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# LOWER ANTI Q-FUZZY GROUP AND ITS LOWER LEVEL SUBGROUPS

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*ABSTRACT*

*In this paper, we define the algebraic structures of a lower anti Q-fuzzy subgroup and some related properties are investigated. We establish the relation between upper Q-fuzzy group and lower anti Q-fuzzy group of a group. The purpose of this study is to implement the fuzzy set theory and group theory in upper Q-fuzzy groups and lower anti Q-fuzzy groups. Characterizations of lower level subsets of a lower anti Q-fuzzy group of a group are given. We also discussed the relation between a given a lower anti Q-fuzzy group and its lower level sub groups and investigate the conditions under which a given group has a properly inclusive chain of sub groups. In particular, we formulate how to structure a lower anti Q-fuzzy group by a given chain of sub groups.*

***Keywords:*** *Fuzzy set, Q-fuzzy set, fuzzy subgroup, Q-fuzzy subgroup, anti-Q fuzzy subgroup, upper and lower anti-Q fuzzy subgroup, lower level subsets .*

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## 1. INTRODUCTION

The notion of fuzzy sets was introduced by L.A. Zadeh [13]. Fuzzy set theory has been developed in many directions by many researchers and has evoked great interest among mathematicians working in different fields of mathematics. In 1971, Rosenfield [10] introduced the concept of fuzzy subgroup. R. Biswas [1] introduced the concept of anti- fuzzy subgroups of groups. K. H. Kim [5] introduced the concept of intuitionistic Q-fuzzy semi prime ideals in semi groups and Osman kazanci, sultan yamark and serife yilmaz [8] introduced the concept of intuitionistic Q-fuzzy R-subgroups of near rings. A.Solairaju and R.Nagarajan[12] introduced and defined a new algebraic structure of Q-fuzzy groups. R. Muthuraj, M.S. Muthuraman, P. M.Sithar selvam [7] introdurced the concept of anti Q-fuzzy groups. In this paper we define a new algebraic structure of lower anti Q-fuzzy subgroups and study some their related properties.

## 2. PRELIMINARIES

In this section we site the fundamental definitions that will be used in the sequel.

**2.1 Definition: [13]** Let S be any non empty set. A fuzzy subset A of S is a function A: S → [0, 1].

**2.2 Definition: [10]** Let G be a group. A fuzzy subset A of G is called a fuzzy subgroup if for x, y ∈ G,

i. $A(xy) \geq \min \{ A(x) , A(y)\}$,
ii. $A(x^{-1}) = A(x)$.

**2.3 Definition: [1]** Let G be a group. A fuzzy subset A of G is called an anti fuzzy subgroup if for x, y ∈ G,

i. $A(xy) \leq \max \{ A(x) , A(y)\}$,
ii. $A(x^{-1}) = A(x)$.

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**2.4 Definition [12]** Let Q and G be any two sets. A mapping A: G x Q → [0, 1] is called a Q–fuzzy set in G.

**2.5 Definition [12]** A Q-fuzzy set 'A' is called Q-fuzzy group of a group G if for x, y ∈ G, q∈ Q,

i. $A(xy, q) \geq \min\{A(x, q), A(y, q)\}$
ii. $A(x^{-1}, q) = A(x, q)$ .

**2.6 Definition:** A Q-fuzzy set 'A' is called an upper Q-fuzzy group of a group G if for x, y ∈ G , q∈ Q,

i. $A(xy, q) \geq \min\{A(x, q), A(y, q)\}$ ,
ii. $A(x^{-1}, q) = A(x, q)$ ,
iii. $A(e, q) = 1$.

**2.7 Definition: [7]** A Q-fuzzy set 'A' is called an anti Q-fuzzy group of a group G if for x, y ∈ G, q∈ Q,

i. $A(xy, q) \leq \max\{A(x, q), A(y, q)\}$
ii. $A(x^{-1}, q) = A(x, q)$ .

**2.8 Definition:** A Q-fuzzy set 'A' is called a lower anti Q-fuzzy group of a group G if for x, y ∈ G, q∈ Q,

i. $A(xy, q) \leq \max\{A(x, q), A(y, q)\}$ ,
ii. $A(x^{-1}, q) = A(x, q)$ ,
iii. $A(e, q) = 0$.

**2.9 Definition:** An lower anti Q-fuzzy group of a group G is said to be Normal if $A(xy, q) = A(yx, q)$ for x, y ∈ G, q∈ Q.

**3. PROPERTIES OF LOWER ANTI Q-FUZZY SUBGROUPS**

In this section, we discuss some of the properties of lower anti Q-fuzzy subgroups.

**3.1 Theorem:** Let 'A' be an lower anti Q-fuzzy subgroup of a group G then

i. $A(x, q) \geq 0$ for all x ∈ G, q ∈ Q.
ii. The subset $H = \{x \in G \,/\, A(x, q) = 0\}$ is a subgroup of G.

**Proof:**

i. Let x∈ G and q ∈ Q.

$$
\begin{aligned}
A(x, q) &= \max\{A(x, q), A(x, q)\} \\
&= \max\{A(x, q), A(x^{-1}, q)\} \\
&\geq A(xx^{-1}, q) \\
&= A(e, q). \\
A(x, q) &\geq 0.
\end{aligned}
$$

ii. Let $H = \{x \in G \,/\, A(x, q) = 0\}$.

Clearly H is non-empty as e∈ H. Let x , y ∈ H.

Then, $A(x, q) = A(y, q) = 0$.

$$
\begin{aligned}
A(xy^{-1}, q) &\leq \max\{A(x, q), A(y^{-1}, q)\} \\
&= \max\{A(x, q), A(y, q)\} \\
&= \max\{0, 0\} \\
&= 0.
\end{aligned}
$$

That is, $A(xy^{-1}, q) \leq 0$ and obviously $A(xy^{-1}, q) \geq A(e, q) = 0$.

Hence, $A(xy^{-1}, q) = 0$ and $xy^{-1} \in H$.

Clearly, H is a subgroup of G.

**3.2 Theorem:** If 'A' is an upper Q-fuzzy subgroup of G, iff $A^C$ is a lower anti Q-fuzzy subgroup of G.

**Proof:** Suppose A is an upper Q-fuzzy subgroup of G. Then for all x, y ∈ G and q ∈ Q ,

$$
\begin{aligned}
& A(xy, q) \geq \min\{A(x, q), A(y, q)\} \\
\Leftrightarrow\ & 1 - A^c(xy, q) \geq \min\{(1 - A^c(x, q)), (1 - A^c(y, q))\} \\
\Leftrightarrow\ & A^c(xy, q) \leq 1 - \min\{(1 - A^c(x, q)), (1 - A^c(y, q))\}
\end{aligned}
$$

$\Leftrightarrow \; A^c(xy, q) \leq \max\{A^c(x, q), A^c(y, q)\}$.

We have, $A(x, q) = A(x^{-1}, q)$ for all x in G and $q \in Q$,

$\Leftrightarrow \; 1 - A^c(x, q) = 1 - A^c(x^{-1}, q)$ .

Therefore, $A^c(x, q) = A^c(x^{-1}, q)$.

Also, $A^c(e, q) = 1 - A(e, q) = 0$

Hence $A^c$ is a lower anti Q-fuzzy subgroup of G.

**3.3 Theorem:** Let A be any lower anti Q-fuzzy subgroup of a group G with identity e. Then $A(xy^{-1}, q) = 0 \Rightarrow A(x, q) = A(y, q)$ for all x, y in G and $q \in Q$.

**Proof:** Given A is a lower anti Q-fuzzy subgroup of G and $A(xy^{-1}, q) = 0$ .
Then for all x, y in G and $q \in Q$,

$$\begin{aligned} A(x, q) &= A(x(y^{-1}y), q) \\ &= A((xy^{-1})y, q) \\ &\leq \max\{A(xy^{-1}, q), A(y, q)\} \\ &= \max\{0, A(y, q)\} \\ &= A(y, q). \end{aligned}$$

That is, $A(x, q) \leq A(y, q)$.

Now, $A(y, q) = A(y^{-1}, q)$ , since A is a lower anti Q-fuzzy subgroup of G.

$$\begin{aligned} &= A(ey^{-1}, q) \\ &= A((x^{-1}x)y^{-1}, q) \\ &= A(x^{-1}(xy^{-1}), q) \\ &\leq \max\{A(x^{-1}, q), A(xy^{-1}, q)\} \\ &= \max\{A(x, q), 0\} \\ &= A(x, q). \end{aligned}$$

That is, $A(y, q) \leq A(x, q)$.

Hence, $A(x, q) = A(y, q)$.

**3.4 Theorem:** A is a lower anti Q-fuzzy subgroup of a group G if and only if $A(xy^{-1}, q) \leq \max\{A(x, q), A(y, q)\}$, for all x, y in G and $q \in Q$.

**Proof:** Let A be a lower anti Q-fuzzy subgroup of a group G. Then for all x ,y in G and $q \in Q$,

$A(xy, q) \leq \max\{A(x, q), A(y, q)\}$ and $A(x, q) = A(x^{-1}, q)$ with $A(e, q) = 0$.

Now, $A(xy^{-1}, q) \leq \max\{A(x, q), A(y^{-1}, q)\}$.

$= \max\{A(x, q), A(y, q)\}$

$\Leftrightarrow \; A(xy^{-1}, q) \leq \max\{A(x, q), A(y, q)\}$.

## 4. PROPERTIES OF LOWER LEVEL SUBSETS OF A LOWER ANTI Q-FUZZY SUBGROUP

In this section, we introduce the concept of lower level subset of a lower anti Q-fuzzy subgroup and discuss some of its properties.

**4.1 Definition:** Let A be a lower anti Q-fuzzy group of a group G. For any $t \in [0, 1]$, we define the lower level subset of A is the set, $L(A; t) = \{x \in G \,/\, A(x, q) \leq t \text{ and } q \in Q\}$.

**4.1 Theorem:** Let A be a lower anti Q-fuzzy subgroup of a group G. Then for $t \in [0, 1]$ such that $t \geq 0$, L (A; t) is a subgroup of G.

**Proof:** L (A; 0) is non empty as $e \in L(A; 0)$.

For all $x, y \in L(A; t)$ and $q \in Q$, we have, $A(x, q) \leq t$ ; $A(y, q) \leq t$.

Now, $A(xy^{-1}, q) \leq \max\{A(x, q), A(y, q)\}$.

$A(xy^{-1}, q) \leq \max\{t, t\}$.

$A(xy^{-1}, q) \leq t$.

$x y^{-1} \in L(A; t)$.

Hence L (A; t) is a subgroup of G.

**4.2 Theorem:** Let G be a group and A be a Q-fuzzy subset of G such that L (A; t) is a subgroup of G. For $t \in [0, 1]$ such that $t \geq 0$, A is a lower anti Q-fuzzy subgroup of G.

**Proof:** Let x, y in G and $q \in Q$, let $A(x, q) = t_1$ and $A(y, q) = t_2$.

Suppose $t_1 < t_2$, then $x, y \in L(A; t_2)$.

As L (A; $t_2$) is a subgroup of G, $x y^{-1} \in L(A; t_2)$.

Hence,
$$A(x y^{-1}, q) \leq t_2 = \max\{t_1, t_2\} \leq \max\{A(x, q), A(y, q)\}$$

That is, $A(x y^{-1}, q) \leq \max\{A(x, q), A(y, q)\}$.

By Theorem 3.4, A is a lower anti Q-fuzzy subgroup of G.

**4.2 Definition:** Let A be a lower anti Q-fuzzy subgroup of a group G. The subgroups L (A; t) for $t \in [0, 1]$ and $t \geq 0$, are called lower level subgroups of A.

**4.3 Theorem:** Let A be a lower anti Q-fuzzy subgroup of a group G. If two lower level subgroups ( A ; $t_1$), L ( A ; $t_2$), for, $t_1, t_2 \in [0,1]$ and $t_1, t_2 \geq 0$ with $t_1 < t_2$ of A are equal then there is no x in G such that $t_1 < A(x, q) \leq t_2$.

**Proof:** Let $L(A; t_1) = L(A; t_2)$.

Suppose there exists a $x \in G$ such that $t_1 < A(x, q) \leq t_2$ then $L(A; t_1) \subseteq L(A; t_2)$.

Then $x \in L(A; t_2)$, but $x \notin L(A; t_1)$, which contradicts the assumption that, $L(A; t_1) = L(A; t_2)$.Hence there is no x in G such that $t_1 < A(x, q) \leq t_2$.

Conversely, suppose that there is no x in G such that $t_1 < A(x, q) \leq t_2$.

Then, by definition, $L(A; t_1) \subseteq L(A; t_2)$.

Let $x \in L(A; t_2)$ and there is no x in G such that $t_1 < A(x, q) \leq t_2$.

Hence $x \in L(A; t_1)$ and $L(A; t_2) \subseteq L(A; t_1)$.

Hence $L(A; t_1) = L(A; t_2)$.

**4.4 Theorem:** A Q-fuzzy subset A of G is a lower anti Q-fuzzy subgroup of a group G if and only if the lower level subsets L (A; t), $t \in$ Image A, are subgroups of G.

**Proof:** It is clear.

**4.5 Theorem:** Any subgroup H of a group G can be realized as a lower level subgroup of some lower anti Q-fuzzy subgroup of G.

**Proof:** Let A be a Q-fuzzy subset and $x \in G$ and $q \in Q$.
Define,
$$A(x, q) = \begin{cases} 0 & \text{if } x \in H \\ t & \text{if } x \notin H \text{, where } t \in (0,1]. \end{cases}$$

We shall prove that A is a lower anti Q-fuzzy subgroup of G.

Let $x, y \in G$ and $q \in Q$.

(i) Suppose $x, y \in H$, then $xy \in H$ and $xy^{-1} \in H$. $A(x, q) = 0$, $A(y, q) = 0$, A and $A(xy^{-1}, q) = 0$.
Hence $A(xy^{-1}, q) \leq \max\{A(x, q), A(y, q)\}$.

(i) Suppose $x \in H$ and $y \notin H$, then $xy \notin H$ and $xy^{-1} \notin H$. $A(x, q) = 0$, $A(y, q) = t$ and $A(xy^{-1}, q) = t$.
Hence $A(xy^{-1}, q) \leq \max\{A(x, q), A(y, q)\}$.

(iii) Suppose $x, y \notin H$, then $xy^{-1} \in H$ or $xy^{-1} \notin H$. $A(x, q) = t$, $A(y, q) = t$ and $A(xy^{-1}, q) = 0$ or $t$.
Hence $A(xy^{-1}, q) \leq \max\{A(x, q), A(y, q)\}$.

Since H is a subgroup and $e \in H$, then $A(e, q) = 0$.

Thus in all cases, A is a lower anti Q-fuzzy subgroup of G.

For this lower anti Q-fuzzy subgroup, $L(A; t) = H$.

**Remark:** As a consequence of the Theorem 4.3, the lower level subgroups of a lower anti Q-fuzzy subgroup A of a group G form a chain. Since $0 \leq A(x, q)$ for all x in G and $q \in Q$, therefore $L(A; 0)$ is the smallest and we have the chain :

$\{e\} \subset L(A; 0) \subset L(A; t_1) \subset L(A; t_2) \subset \ldots \subset L(A; t_n) = G$, where $0 < t_1 < t_2 < \ldots\ldots < t_n$.

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