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COMPUTING CERTAIN TOPOLOGICAL INDICES FOR LINE GRAPHS OF TADPOLE GRAPHS, WHEEL GRAPH AND LADDER GRAPH

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ABSTRACT

Topological indices have been used as a special tools in mathematical chemistry to predict physico-chemical properties of certain chemical compounds. Recently various new topological indices have been conceived by researchers such as Sanskruti index $\mathsf{S}(G)$*, fifth* M_1 *and* M_2 *Zagreb indices, fifth* M_1 *and* M_2 *multiplicative Zagreb indices. In this paper, we give explicit formulae for tadpole graph, wheel graph and ladder graph and their corresponding line graphs for the above mentioned topological indices in terms of the order of the original graph* G.

1. INTRODUCTION AND PRELIMINARIES

Let G be a simple graph. The order of a graph is $|V(G)|$, its number of vertices denoted by n. The size of a graph is $|E(G)|$, its number of edges denoted by m. The degree of a vertex v, denoted by $d_G(v)$. The subdivision graph $S_1(G)$ is the graph attained from G by replacing each of its edges by a path of length 2. The line graph $L(G)$ of a graph is the graph derived from G in such a way that the edges in G are replaced by vertices in $L(G)$ and two vertices in $L(G)$ are connected whenever the corresponding edges in G are adjacent [4]. For any number d, we define $V_d = \{u \in V(G) \setminus s_G(u) = d\}$, in which $s_G(u) = \sum_{v \in N_G(u)} d_G(v)$ and

$N_G(u) = \{v \in V(G) \setminus uv \in E(G)\}$.

In structural chemistry and biology, molecular structure descriptors are utilized for modeling information of molecules, which are known as topological indices. Many topological indices are introduced to explain the physical and chemical properties of molecules. Topological Index is a numerical value associated with the molecular graph and is obtained from various graph parameters. One of them is degree based topological indices. A large number of such indices depend only on vertex degree of the molecular graph. One of them is the Sanskruti index, proposed by Hosamani [8]. Which is defined as follows:

$$\mathsf{S}(G) = \sum_{uv \in E(G)} \left(\frac{s_G(u) s_G(v)}{s_G(u) + s_G(v) - 2} \right)^3 . \tag{1}$$

Recently, the first neighborhood Zagreb index was introduced and studied by Basavangoud *et.al*, [1]

$$NM_1(G) = \sum_{u \in V(G)} S_G(u)^2 . \tag{2}$$

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The fifth M_1 and M_2 Zagreb indices were introduced by Graovac *et.al*, [2], defined as:

$$M_1G_5(G) = \sum_{uv\in E(G)} [S_G(u) + S_G(v)]. \tag{3}$$

$$M_2G_5(G) = \sum_{uv\in E(G)} [S_G(u) \cdot S_G(v)]. \tag{4}$$

The fifth multiplicative M_1 and M_2 Zagreb indices were proposed by Kulli in [9], defined as

$$M_1G_5\prod(G) = \prod_{uv\in E(G)} [S_G(u) + S_G(v)]. \tag{5}$$

$$M_2G_5\prod(G) = \prod_{uv\in E(G)} [S_G(u) \cdot S_G(v)]. \tag{6}$$

For more information on topological indices we refer the articles [3,5,6,10].

Now we define some notions of the graph theory. The subdivision graph $S(G)$ is the graph obtained from G by replacing each of its edge by a path of length 2. The line graph $L(G)$ of graph G is the graph whose vertices are the edges of G, two vertices e and f are incident if and only if they have a common end vertex in G. The tadpole graph $T_{n,k}$ is the graph obtained by joining a cycle of n vertices with a path of length k. A ladder graph L_n is obtained by taking cartesian product of two paths $P_n \times P_2$. A wheel graph W_n of order n composed of a vertex, which will be called the hub, adjacent to all vertices of a cycle of order n, i.e $W_n = C_{n-1} + K_1$.

2. TOPOLOGICAL INDICES OF THE TADPOLE GRAPH AND ITS LINE GRAPH

In 2011, Ranjini *et.al.* calculated the explicit expressions for the Shultz index of the subdivision graphs of the tadpole graph, wheel, helm and ladder graphs [13]. They also studied the Zagreb indices of the line graph of tadpole, wheel and ladder graphs with subdivision in [12]. In 2015, Su and Xu calculated the general sum-connectivity index and co-index for the line graph of tadpole, wheel and ladder graphs with subdivision [14]. In the same year (2015) M. F. Nadeem *et.al.* computed the ABC_4 and GA_5 indices for the same graphs [11].

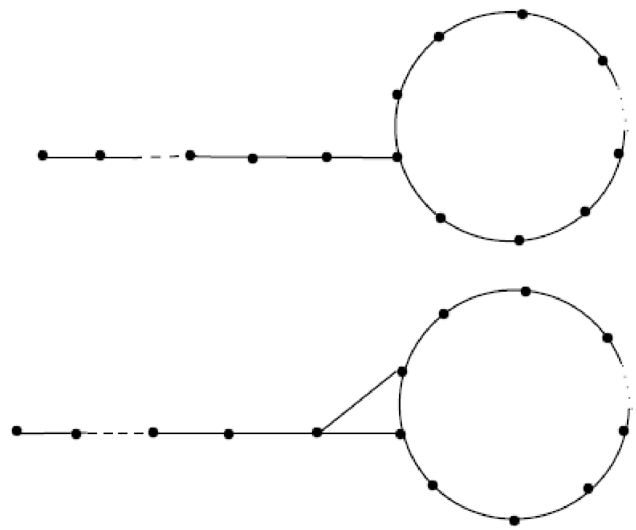

Fig.1. (a) The tadpole graph $T_{n,k}$; (b) the line graph $L(T_{n,k})$ of tadpole graph

Table-1: The edge partition of tadpole graph $T_{n,k}$.

(S_u, S_v) where $uv \in E(G)$	(4,4)	(5,6)	(3,2)
Number of edges	$n+k-4$	3	1

Table-2: The edge partition of line graph of tadpole graph $T_{n,k}$.

(S_u, S_v) where $uv \in E(G)$	(4,4)	(5,8)	(8,8)	(2,3)
Number of edges	$n+k-6$	3	3	1

Theorem 1:1 *Let* $G = T_{n,k}$ *be the tadpole graph. Then*

1. $S(G) = (n+k-4)(18.9572) + 119.1077$
2. $M_1G_5(G) = 8(n+k)+6$
3. $M_2G_5(G) = 16(n+k)+32$
4. $M_1G_5\prod(G) = 1320(n+k)-5280$
5. $M_2G_5\prod(G) = 8640(n+k)-34560$

Proof: Let G be a tadpole graph $T_{n,k}$ with $|V(G) = n+k| = E(G)$. Since G is unicyclic therefore G has same order and size. Now consider the following cases:

Case-1: Consider the Sanskruti index $S(G) = \sum_{uv\in E(G)} \left(\frac{s_G(u)s_G(v)}{s_G(u)+s_G(v)-2} \right)^3$. By using the information in Table 1, we have

$$\begin{aligned} S(G) &= \sum_{uv\in E(G)} \left(\frac{s_G(u)s_G(v)}{s_G(u)+s_G(v)-2} \right) \\ &= (n+k-4)\left(\frac{4\times 4}{4+4-2}\right)^3 + 3\left(\frac{5\times 6}{5+6-2}\right)^3 + 1\left(\frac{3\times 2}{3+2-2}\right)^3 \\ &= (n+k-4)(2.6666)^3 + 3(3.3333)^3 + 1(2)^3 \\ &= (n+k-4)(18.9572) + 119.1077. \end{aligned}$$

Case-2: Consider the fifth M_1 Zagreb Index, $M_1G_5(G) = \sum_{uv\in E(G)} [S_G(u)+S_G(v)]$. Again by using the information in Table 1, we have

$$\begin{aligned} M_1G_5(G) &= \sum_{uv\in E(G)} [S_G(u)+S_G(v)] \\ &= (n+k-4)(4+4) + 3(5+6) + 1(3+2) \\ &= 8(n+k)+6. \end{aligned}$$

Case-3: Consider the fifth M_2 Zagreb Index, $M_2G_5(G) = \sum_{uv\in E(G)} [S_G(u)S_G(v)]$. Again by using the information in Table 1, we have

$$\begin{aligned} M_1G_5(G) &= \sum_{uv\in E(G)} [S_G(u)S_G(v)] \\ &= (n+k-4)(4\times 4) + 3(5\times 6) + 1(3\times 2) \\ &= 16(n+k+2). \end{aligned}$$

Case-4: Consider the The fifth multiplicative M_1 Zagreb index $M_1G_5\prod(G) = \prod_{uv\in E(G)} [S_G(u)+S_G(v)]$. Using Table 1, we have

$$\begin{aligned} M_1G_5\prod(G) &= \prod_{uv\in E(G)} [S_G(u)+S_G(v)] \\ &= (n+k-4)(4+4)\times 3(5+6)\times 1(3+2) \\ &= 1320(n+k)-5280. \end{aligned}$$

Consider the The fifth multiplicative M_2 Zagreb index $M_2G_5\prod(G) = \prod_{uv\in E(G)}[S_G(u)S_G(v)]$. Using Table 1, we have

$$\begin{aligned}
M_2G_5\prod(G) &= \prod_{uv\in E(G)}[S_G(u)S_G(v)]\\
&= (n+k-4)(4\times 4)\times 3(5\times 6)\times 1(3\times 2)\\
&= 8640(n+k)(34560).
\end{aligned}$$

Theorem 2:2 *Let* $H = L(T_{n,k})$ *be the line graph of tadpole graph* $T_{n,k}$. *Then*

1. $S(H) = (18.9615)(n+k-6)+457.84$
2. $M_1G_5(H) = 8(n+k)+44$
3. $M_2G_5(H) = 16(n+k)+222$
4. $M_1G_5\prod(H) = 74880(n+k-6)$
5. $M_2G_5\prod(H) = (n+k-6)(2211840)$

Proof: Let G be a tadpole graph $T_{n,k}$ with $|V(G) = n+k| = E(G)$ and let $H = L(T_{n,k})$ be the line graph of tadpole graph. Now consider the following cases:

Case-1: Consider the Sanskruti index $S(G) = \sum_{uv\in E(G)}\left(\frac{s_G(u)s_G(v)}{s_G(u)+s_G(v)-2}\right)^3$. By using the information in Table 2, we have

$$\begin{aligned}
S(H) &= \sum_{uv\in E(G)}\left(\frac{s_G(u)s_G(v)}{s_G(u)+s_G(v)-2}\right)^3\\
&= (n+k-6)\left(\frac{4\times 4}{4+4-2}\right)^3 + 3\left(\frac{5\times 8}{5+8-2}\right)^3 + 3\left(\frac{8\times 8}{8+8-2}\right)^3 + 1\left(\frac{2\times 3}{2+3-2}\right)^3\\
&= (n+k-6)\left(\frac{16}{6}\right)^3 + 3\left(\frac{40}{11}\right)^3 + 3\left(\frac{64}{14}\right)^3 + 1\left(\frac{6}{3}\right)^3\\
&= (n+k-6)(2.6666)^3 + 3(3.6363)^3 + 3(4.5714)^3 + 1(3)^3\\
&= (n+k-6)(18.9615) + 457.84.
\end{aligned}$$

Case-2: Consider the fifth M_1 Zagreb Index, $M_1G_5(G) = \sum_{uv\in E(G)}[S_G(u)+S_G(v)]$. Again by using the information in Table 2, we have

$$\begin{aligned}
M_1G_5(G) &= \sum_{uv\in E(G)}[S_G(u)+S_G(v)]\\
&= (n+k-6)(4+4) + 3(5+8) + 3(8+8) + 1(2+3)\\
&= 8(n+k)+44.
\end{aligned}$$

Case-3: Consider the fifth M_2 Zagreb Index, $M_2G_5(G) = \sum_{uv\in E(G)}[S_G(u)S_G(v)]$. Again by using the information in Table 2, we have

$$\begin{aligned}
M_1G_5(G) &= \sum_{uv\in E(G)}[S_G(u)S_G(v)]\\
&= (n+k-6)(4\times 4) + 3(5\times 8) + 3(8\times 8) + 1(2\times 3)\\
&= 16(n+k)+222.
\end{aligned}$$

Case-4: Consider the The fifth multiplicative M_1 Zagreb index $M_1G_5\prod(G) = \prod_{uv\in E(G)}[S_G(u)+S_G(v)]$. Using Table 2, we have

$$M_1G_5\prod(G) = \prod_{uv\in E(G)}[S_G(u)+S_G(v)]$$
$$= (n+k-4)(4+4)\times 3(5+8)\times 3(8+8)\times 1(2+3)$$
$$= 74880(n+k-6)$$

Case-5: Consider the The fifth multiplicative M_2 Zagreb index $M_2G_5\prod(G) = \prod_{uv\in E(G)}[S_G(u)S_G(v)]$. Using Table 2, we have

$$M_2G_5\prod(G) = \prod_{uv\in E(G)}[S_G(u)S_G(v)]$$
$$= (n+k-4)(4\times 4)\times 3(5\times 8)\times 3(8\times 8)\times 1(2\times 3)$$
$$= (n+k-6)(2211840).$$

3. TOPOLOGICAL INDICES OF THE WHEEL GRAPH AND ITS LINE GRAPH

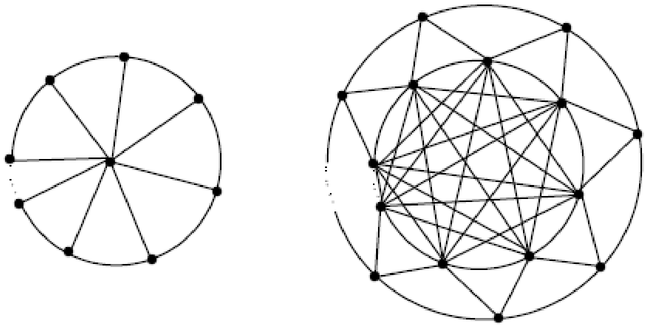

Fig.2. The Wheel graph W_n and its line graph $L(W_n)$

Table-3: The edge partition wheel graph W_n.

(S_u, S_v) where $uv \in E(G)$	$(n+5, n+5)$	$(n+5, 3(n-1))$
Number of edges	$n-1$	$n-1$

Table-4: The edge partition of line graph of wheel graph W_n.

(S_u, S_v) where $uv \in E(G)$	$(2n+8, 2n+8)$	$(2n+8, n(n-2)+8)$	$(n(n-2)+8, n(n-2)+8)$
Number of edges	$n-1$	$2(n-1)$	$\frac{(n-1)(n-2)}{2}$

Theorem 3:3 *Let* $G = W_n$ *be the wheel graph. Then*

1. $(G) = (n-1)\left[\frac{(n+5)^6}{(2n+8)^3}+\frac{[(n+5)(3n-3)]^3}{64n^3}\right]$
2. $M_1G_5(G) = 6(n-1)(n+2)$
3. $M_2G_5(G) = (n-1)(n+5)(4n+3)$
4. $M_1G_5\prod(G) = (n-1)^2(2n+10)(4n+2)$
5. $M_2G_5\prod(G) = (n-1)^2(n+5)^3(3n-3)$

Proof: Let $G = W_n$ be a wheel graph of order n. Now consider the following cases:

Case-1: Consider the Sanskruti index $\mathsf{S}(G) = \sum_{uv\in E(G)} \left(\frac{s_G(u)s_G(v)}{s_G(u)+s_G(v)-2} \right)^3$. By using the information in Table 3, we have

$$\begin{aligned}
\mathsf{S}(G) &= \sum_{uv\in E(G)} \left(\frac{s_G(u)s_G(v)}{s_G(u)+s_G(v)-2} \right)^3 \\
&= (n-1)\left(\frac{(n+5)(n+5)}{n+5+n+5-2} \right)^3 + (n-1)\left(\frac{(n+5)(3n-3)}{n+53n-3-2} \right)^3 \\
&= (n-1)\left[\frac{(n+5)^6}{(2n+8)^3} + \frac{[(n+5)(3n-3)]^3}{64n^3} \right].
\end{aligned}$$

Case-2: Consider the fifth M_1 Zagreb Index, $M_1G_5(G) = \sum_{uv\in E(G)} [S_G(u)+S_G(v)]$. Again by using the information in Table 3, we have

$$\begin{aligned}
M_1G_5(G) &= \sum_{uv\in E(G)} [S_G(u)+S_G(v)] \\
&= (n-1)(n+5+n+5)+(n-1)(n+5+3n-3) \\
&= 6(n-1)(n+2).
\end{aligned}$$

Case-3: Consider the fifth M_2 Zagreb Index, $M_2G_5(G) = \sum_{uv\in E(G)} [S_G(u)S_G(v)]$. Again by using the information in Table 3, we have

$$\begin{aligned}
M_1G_5(G) &= \sum_{uv\in E(G)} [S_G(u)S_G(v)] \\
&= (n+5)^2(n-1)+(n+5)(3n-2)(n-1) \\
&= (n-1)(n+5)(4n+3).
\end{aligned}$$

Case-4: Consider the The fifth multiplicative M_1 Zagreb index $M_1G_5\prod(G) = \prod_{uv\in E(G)} [S_G(u)+S_G(v)]$. Using Table 3, we have

$$\begin{aligned}
M_1G_5\prod(G) &= \prod_{uv\in E(G)} [S_G(u)+S_G(v)] \\
&= (n-1)(n+5+n+5)\times(n-1)(n+5+3n-3) \\
&= (n-1)^2(4n+2)(2n+10).
\end{aligned}$$

Case-5: Consider the The fifth multiplicative M_2 Zagreb index $M_2G_5\prod(G) = \prod_{uv\in E(G)} [S_G(u)S_G(v)]$. Using Table 3, we have

$$\begin{aligned}
M_2G_5\prod(G) &= \prod_{uv\in E(G)} [S_G(u)S_G(v)] \\
&= (n+5)(n+5)(n-1)\times(n-1)(n+5)(3n-3) \\
&= (3n-3)(n-1)^2(n+5)^3.
\end{aligned}$$

Theorem 4:4 *Let* $H = L(W_n)$ *be the line graph of wheel graph* W_n. *Then*

1. $(H) = \frac{(2n+8)^6(n-1)}{4n+14} + \frac{2(2n+8)^3(n^2-2n+8)^3(n-1)}{n^2+14} + \frac{(n^2-2n+8)^6(n-1)(n-2)}{2[(n^2-2n+8)-2]}$
2. $M_1G_5(H) = (4n+16)(n-1)+2(n^2+16)(n-1)+(n-1)(n-2)(n^2-2n+8)$
3. $M_2G_5(H) = (2n+8)^2(n-1)+2(2n+8)(n^2-2n+8)(n-1)+\frac{(n-1)(n-2)}{2}(n^2-2n+8)^2$

4. $M_1G_5\prod(H) = (n-1)(n-2)(n^2-2n+8)^2(n-1)^2(4n+16)(n^2+16)$
5. $M_2G_5\prod(H) = (n^2-2n+8)(n-1)(n-2)(n^2-2n+8)(2n+8)(n-1)(2n+8)^2(n-1)$

Proof: Let $G = W_n$ be a wheel graph of order n and let $H = L(W_n)$ be the line graph of wheel graph. Now consider the following cases:

Case-1: Consider the Sanskruti index $S(G) = \sum_{uv\in E(G)}\left(\frac{s_G(u)s_G(v)}{s_G(u)+s_G(v)-2}\right)^3$. By using the information in Table 4, we have

$$S(G) = \sum_{uv\in E(G)}\left(\frac{s_G(u)s_G(v)}{s_G(u)+s_G(v)-2}\right)^3$$
$$= (n-1)\left(\frac{(2n+8)^2}{2n+8+2n+8-2}\right)^3 + 2(n-1)\left(\frac{(2n+8)(n^2-2n+8)}{2n+8+n^2-2n+8-2}\right)^3$$
$$+\frac{(n-1)(n-2)}{2}\left(\frac{((n^2-2n+8)^2}{n^2-2n+8++n^2-2n+8-2}\right)^3$$
$$= \frac{(2n+8)^6(n-1)}{4n+14} + \frac{2(2n+8)^3(n^2-2n+8)^3(n-1)}{n^2+14} + \frac{(n^2-2n+8)^6(n-1)(n-2)}{2[(n^2-2n+8)-2]}.$$

Case-2: Consider the fifth M_1 Zagreb Index, $M_1G_5(G) = \sum_{uv\in E(G)}[S_G(u)+S_G(v)]$. Again by using the information in Table 4, we have

$$M_1G_5(G) = \sum_{uv\in E(G)}[S_G(u)+S_G(v)]$$
$$= (n-1)(2n+8+2n+8) + 2(n-1)(2n+8+n^2-2n+8)$$
$$+\frac{(n-1)(n-2)}{2}(n^2-2n+8+n^2-2n+8 \qquad)$$
$$= 4n+16)(n-1) + 2(n^2+16)(n-1) + (n-1)(n-2)(n^2-2n+8).$$

Case-3: Consider the fifth M_2 Zagreb Index, $M_2G_5(G) = \sum_{uv\in E(G)}[S_G(u)S_G(v)]$. Again by using the information in Table 4, we have

$$M_1G_5(G) = \sum_{uv\in E(G)}[S_G(u)S_G(v)]$$
$$= (n-1)(2n+8)^2 + 2(n-1)(2n+8)(n^2-2n+8) + \frac{(n-1)(n-2)}{2}(n^2-2n+8)^2.$$

Case-4: Consider the The fifth multiplicative M_1 Zagreb index $M_1G_5\prod(G) = \prod_{uv\in E(G)}[S_G(u)+S_G(v)]$. Using Table 4, we have

$$M_1G_5\prod(G) = \prod_{uv\in E(G)}[S_G(u)+S_G(v)]$$
$$= (n-1)(2n+8+2n+8)\times 2(n-1)(2n+8+n^2-2n+8)$$
$$\times\frac{(n-1)(n-2)}{2}(n^2-2n+8+n^2-2n+8 \qquad)$$
$$= (n-1)(n-2)(n^2-2n+8)^2(n-1)^2(4n+16)(n^2+16).$$

Case-5: Consider the The fifth multiplicative M_2 Zagreb index $M_2G_5\prod(G) = \prod_{uv\in E(G)}[S_G(u)S_G(v)]$. Using Table 4, we have

$$M_2G_5\prod(G) = \prod_{uv\in E(G)}[S_G(u)S_G(v)]$$
$$= (n-1)(2n+8)^2 \times 2(n-1)(2n+8)(n^2-2n+8)\times\frac{(n-1)(n-2)}{2}(n^2-2n+8)^2$$
$$= (n^2-2n+8)(n-1)(n-2)(n^2-2n+8)(2n+8)(n-1)(2n+8)^2(n-1).$$

4. TOPOLOGICAL INDICES OF THE LADDER GRAPH AND ITS LINE GRAPH

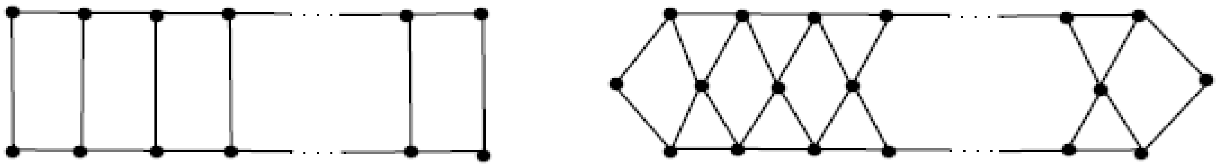

Fig. 3. The ladder graph $P_n \times P_2$ and its line graph $L(P_n \times P_2)$

Table-5: The edge partition of the ladder graph $P_n \times P_2; n \geq 4$.

(S_u, S_v) where $uv \in E(G)$	(5,5)	(5,8)	(8,9)	(8,8)	(9,9)
Number of edges	2	4	4	2	$3n-14$

Table-6: The edge partition of the line graph of ladder graph $P_n \times P_2; n \geq 6$.

(S_u, S_v) where $uv \in E(G)$	(6,10)	(10,15)	(10,14)	(14,15)	(15,16)	(16,16)
Number of edges	4	4	4	4	8	$6n-32$

Theorem 5:5 *Let* $G = P_n \times P_2$ *be the ladder graph. Then*

1. $S(G) = (129.7463)(3n-14)+886.7929$
2. $M_1G_5(G) = 54n-80$
3. $M_2G_5(G) = 243n-80$
4. $M_1G_5\prod(G) = 18(3n-14)(2263040)$
5. $M_2G_5\prod(G) = 81(3n-4)(294912000)$

Proof: Let $G = P_n \times P_2$ be a ladder graph with $|V(G)| = 2n$ and $|E(G)| = 3n-2$. Since it is easy to calculate the topological indices value for G up to $n = 3$. Therefore we consider $n \geq 4$. Now consider the following cases:

Case-1: Consider the Sanskruti index $S(G) = \sum_{uv\in E(G)}\left(\frac{s_G(u)s_G(v)}{s_G(u)+s_G(v)-2}\right)^3$. By using the information in Table 5, we have

$$S(G) = \sum_{uv\in E(G)}\left(\frac{s_G(u)s_G(v)}{s_G(u)+s_G(v)-2}\right)^3$$
$$= 2\left(\frac{5times5}{5+5-2}\right)^3 + 4\left(\frac{5times8}{5+8-2}\right)^3 + 4\left(\frac{8times9}{8+9-2}\right)^3 + 2\left(\frac{8times8}{8+8-2}\right)^3 + (3n-4\left(\frac{9times9}{9+9-2}\right)^3$$
$$= (129.7463)(3n-14)+886.7929.$$

Case-2: Consider the fifth M_1 Zagreb Index, $M_1G_5(G) = \sum_{uv\in E(G)}[S_G(u)+S_G(v)]$. Again by using the information in Table 5, we have

$$M_1G_5(G) = \sum_{uv\in E(G)}[S_G(u)+S_G(v)]$$
$$= 2(5+5)+4(5+8)+4(8+9)+2(8+8)+(3n-4)(9+9)$$
$$= 54n-80.$$

Case-3: Consider the fifth M_2 Zagreb Index, $M_2G_5(G) = \sum_{uv\in E(G)}[S_G(u)S_G(v)]$. Again by using the information in Table 5, we have

$$M_1G_5(G) = \sum_{uv\in E(G)}[S_G(u)S_G(v)]$$
$$= 2(5\times5)+4(5\times8)+4(8\times9)+2(8\times8)+(3n-4)(9\times9)$$
$$= 243n-508.$$

Case-4: Consider the The fifth multiplicative M_1 Zagreb index $M_1G_5\prod(G) = \prod_{uv\in E(G)}[S_G(u)+S_G(v)]$. Using Table 5, we have

$$M_1G_5\prod(G) = \prod_{uv\in E(G)}[S_G(u)+S_G(v)]$$
$$= 2(5+5)\times4(5+8)\times4(8+9)\times2(8+8)\times(3n-4)(9+9)$$
$$=18(3n-14)(2263040).$$

Case-5: Consider the The fifth multiplicative M_2 Zagreb index $M_2G_5\prod(G) = \prod_{uv\in E(G)}[S_G(u)S_G(v)]$. Using Table 5, we have

$$M_2G_5\prod(G) = \prod_{uv\in E(G)}[S_G(u)S_G(v)]$$
$$= 2(5\times5)\times4(5\times8)\times4(8\times9)\times2(8\times8)\times(3n-4)(9\times9)$$
$$= 81(3n-4)(294912000).$$

Theorem 6:6 *Let* $H = L(P_n\times P_2)$ *be the line graph of ladder graph* $P_n\times P_2$. *Then*

1. $S(H) = (621.3735)(6n-32)+8871.0265$
2. $M_1G_5(H) = 192n-375$
3. $M_2G_5(H) = 1536n-3882$
4. $M_1G_5\prod(H) = 248(6n-32)(71270400)$
5. $M_2G_5\prod(H) = (6n-32)(80640000)(412876800)$

Proof: Let $G = P_n\times P_2$ be a ladder graph with $|V(G)| = 2n$ and $|E(G)| = 3n-2$ and let $H = L(P_n\times P_2)$ be the line graph of $P_n\times P_2$. Since it is easy to calculate the topological indices value for G up to $n=3$. Therefore we consider $n\ge4$. Now consider the following cases:

Case-1: Consider the Sanskruti index $S(G) = \sum_{uv\in E(G)}\left(\frac{s_G(u)s_G(v)}{s_G(u)+s_G(v)-2}\right)^3$. By using the information in Table 6, we have

$$
\begin{aligned}
S(G) &= \sum_{uv\in E(G)}\left(\frac{s_G(u)s_G(v)}{s_G(u)+s_G(v)-2}\right)^3\\
&= 4\left(\frac{6times10}{6+10-2}\right)^3+4\left(\frac{10times15}{10+15-2}\right)^3+4\left(\frac{10times14}{10+14-2}\right)^3\\
&+4\left(\frac{14times15}{14+15-2}\right)^3+8\left(\frac{15times16}{15+16-2}\right)^3+(6n-32)\left(\frac{16times16}{16+16-2}\right)^3\\
&= (621.3735)(6n-32)+8871.0265.
\end{aligned}
$$

Case-2: Consider the fifth M_1 Zagreb Index, $M_1G_5(G) = \sum_{uv\in E(G)}[S_G(u)+S_G(v)]$. Again by using the information in Table 6, we have

$$
\begin{aligned}
M_1G_5(G) &= \sum_{uv\in E(G)}[S_G(u)+S_G(v)]\\
&= 4(6+10)+5(10+15)+4(10+14)+4(14+15)+8(15+16)+(6n-32)(16+16)\\
&= 192n-375.
\end{aligned}
$$

Case-3: Consider the fifth M_2 Zagreb Index, $M_2G_5(G) = \sum_{uv\in E(G)}[S_G(u)S_G(v)]$. Again by using the information in Table 6, we have

$$
\begin{aligned}
M_1G_5(G) &= \sum_{uv\in E(G)}[S_G(u)S_G(v)]\\
&= 4(6\times10)+5(10\times15)+4(10\times14)+4(14\times15)+8(15\times16)+(6n-32)(16\times16)\\
&= 1536n-3882.
\end{aligned}
$$

Case-4: Consider the The fifth multiplicative M_1 Zagreb index $M_1G_5\prod(G) = \prod_{uv\in E(G)}[S_G(u)+S_G(v)]$. Using Table 6, we have

$$
\begin{aligned}
M_1G_5\prod(G) &= \prod_{uv\in E(G)}[S_G(u)+S_G(v)]\\
&= 4(6+10)\times5(10+15)\times4(10+14)\times4(14+15)\\
&\times8(15+16)\times(6n-32)(16+16)\\
&= (248)(6n-32)(71270400).
\end{aligned}
$$

Case-5: Consider the The fifth multiplicative M_2 Zagreb index $M_2G_5\prod(G) = \prod_{uv\in E(G)}[S_G(u)S_G(v)]$. Using Table 5, we have

$$
\begin{aligned}
M_2G_5\prod(G) &= \prod_{uv\in E(G)}[S_G(u)S_G(v)]\\
&= 4(6\times10)\times5(10\times15)\times4(10\times14)\times4(14\times15)\times\ 8(15\times16)\times(6n-32)(16\times16)\\
&= (6n-32)(80640000)(412876800).
\end{aligned}
$$

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