ABSTRACT
Topological indices have been used as a special tools in mathematical chemistry to predict physico-chemical properties of certain chemical compounds. Recently various new topological indices have been conceived by researchers such as Sanskruti index $S(G)$, fifth $M_1$ and $M_2$ Zagreb indices, fifth $M_1$ and $M_2$ multiplicative Zagreb indices. In this paper, we give explicit formulae for tadpole graph, wheel graph and ladder graph and their corresponding line graphs for the above mentioned topological indices in terms of the order of the original graph $G$.

1. INTRODUCTION AND PRELIMINARIES
Let $G$ be a simple graph. The order of a graph is $|V(G)|$, its number of vertices denoted by $n$. The size of a graph is $|E(G)|$, its number of edges denoted by $m$. The degree of a vertex $v$, denoted by $d_G(v)$. The subdivision graph $S_1(G)$ is the graph attained from $G$ by replacing each of its edges by a path of length 2. The line graph $L(G)$ of a graph is the graph derived from $G$ in such a way that the edges in $G$ are replaced by vertices in $L(G)$ and two vertices in $L(G)$ are connected whenever the corresponding edges in $G$ are adjacent [4]. For any number $d$, we define $V_d = \{u \in V(G) \setminus s_G(u) = d\}$, in which $s_G(u) = \sum_{v \in N_G(u)} d_G(v)$ and $N_G(u) = \{v \in V(G) \setminus uv \in E(G)\}$.

In structural chemistry and biology, molecular structure descriptors are utilized for modeling information of molecules, which are known as topological indices. Many topological indices are introduced to explain the physical and chemical properties of molecules. Topological Index is a numerical value associated with the molecular graph and is obtained from various graph parameters. One of them is degree based topological indices. A large number of such indices depend only on vertex degree of the molecular graph. One of them is the Sanskruti index, proposed by Hosamani [8]. Which is defined as follows:

$$S(G) = \sum_{uv \in E(G)} \left(\frac{s_G(u)s_G(v)}{s_G(u) + s_G(v) - 2}\right)^3. \quad (1)$$

Recently, the first neighborhood Zagreb index was introduced and studied by Basavangoud et.al, [1]

$$NM_1(G) = \sum_{uv \in E(G)} s_G(u)^2. \quad (2)$$

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The fifth $M_1$ and $M_2$ Zagreb indices were introduced by Graovac et al. \cite{2}, defined as:

$$M_1 G_5(G) = \sum_{u,v \in E(G)} [S_G(u) + S_G(v)]. \quad (3)$$

$$M_2 G_5(G) = \sum_{u,v \in E(G)} [S_G(u) \cdot S_G(v)]. \quad (4)$$

The fifth multiplicative $M_1$ and $M_2$ Zagreb indices were proposed by Kulli in \cite{9}, defined as

$$M_1 G_5 \prod(G) = \prod_{u,v \in E(G)} [S_G(u) + S_G(v)]. \quad (5)$$

$$M_2 G_5 \prod(G) = \prod_{u,v \in E(G)} [S_G(u) \cdot S_G(v)]. \quad (6)$$

For more information on topological indices we refer the articles \cite{3,5,6,10}.

Now we define some notions of the graph theory. The subdivision graph $S(G)$ is the graph obtained from $G$ by replacing each of its edge by a path of length 2. The line graph $L(G)$ of graph $G$ is the graph whose vertices are the edges of $G$, two vertices $e$ and $f$ are incident if and only if they have a common end vertex in $G$. The tadpole graph $T_{n,k}$ is the graph obtained by joining a cycle of $n$ vertices with a path of length $k$. A ladder graph $L_n$ is obtained by taking cartesian product of two paths $P_n \times P_2$. A wheel graph $W_n$ of order $n$ composed of a vertex, which will be called the hub, adjacent to all vertices of a cycle of order $n$, i.e $W_n = C_{n-1} + K_1$.

2. TOPOLOGICAL INDICES OF THE TADPOLE GRAPH AND ITS LINE GRAPH

In 2011, Ranjini et al. calculated the explicit expressions for the Shultz index of the subdivision graphs of the tadpole graph, wheel, helm and ladder graphs \cite{13}. They also studied the Zagreb indices of the line graph of tadpole, wheel and ladder graphs with subdivision in \cite{12}. In 2015, Su and Xu calculated the general sum-connectivity index and co-index for the line graph of tadpole, wheel and ladder graphs with subdivision \cite{14}. In the same year (2015) M. F. Nadeem et al. computed the $ABC_4$ and $GA_5$ indices for the same graphs \cite{11}.

![Diagram of a tadpole graph and its line graph](image)

Fig. 1. (a) The tadpole graph $T_{n,k}$; (b) the line graph $L(T_{n,k})$ of tadpole graph

<table>
<thead>
<tr>
<th>Table-1: The edge partition of tadpole graph $T_{n,k}$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(S_u, S_v)$ where $uv \in E(G)$</td>
</tr>
<tr>
<td>(4,4) (5,6) (3,2)</td>
</tr>
<tr>
<td>Number of edges</td>
</tr>
<tr>
<td>$n + k - 4$</td>
</tr>
<tr>
<td>3 1</td>
</tr>
</tbody>
</table>
**Theorem 1:** Let \( G = T_{n,k} \) be the tadpole graph. Then

1. \( S(G) = (n+k-4)(18.9572) + 119.1077 \)
2. \( M_1 G_5(G) = 8(n+k) + 6 \)
3. \( M_2 G_5(G) = 16(n+k) + 32 \)
4. \( M_1 G_5 \prod (G) = 1320(n+k) - 5280 \)
5. \( M_2 G_5 \prod (G) = 8640(n+k) - 34560 \)

**Proof:** Let \( G \) be a tadpole graph \( T_{n,k} \) with \(|V(G)| = n+k = E(G)|. Since \( G \) is unicyclic therefore \( G \) has same order and size. Now consider the following cases:

**Case-1:** Consider the Sanskruti index \( S(G) = \sum_{uv \in E(G)} \left( \frac{s_G(u)s_G(v)}{s_G(u)+s_G(v)-2} \right)^3 \). By using the information in Table 1, we have

\[
S(G) = \sum_{uv \in E(G)} \left( \frac{s_G(u)s_G(v)}{s_G(u)+s_G(v)-2} \right) = (n+k-4) \left( \frac{4 \times 4}{5+6-2} \right) + 3 \left( \frac{5 \times 6}{5+6-2} \right) + 1 \left( \frac{3 \times 2}{3+2-2} \right) = (n+k-4)(2.6666) + 3(3.3333) + 1(2) = (n+k-4)(18.9572) + 119.1077.
\]

**Case-2:** Consider the fifth \( M_1 \) Zagreb Index, \( M_1 G_5(G) = \sum_{uv \in E(G)} [S_G(u) + S_G(v)] \). Again by using the information in Table 1, we have

\[
M_1 G_5(G) = \sum_{uv \in E(G)} [S_G(u) + S_G(v)] = (n+k-4)(4+4) + 3(5+6) + 1(3+2) = 8(n+k) + 6.
\]

**Case-3:** Consider the fifth \( M_2 \) Zagreb Index, \( M_2 G_5(G) = \sum_{uv \in E(G)} [S_G(u)S_G(v)] \). Again by using the information in Table 1, we have

\[
M_2 G_5(G) = \sum_{uv \in E(G)} [S_G(u)S_G(v)] = (n+k-4)(4 \times 4) + 3(5 \times 6) + 1(3 \times 2) = 16(n+k + 2).
\]

**Case-4:** Consider the The fifth multiplicative \( M_1 \) Zagreb index \( M_1 G_5 \prod (G) = \prod_{uv \in E(G)} [S_G(u) + S_G(v)] \). Using Table 1, we have

\[
M_1 G_5 \prod (G) = \prod_{uv \in E(G)} [S_G(u) + S_G(v)] = (n+k-4)(4+4) \times 3(5+6) \times 1(3+2) = 1320(n+k) - 5280.
\]
Consider the The fifth multiplicative $M_2$ Zagreb index $M_2 G_2 \prod(G) = \prod_{u \in E(G)} [S^S_G(u)S^S_G(v)]$. Using Table 1, we have

$$M_2 G_2 \prod(G) = \prod_{u \in E(G)} [S^S_G(u)S^S_G(v)] = (n + k - 4)(4 \times 4) \times 3(5 \times 6) \times 1(3 \times 2) = 8640(n + k)(34560).$$

**Theorem 2:** Let $H = L(T_{n,k})$ be the line graph of tadpole graph $T_{n,k}$. Then

1. $S(H) = (18.9615)(n + k - 6) + 457.84$
2. $M_1 G_3(H) = 8(n + k) + 44$
3. $M_2 G_3(H) = 16(n + k) + 222$
4. $M_1 G_5(H) = 74880(n + k - 6)$
5. $M_2 G_5(H) = (n + k - 6)(2211840)$

**Proof:** Let $G$ be a tadpole graph $T_{n,k}$ with $|V(G)| = n + k = E(G)$ and let $H = L(T_{n,k})$ be the line graph of tadpole graph. Now consider the following cases:

**Case-1:** Consider the Sanskruti index $S(G) = \sum_{u \in E(G)} \left( \frac{s^S_G(u) s^S_G(v)}{s^S_G(u) + s^S_G(v) - 2} \right)^3$. By using the information in Table 2, we have

$$S(H) = \sum_{u \in E(G)} \left( \frac{s^S_G(u) s^S_G(v)}{s^S_G(u) + s^S_G(v) - 2} \right)^3 = (n + k - 6)\left(\frac{4 \times 4}{4 + 4 - 2}\right)^3 + 3\left(\frac{5 \times 8}{5 + 8 - 2}\right)^3 + 3\left(\frac{8 \times 8}{8 + 8 - 2}\right)^3 + 1\left(\frac{2 \times 3}{2 + 3 - 2}\right)^3 = (n + k - 6)(16.6666)^3 + 3(3.6363)^3 + 3(4.5714)^3 + 3(3)^3 = (n + k - 6)(18.9615) + 457.84.$$

**Case-2:** Consider the fifth $M_1$ Zagreb Index, $M_1 G_5(G) = \sum_{u \in E(G)} [S_G(u) + S_G(v)]$. Again by using the information in Table 2, we have

$$M_1 G_5(G) = \sum_{u \in E(G)} [S_G(u) + S_G(v)] = (n + k - 6)(4 + 4) + 3(5 + 8) + 3(8 + 8) + 1(2 + 3) = 8(n + k) + 44.$$

**Case-3:** Consider the fifth $M_2$ Zagreb Index, $M_2 G_5(G) = \sum_{u \in E(G)} [S_G(u)S_G(v)]$. Again by using the information in Table 2, we have

$$M_2 G_5(G) = \sum_{u \in E(G)} [S_G(u)S_G(v)] = (n + k - 6)(4 \times 4) + 3(5 \times 8) + 3(8 \times 8) + 1(2 \times 3) = 16(n + k) + 222.$$
Case-4: Consider the The fifth multiplicative $M_1$ Zagreb index $M_1 G_5 \prod_{uv \in E(G)}(G) = \prod_{uv \in E(G)} [S_{G} (u) + S_{G} (v)]$. Using Table 2, we have

$$M_1 G_5 \prod_{uv \in E(G)}(G) = \prod_{uv \in E(G)} [S_{G} (u) + S_{G} (v)]$$

$$= (n + k - 4)(4 + 4) \times 3(5 + 8) \times 3(8 + 8) \times 1(2 + 3)$$

$$= 74880(n + k - 6)$$

Case-5: Consider the The fifth multiplicative $M_2$ Zagreb index $M_2 G_5 \prod_{uv \in E(G)}(G) = \prod_{uv \in E(G)} [S_{G} (u)S_{G} (v)]$. Using Table 2, we have

$$M_2 G_5 \prod_{uv \in E(G)}(G) = \prod_{uv \in E(G)} [S_{G} (u)S_{G} (v)]$$

$$= (n + k - 4)(4 \times 4) \times 3(5 \times 8) \times 3(8 \times 8) \times 1(2 \times 3)$$

$$= (n + k - 6)(2211840).$$

3. TOPOLOGICAL INDICES OF THE WHEEL GRAPH AND ITS LINE GRAPH

![Wheel graph and its line graph](image)

Table-3: The edge partition wheel graph $W_n$.

<table>
<thead>
<tr>
<th>$S_u, S_v$ where $uv \in E(G)$</th>
<th>$(n + 5, n + 5)$</th>
<th>$(n + 5, 3(n - 1))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of edges</td>
<td>$n - 1$</td>
<td>$n - 1$</td>
</tr>
</tbody>
</table>

Table-4: The edge partition of line graph of wheel graph $W_n$.

<table>
<thead>
<tr>
<th>$S_u, S_v$ where $uv \in E(G)$</th>
<th>$(2n + 8, 2n + 8)$</th>
<th>$(2n + 8, n(n - 2) + 8)$</th>
<th>$(n(n - 2) + 8, n(n - 2) + 8)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of edges</td>
<td>$n - 1$</td>
<td>$2(n - 1)$</td>
<td>$(n - 1)(n - 2)$</td>
</tr>
</tbody>
</table>

**Theorem 3:3** Let $G = W_n$ be the wheel graph. Then

1. $(G) = (n - 1) \left[ \frac{(n + 5)^6}{(2n + 8)^3} + \frac{(n + 5)(3n - 3)^3}{64n^3} \right]$
2. $M_1 G_5 (G) = 6(n - 1)(n + 2)$
3. $M_2 G_5 (G) = (n - 1)(n + 5)(4n + 3)$
4. $M_1 G_5 \prod (G) = (n - 1)^2(2n + 10)(4n + 2)$
5. $M_2 G_5 \prod (G) = (n - 1)^2(n + 5)^3(3n - 3)$

**Proof:** Let $G = W_n$ be a wheel graph of order $n$. Now consider the following cases:
Case-1: Consider the Sanskruti index $S(G) = \sum_{uv \in E(G)} \left( \frac{s_G(u)s_G(v)}{s_G(u) + s_G(v) - 2} \right)^3$. By using the information in Table 3, we have

$$S(G) = \sum_{uv \in E(G)} \left( \frac{s_G(u)s_G(v)}{s_G(u) + s_G(v) - 2} \right)^3 = (n-1) \left( \frac{(n+5)(n+5)}{n+5+n+5-2} \right)^3 + (n-1) \left( \frac{(n+5)(3n-3)}{n+5n+5-2} \right)^3$$

$$= (n-1) \left[ \frac{(n+5)^6}{(2n+8)^3} + \frac{[(n+5)(3n-3)]^3}{64n^3} \right].$$

Case-2: Consider the fifth $M_1$ Zagreb Index, $M_1G_3(G) = \sum_{uv \in E(G)} [S_G(u) + S_G(v)]$. Again by using the information in Table 3, we have

$$M_1G_3(G) = \sum_{uv \in E(G)} [S_G(u) + S_G(v)] = (n-1)(n+5+n+5) + (n-1)(n+5+3n-3) = 6(n-1)(n+2).$$

Case-3: Consider the fifth $M_2$ Zagreb Index, $M_2G_3(G) = \sum_{uv \in E(G)} [S_G(u)S_G(v)]$. Again by using the information in Table 3, we have

$$M_2G_3(G) = \sum_{uv \in E(G)} [S_G(u)S_G(v)] = (n+5)^2(n-1) + (n+5)(3n-2)(n-1) = (n-1)(n+5)(4n+3).$$

Case-4: Consider the fifth multiplicative $M_1$ Zagreb index $M_1G_3 \prod(G) = \prod_{uv \in E(G)} [S_G(u) + S_G(v)]$. Using Table 3, we have

$$M_1G_3 \prod(G) = \prod_{uv \in E(G)} [S_G(u) + S_G(v)] = (n-1)(n+5+n+5) \times (n-1)(n+5+3n-3) = (n-1)^2(4n+2)(2n+10).$$

Case-5: Consider the fifth multiplicative $M_2$ Zagreb index $M_2G_3 \prod(G) = \prod_{uv \in E(G)} [S_G(u)S_G(v)]$. Using Table 3, we have

$$M_2G_3 \prod(G) = \prod_{uv \in E(G)} [S_G(u)S_G(v)] = (n+5)(n+5)(n-1) \times (n-1)(n+5)(3n-3) = (3n-3)(n-1)^2(n+5)^3.$$

Theorem 4:4 Let $H = L(W_n)$ be the line graph of wheel graph $W_n$. Then

1. $(H) = \frac{(2n+8)^6(n-1)}{4n+14} + \frac{2(2n+8)^3(n^2-2n+8)(n-1)}{n^2+14} + \frac{(n^2-2n+8)(n-1)(n-2)}{2[(n^2-2n+8)-2]}$  
2. $M_1G_3(H) = (4n+16)(n-1) + 2(n^2+16)(n-1) + (n-1)(n-2)(n^2-2n+8)$  
3. $M_2G_3(H) = (2n+8)^2(n-1) + 2(2n+8)(n^2-2n+8)(n-1) + \frac{(n-1)(n-2)}{2}(n^2-2n+8)^2$
4. \( M_1G_3 \prod (H) = (n-1)(n-2)(n^2 - 2n + 8)^2(n-1)^2(4n+16)(n^2 + 16) \)
5. \( M_2G_3 \prod (H) = (n^2 - 2n + 8)(n-1)(n-2)(n^2 - 2n + 8)(2n+8)(n-1)(2n+8)^2(n-1) \)

**Proof:** Let \( G = W_n \) be a wheel graph of order \( n \) and let \( H = L(W_n) \) be the line graph of wheel graph. Now consider the following cases:

**Case-1:** Consider the Sanskruti index \( S(G) = \sum_{uv \in E(G)} \left( \frac{s_G(u)s_G(v)}{s_G(u) + s_G(v) - 2} \right)^3 \). By using the information in Table 4, we have

\[
S(G) = \sum_{uv \in E(G)} \left( \frac{s_G(u)s_G(v)}{s_G(u) + s_G(v) - 2} \right)^3 \\
= (n-1)\left( \frac{(2n+8)^2}{2n+8+2n+8-2} \right)^3 + 2(n-1)\left( \frac{(2n+8)(n^2 - 2n + 8)}{2n+8+n^2 - 2n + 8 - 2} \right)^3 \\
+ \frac{(n-1)(n-2)}{2} \left( \frac{(n^2 - 2n + 8)^2}{n^2 - 2n + 8 + n^2 - 2n + 8 - 2} \right)^3 \\
= \frac{(2n+8)^6(n-1)}{4n+14} + \frac{2(2n+8)^3(n^2 - 2n + 8)^3(n-1)}{n^2 + 14} + \frac{(n^2 - 2n + 8)^6(n-1)(n-2)}{2([n^2 - 2n + 8] - 2)}.
\]

**Case-2:** Consider the fifth \( M_1 \) Zagreb Index, \( M_1G_3(G) = \sum_{uv \in E(G)} [S_G(u) + S_G(v)] \). Again by using the information in Table 4, we have

\[
M_1G_3(G) = \sum_{uv \in E(G)} [S_G(u) + S_G(v)] \\
= (n-1)(2n+8 + 2n+8) + 2(n-1)(2n+8 + n^2 - 2n + 8) \\
+ \frac{(n-1)(n-2)}{2} (n^2 - 2n + 8 + n^2 - 2n + 8) \\
= 4n+16)(n-1) + 2(n^2 + 16)(n-1) + (n-1)(n-2)(n^2 - 2n + 8).
\]

**Case-3:** Consider the fifth \( M_2 \) Zagreb Index, \( M_2G_3(G) = \sum_{uv \in E(G)} [S_G(u)S_G(v)] \). Again by using the information in Table 4, we have

\[
M_2G_3(G) = \sum_{uv \in E(G)} [S_G(u)S_G(v)] \\
= (n-1)(2n+8)^2 + 2(n-1)(2n+8)(n^2 - 2n + 8) + \frac{(n-1)(n-2)}{2} (n^2 - 2n + 8)^2.
\]

**Case-4:** Consider the The fifth multiplicative \( M_1 \) Zagreb index \( M_1G_3 \prod (G) = \prod_{uv \in E(G)} [S_G(u) + S_G(v)] \). Using Table 4, we have

\[
M_1G_3 \prod (G) = \prod_{uv \in E(G)} [S_G(u) + S_G(v)] \\
= (n-1)(2n+8 + 2n+8) \times 2(n-1)(2n+8 + n^2 - 2n + 8) \\
\times \frac{(n-1)(n-2)}{2} (n^2 - 2n + 8 + n^2 - 2n + 8) \\
= (n-1)(n-2)(n^2 - 2n + 8)^2(n-1)^2(4n+16)(n^2 + 16).
\]

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Case-5: Consider the The fifth multiplicative $M_2$ Zagreb index $M_2G_5\prod(G) = \prod_{u,v \in E(G)} [S_G(u)S_G(v)]$. Using Table 4, we have

$$M_2G_5\prod(G) = \prod_{u,v \in E(G)} [S_G(u)S_G(v)]$$

$$= (n-1)(2n+8)^2 \times (n-1)(2n+8)(n^2 - 2n + 8) \times \frac{(n-1)(n-2)}{2} (n^2 - 2n + 8)^2$$

$$= (n^2 - 2n + 8)(n-1)(n-2)(n^2 - 2n + 8)(2n+8)(n-1)(2n+8)^2(n-1).$$

4. TOPOLOGICAL INDICES OF THE LADDER GRAPH AND ITS LINE GRAPH

![Ladder Graph and Line Graph](image)

**Case-1:** Consider the Sanskruti index $S(G) = \sum_{u,v \in E(G)} \left( \frac{s_G(u)s_G(v)}{s_G(u) + s_G(v) - 2} \right)^3$. By using the information in Table 5, we have

$$S(G) = \sum_{u,v \in E(G)} \left( \frac{s_G(u)s_G(v)}{s_G(u) + s_G(v) - 2} \right)^3$$

$$= 2 \left( \frac{5 \times 5}{5+5-2} \right)^3 + 4 \left( \frac{5 \times 8}{5+8-2} \right)^3 + 4 \left( \frac{8 \times 9}{8+9-2} \right)^3 + 2 \left( \frac{8 \times 8}{8+8-2} \right)^3 + (3n-4) \left( \frac{9 \times 9}{9+9-2} \right)^3$$

$$= (129.7463)(3n-14) + 886.7929.$$
Case-2: Consider the fifth $M_1$ Zagreb Index, $M_1G_3(G) = \sum_{uv \in E(G)} [S_G(u) + S_G(v)]$. Again by using the information in Table 5, we have
\[
M_1G_3(G) = \sum_{uv \in E(G)} [S_G(u) + S_G(v)] \\
= 2(5 + 5) + 4(5 + 8) + 4(8 + 9) + 2(8 + 8) + (3n - 4)(9 + 9) \\
= 54n - 80.
\]

Case-3: Consider the fifth $M_2$ Zagreb Index, $M_2G_3(G) = \sum_{uv \in E(G)} [S_G(u)S_G(v)]$. Again by using the information in Table 5, we have
\[
M_2G_3(G) = \sum_{uv \in E(G)} [S_G(u)S_G(v)] \\
= 2(5(5) + 4(5)(8) + 4(8)(9) + 2(8)(8) + (3n - 4)(9)(9) \\
= 243n - 508.
\]

Case-4: Consider the fifth multiplicative $M_1$ Zagreb index $M_1G_3(\prod(G)) = \prod_{uv \in E(G)} [S_G(u) + S_G(v)]$. Using Table 5, we have
\[
M_1G_3(\prod(G)) = \prod_{uv \in E(G)} [S_G(u) + S_G(v)] \\
= 2(5(5) + 4(5)(8) + 4(8)(9) + 2(8)(8) + (3n - 4)(9)(9) \\
= 243n - 508.
\]

Case-5: Consider the fifth multiplicative $M_2$ Zagreb index $M_2G_3(\prod(G)) = \prod_{uv \in E(G)} [S_G(u)S_G(v)]$. Using Table 5, we have
\[
M_2G_3(\prod(G)) = \prod_{uv \in E(G)} [S_G(u)S_G(v)] \\
= 2(5(5) + 4(5)(8) + 4(8)(9) + 2(8)(8) + (3n - 4)(9)(9) \\
= 81(3n - 4)(294912000).
\]

Theorem 6.6: Let \( H = L(P_n \times P_2) \) be the line graph of ladder graph \( P_n \times P_2 \). Then

1. \( S(H) = (621.3735)(6n - 32) + 8871.0265 \)
2. \( M_1G_5(H) = 192n - 375 \)
3. \( M_2G_5(H) = 1536n - 3882 \)
4. \( M_1G_5(\prod(H)) = 248(6n - 32)(71270400) \)
5. \( M_2G_5(\prod(H)) = (6n - 32)(80640000)(412876800) \)

Proof: Let \( G = P_n \times P_2 \) be a ladder graph with \( |V(G)| = 2n \) and \( |E(G)| = 3n - 2 \) and let \( H = L(P_n \times P_2) \) be the line graph of \( P_n \times P_2 \). Since it is easy to calculate the topological indices value for \( G \) up to \( n = 3 \). Therefore we consider \( n \geq 4 \). Now consider the following cases:

Case-1: Consider the Sanskruti index \( S(G) = \sum_{uv \in E(G)} \left( \frac{s_G(u)s_G(v)}{s_G(u) + s_G(v) - 2} \right)^3 \). By using the information in Table 6, we have
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Computing Certain Topological Indices for Line Graphs of Tadpole Graphs, wheel Graph and Ladder… / IJMA- 10(12), Dec.-2019.

$$S(G) = \sum_{uv \in E(G)} \left( \frac{s_{G}(u)s_{G}(v)}{s_{G}(u) + s_{G}(v) - 2} \right)^3$$

$$= 4 \left( \frac{6 \times 10^{10}}{6 + 10 - 2} \right)^3 + 4 \left( \frac{10 \times 15}{10 + 15 - 2} \right)^3 + 4 \left( \frac{14 \times 15}{10 + 14 - 2} \right)^3$$

$$+ 4 \left( \frac{14 \times 15}{14 + 15 - 2} \right)^3 + 8 \left( \frac{15 \times 16}{15 + 16 - 2} \right)^3 + (6n - 32) \left( \frac{16 \times 16}{16 + 16 - 2} \right)^3$$

$$= (621.3735)(6n - 32) + 8871.0265.$$  

**Case-2:** Consider the fifth $M_1$ Zagreb Index, $M_1G_2(G) = \sum_{uv \in E(G)} [S_{G}(u) + S_{G}(v)]$. Again by using the information in Table 6, we have

$$M_1G_2(G) = \sum_{uv \in E(G)} [S_{G}(u) + S_{G}(v)]$$

$$= 4(6 + 10) + 5(10 + 15) + 4(10 + 14) + 4(14 + 15) + 8(15 + 16) + (6n - 32)(16 + 16)$$

$$= 192n - 375.$$  

**Case-3:** Consider the fifth $M_2$ Zagreb Index, $M_2G_5(G) = \sum_{uv \in E(G)} [S_{G}(u)S_{G}(v)]$. Again by using the information in Table 6, we have

$$M_2G_5(G) = \sum_{uv \in E(G)} [S_{G}(u)S_{G}(v)]$$

$$= 4(6 \times 10) + 5(10 \times 15) + 4(10 \times 14) + 4(14 \times 15) + 8(15 \times 16) + (6n - 32)(16 \times 16)$$

$$= 1536n - 3882.$$  

**Case-4:** Consider the The fifth multiplicative $M_1$ Zagreb index $M_1G_5 \prod(G) = \prod_{uv \in E(G)} [S_{G}(u) + S_{G}(v)]$. Using Table 6, we have

$$M_1G_5 \prod(G) = \prod_{uv \in E(G)} [S_{G}(u) + S_{G}(v)]$$

$$= 4(6 + 10) \times 5(10 + 15) \times 4(10 + 14) \times 4(14 + 15) \times 8(15 + 16) \times (6n - 32)(16 + 16)$$

$$= (248)(6n - 32)(71270400).$$  

**Case-5:** Consider the The fifth multiplicative $M_2$ Zagreb index $M_2G_5 \prod(G) = \prod_{uv \in E(G)} [S_{G}(u)S_{G}(v)]$. Using Table 5, we have

$$M_2G_5 \prod(G) = \prod_{uv \in E(G)} [S_{G}(u)S_{G}(v)]$$

$$= 4(6 \times 10) \times 5(10 \times 15) \times 4(10 \times 14) \times 4(14 \times 15) \times 8(15 \times 16) \times (6n - 32)(16 \times 16)$$

$$= (6n - 32)(80640000)(412876800).$$  

REFERENCES


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