

## SOME RESULTS ON DOUBLE SEQUENCE THEOREMS IN METRIZABLE SPACES

PRALAHAD MAHGAONKAR\*

Ballari Institute of Technology and Management.

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### ABSTRACT

*In this paper, we have discussed some generalized results in double sequence theorems on metrizable spaces and also some new concepts of generalized metric spaces.*

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### INTRODUCTION

The discussion of some equivalence metrication theorems, modified single sequence theorem and modified double sequence theorems has been studied by Nigata [1]. Here we also defined metric topologies, Before that, however, we want to give a name to those topological spaces whose topologies are metric topologies. For the sake of compactness we have studied and the proof given by Mattin [3]. Some of authors studied various types of theorems in metric spaces. In this paper, we have establish and generalized results which is observed in results in [2]. Further we have added some illustrative examples and results.

**Definition of  $T_1$  spaces:** A  $T_1$ -space is a topological space in which given any pair of disjoint points, each has a neighbourhood which does not contain the other.

It is obvious that any subspace of  $T_1$ -space is also a  $T_1$ -space.

### MAIN RESULTS

In the present section, clearly, it validates the equivalences of metrization conditions of Bing's Theorem, Nagata Theorem, and Double sequence Theorems. Symmetrically which follows only from Urysohn's Theorem. Hence therefore we studied the following theorems based on  $T_1$ -spaces.

**Theorem 1:** If a topological space  $\tau$  then

- (i). Is a  $T_1$ -space
- (ii). is a regular space
- (iii). has a  $\sigma$  – discrete base then  $\tau$  is a metrizable space

**Theorem 2:** If a topological space  $\tau$  then

- (iv). is a  $T_1$ -space
- (v). is a regular space
- (vi). has a  $\sigma$  – locally finite base then  $\tau$  is a metrizable space.

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**Corresponding Author: Pralahad Mahgaonkar\***  
**Ballari Institute of Technology and Management.**

**Theorem 3:** If a topological space  $\tau$  then

- (vii). is a  $T_1$ -space
- (viii). there exists a sequence  $u_1, u_2, u_3, \dots$  of open covering such that
- (ix).  $u_1 > u_2^* > u_2 > u_3^* > \dots$  can be repaced as  $u_1 > u_1^\Delta > u_2 > u_2^\Delta \dots$
- (x).  $\{S(p, u_n) : n = 1, 2, 3, \dots\}$  is a neighbourhood basis at each point of "p" of R then R is a metrizable.

**Proof:**

Theorem (1)  $\Rightarrow$  Theorem (2) is obvious thus implies.

Theorem (3)  $\Rightarrow$  Theorem (2) have been proved by Nagata[1]

It is enough to show that conditions of theorem 3 imply the conditions of theorem 1. which we do now.

We assume theorem 3.

$$(vii) \Rightarrow (i) \quad (1)$$

We now prove (viii)

Let  $N(p)$  be neighbourhood of a point  $p$ , There exists from (x) and  $n \in \mathbb{N}$  such that

$$S(p, u_n) \subset N(p) \quad (2)$$

Now consider  $S(p, u_{n+1})$  then  $S(p, u_n)$  is closed neighbourhood of p.

$$\text{Let } b \in \overline{S(p, u_{n+1})}. \quad (3)$$

Every  $G_{n+1}$  such that  $b \in G_{n+1}$ .  $G_{n+1} \in u_{n+1}$  call them as  $G_{n+1}(b)$  which meets  $S(p, u_{n+1})$ , i.e meets to  $G_{n+1}$  which contains "p". Call it as  $G_{n+1}(p)$ .

$$G_{n+1}(p) \cup G_{n+1}(b) \subset u_{n+1}^* \subset u_n \quad (4)$$

There exists a set  $u_n$  which contains "p" for every b.

Now  $S(p, u_n)$  examples contain p and b.

$$b \in S(p, u_n) \quad (5)$$

$$\overline{S(p, u_{n+1})} \in S(p, u_n) \subset N(p). \quad (6)$$

Which established (viii).

We prove (ix) in a series of steps.

**STEP-1:** The following results are obvious

$$\{q \in S_n(p)\} \Rightarrow \{p \in S_n(q)\} \quad (7)$$

If  $q \in S(p, u_{n+1})$ , then from (viii)(ix)(x)

$$S(p, u_{n+1}) \cup S(q, u_{n+1}) \subset S(p, u_n) \quad (8)$$

**STEP-2:** Let u be an open cover in R such that  $U \in u$ .

$$\text{Let } U_n = \{x \in U : x \neq S(p, u_n), p \in (R - U)\} \quad (9)$$

$$\text{We assert that if } q \notin U_{n+1} \quad (10)$$

$$\text{and } x \notin U_{n+1} \quad (11)$$

Further

$$x \notin S_{n+1}(q) \quad (12)$$

For otherwise if  $x \in S_{n+1}(q) = S(q, u_{n+1})$ .

Then for some value of “p” in (R-U),  $q \in S(p, u_{n+1})$  and  $S(p, u_{n+1}) \cap U_{n+1} = \phi$ . Which contradiction, Hence the proof of equation 12.

Now let us consider u is an open cover. Well defined order of u by a relation  $\lambda$ .

Let us consider  $U, V \in u$ , then

$$V \lambda U \quad (13)$$

Or

$$U \lambda V \quad (14)$$

Set

$$V_n^* = V_n - \cup U_{n+1} \quad (15)$$

For all  $U \in u$  such that  $V \lambda U$ .

Therefore

$$U_n^* = U_n - \cup V_{n+1} \quad (16)$$

For all  $V \lambda U$

Then

$$V_n^* \subset R - \cup U_{n+1} \lambda R - U_{n+1} \quad (17)$$

Since  $U_{n+1} \subset U_n$ ,

Thus

$$V_n^* \cap U_n^* = \phi \quad (18)$$

For all the values of  $n=1,2,3,\dots$

$$\text{Let } p \in V_n^* \quad (19)$$

Then  $p \in V_n - \cup U_{n+1}, U \lambda V$ .

$$p \notin U_{n+1} \quad (20)$$

Therefore

$$S_{n+1}(p) \cap U_n^* = \phi \quad (21)$$

$$\text{If } p \in U_n^* \quad (22)$$

Similarly we get

$$S_{n+1}(p) \cap V_n^* = \phi \quad (23)$$

For each  $p \in U_n^*$ , consider the set points for x,

## CONCLUSION

In the present paper, we gave generalized concepts of Nagata and Bing, further the sequence theorems and some double sequence theorems will play an important rule on  $T_1$ -spaces.

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