M=0.5,Q=0.5,β=0.2,Rd=0.5,a=0.1,ξ=π/4,θr=-2

Figs.2a-2b illustrate the variation of velocity and temperature with Grashof number (G). It can observed from the profiles that the axial velocity enhances in the flow region. An increase in G reduces the temperature. This may be attributed to the fact that the thickness of the thermal boundary layer reduces with increasing values of G.

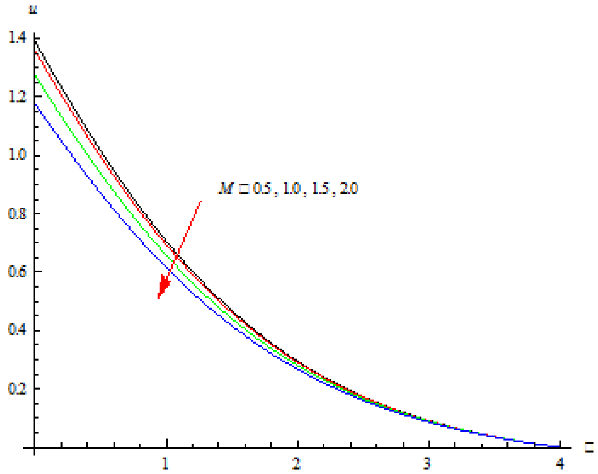

Fig.-3a: Variation of velocity (u) with M
G=2,Q=0.5,β=0.2,Rd=0.5,a=0.1,ξ=π/4,θr=-2

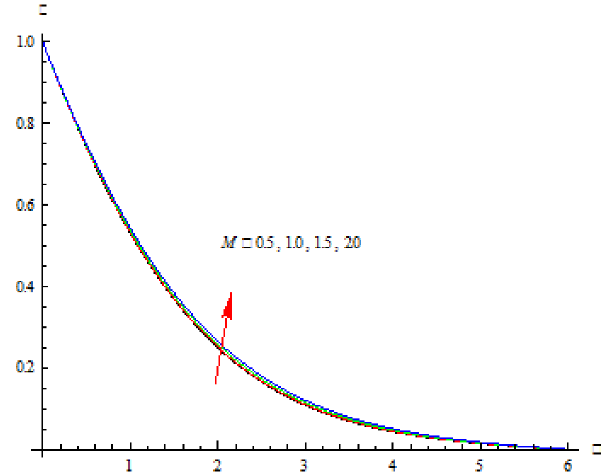

Fig.-3b: Variation of temperature (θ) with M
G=2, Q=0.5,β=0.2,Rd=0.5,a=0.1,ξ=π/4,θr=-2

Fig.3a-3b depict the variation of velocity and temperature with magnetic parameter (M). From the profiles we find that the velocity reduces and temperature enhances with increasing values of magnetic parameter.

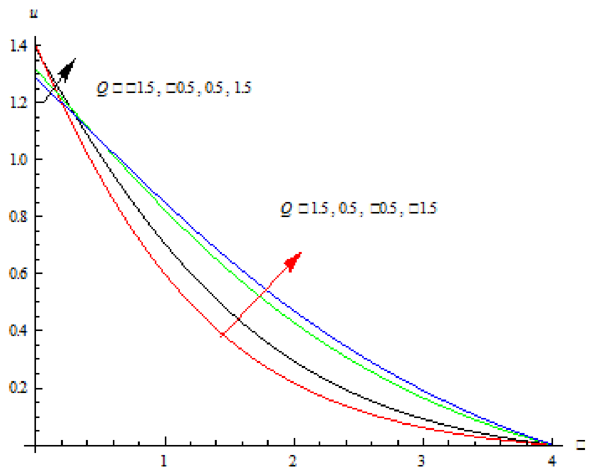

Fig.-4a: Variation of velocity (u) with Q
G=2, M=0.5,β=0.2,Rd=0.5,a=0.1,ξ=π/4,θr=-2

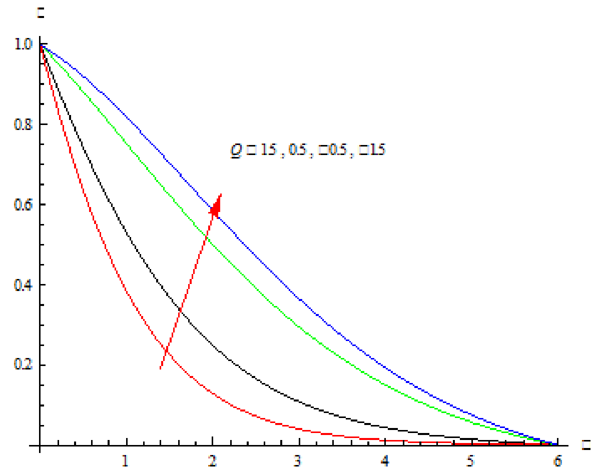

Fig.-4b: Variation of temperature (θ) with Q
G=2, M=0.5,β=0.2,Rd=0.5,a=0.1,ξ=π/4,θr=-2

Figs.4a-4b represent u and θ with heat source parameter (Q). From fig.4a, we find that the an increase in the strength of the heat generating source enhances the velocity in the region (0, 0.5) adjacent to the wall and reduces far away from the wall, while in the case of heat absorbing source, heat is generated in the region (0, 0.5) and the heat energy is absorbed in the region (0.5, 4.0).The temperature decreases with increase in Q>0 and enhances with increase in Q<0(fig. 4b).

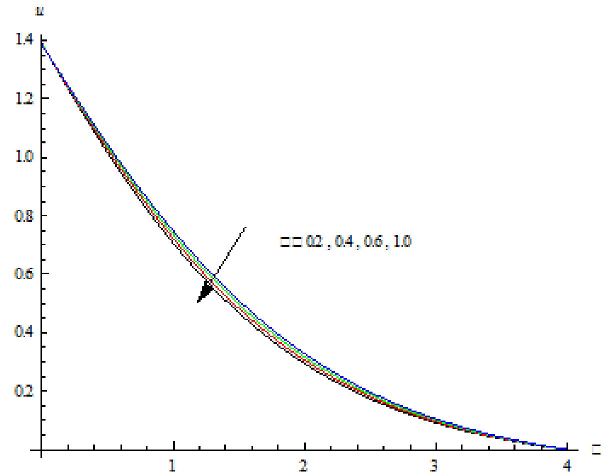

Fig.-5a: Variation of velocity (u) with β
G=2, M=0.5, Q=0.5, Rd=0.5,a=0.1,ξ=π/4,θr=-2

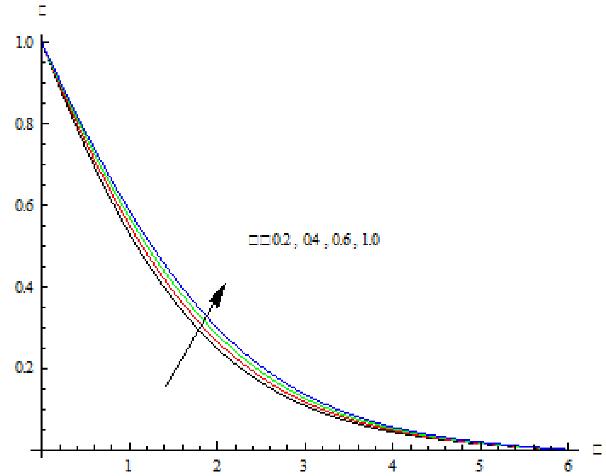

Fig.-5b: Variation of temperature (θ) with β
G=2, M=0.5, Q=0.5, Rd=0.5,a=0.1,ξ=π/4,θr=-2

From 5a-5b represent the effect of thermal conductivity parameter β on the non-dimensional velocity and temperature. Fig.5a shows the variation of velocity with β.In this case the velocity is found to depreciate in the entire flow region .From fig.5b we found that as the thermal conductivity parameter β increases the temperature increases. This is due to the thickening of the thermal boundary layer as a result of increasing values of thermal conductivity.

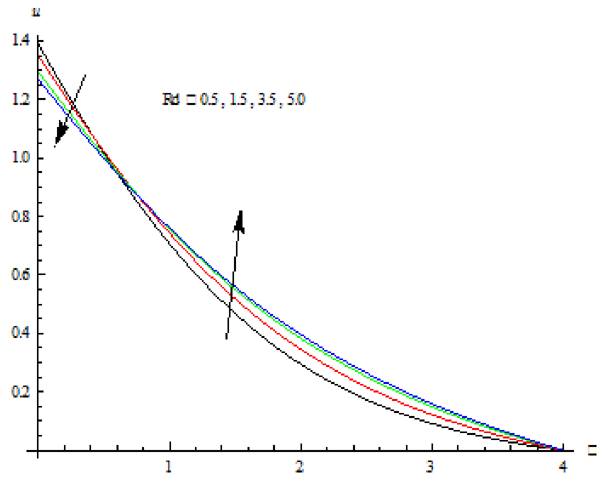

Fig.-6a: Variation of velocity (u) with Rd
G=2, M=0.5, Q=0.5,β=0.2, a=0.1,ξ=π/4,θr=-2

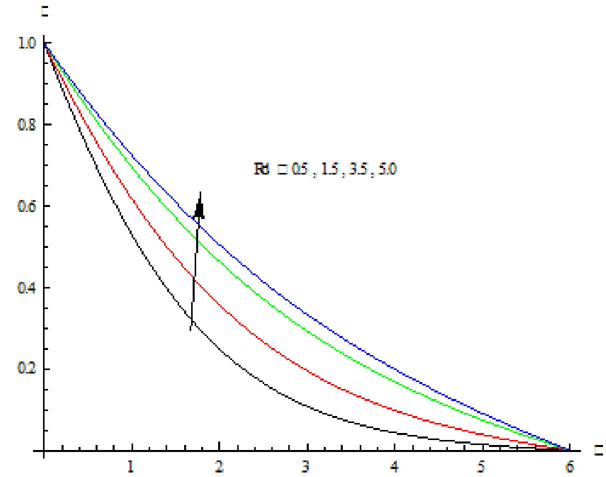

Fig.-6b: Variation of temperature (θ) with Rd
G=2, M=0.5, Q=0.5,β=0.2 ,a=0.1,ξ=π/4,θr=-2

Figs.6a-6b show the variation of velocity and temperature with the influence of radiation parameter (Rd). From fig.6a we find that the velocity reduces in the flow region (0.5) and enhances far away from the boundary. This means that the thickness of the momentum boundary layer reduces with increasing values of Rd. Fig.6b represents the temperature with Rd. It can be seen from the profiles that an increase in Rd leads to thickening of the thermal boundary layer which results in an enhancement of the temperature in the flow region.

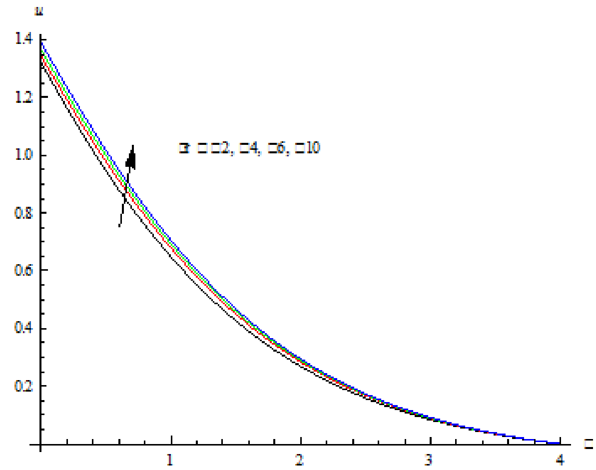

Fig.-7a: Variation of velocity (u) with θr
G=2, M=0.5, Q=0.5,β=0.2,Rd=0.5,a=0.1,ξ=π/4

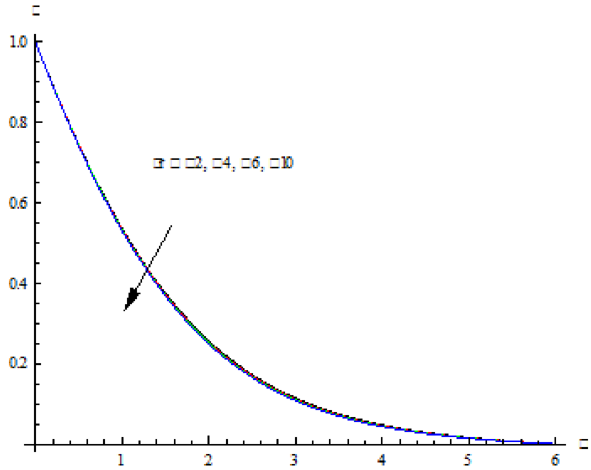

Fig.-7b: Variation of temperature (θ) with θr
G=2, M=0.5, Q=0.5,β=0.2,Rd=0.5,a=0.1,ξ=π/4

The variation of non-dimensional velocity and temperature profiles with η for different values of temperature dependent viscosity parameter(θr) is illustrated in figs.7a-7b. It is found that from fig.7a that the velocity of the fluid enhances in the g flow region. This can be explained physically as the parameter θr increases there is increment in the boundary layer thickness. From fig.7b we notice that the temperature profiles decreases with increasing values of θr. This can be attributed to the fact that an increase in θr de reduces the thickness of the thermal boundary layer which results in a depreciation of the temperature in the flow region.

Figs.8a-8b represent u and θ with amplitude of the wavy surface '*a*'. From fig.9a, we find that an increase in amplitude enhances the velocity in the entire flow region. The temperature increases with increase in '*a*' in the entire flow region.

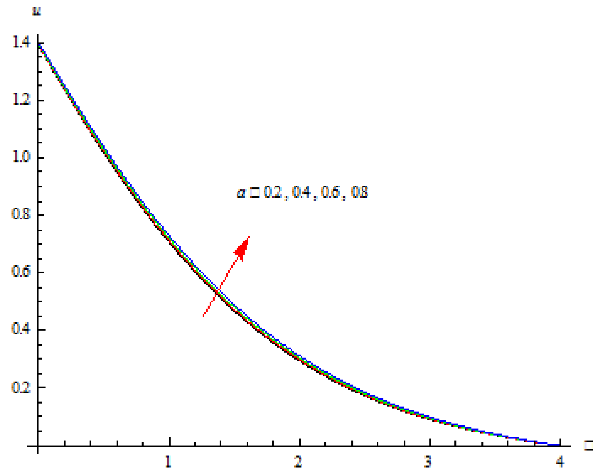

Fig.-8a: Variation of velocity (u) with a
G=2, M=0.5, Q=0.5,β=0.2,Rd=0.5, ξ=π/4,θr=-2

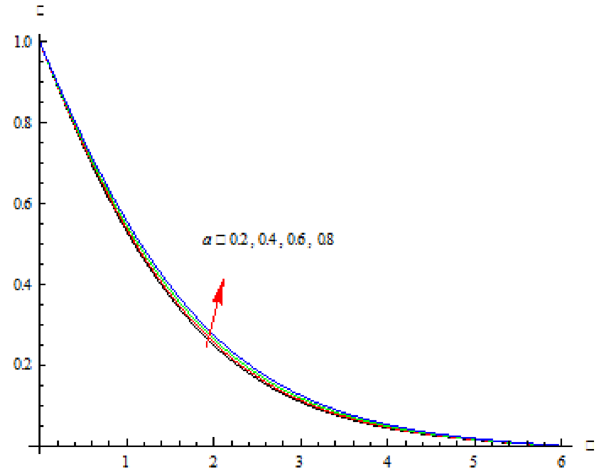

Fig.-8b: Variation of temperature (θ) with a
G=2, M=0.5, Q=0.5,β=0.2,Rd=0.5, ξ=π/4,θr=-2

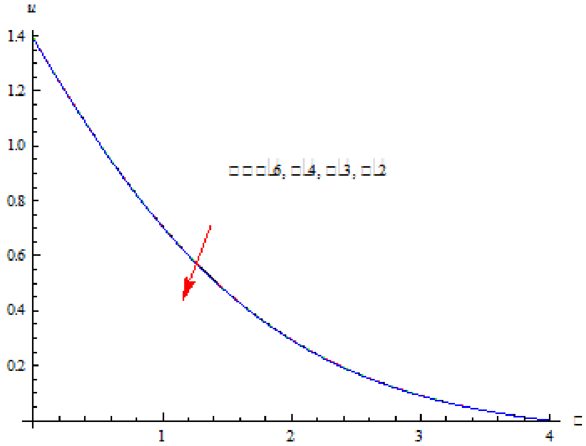

Fig.-9a: Variation of velocity (u) with ξ
G=2, M=0.5, Q=0.5,β=0.2,Rd=0.5,a=0.1, θr=-2

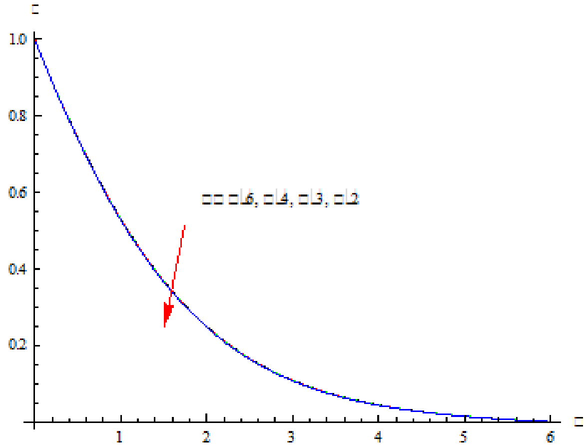

Fig.-9b: Variation of temperature (θ) with ξ
G=2, M=0.5, Q=0.5,β=0.2,Rd=0.5,a=0.1, θr=-2

Figs.9a-9b depict the variation of u and θ.with stream wise coordinate (ξ).It can be seen from the profiles that an increase in stream wise coordinate decreases both the velocity and temperature in the flow region

Nusselt Number at η = 0

The rate of heat transfer(Nu) at the wall is displayed in table.1.From the tabular values we find that rate of heat transfer at the wall increase with increase in Grashof number(G).An increase in the magnetic parameter(M) increases the Nusselt number at the wall .The Nusselt number reduces with increase in viscosity parameter(θr). The variation of Nu with heat source parameter (Q). From the values we find that Nu increases with increase in heat generating source and reduces with heat absorbing source. The Nusselt number reduces with increase in Rd. An increase in thermal conductivity parameter (β) reduces the Nusselt number at the wall. The variation of Nu with amplitude of the wavy surface (*a*) and stream wise coordinate (ξ) shows that Nu decreases with increase in '*a*' and enhances with 'ξ'.

Table – 1

Parameter		**Nu(0)**
M	0.5	0.56015
	1.0	0.55802
	1.5	0.5521
	2.0	0.5252
Q	0.5	0.56015
	1.5	0.85679
	-0.5	-0.2244
	-1.5	-0.1295
β	0.5	0.56015
	1.0	0.52195
	1.5	0.4399
	2.0	0.34394
Rd	0.5	0.56015
	1.5	0.42365
	3.5	0.30886
	5.0	0.2634
θr	-2	0.55301
	-4	0.51847
	-6	0.43856
	-10	0.34394
a	0.2	0.56015
	0.4	0.54851
	0.6	0.53296
	0.8	0.51487
ξ	π/6	0.55836
	π/4	0.56015
	π/3	0.56196
	π/2	0.56381

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