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COMPUTATION OF MULTIPLICATIVE STATUS INDICES OF GRAPHS

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ABSTRACT

A topological index or a graph index is a numerical parameter mathematically derived from the graph structure. In this study, we define the multiplicative vertex status index, multiplicative modified vertex status index, multiplicative F-status index, general multiplicative vertex status index of a graph and compute exact formulas for some standard graphs and friendship graphs.

Keywords: *Multiplicative vertex status index, multiplicative F-status index, graph.*

Mathematics Subject Classification: *05C05, 05C07, 05C035, 05C90.*

1. INTRODUCTION

In this paper, we consider only a finite, simple, connected graph G with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The distance between any two vertices u and v is the length of the shortest path joining u and v and is denoted by $d(u, v)$. The status of a vertex u is defined as the sum of its distance from every other vertex in G and is denoted by $\sigma(u)$. We refer [1] for undefined term and notation.

Topological indices or graph indices have found some applications in chemical documentation, isomer discrimination, QSAR/QSPR study, see [2, 3].

For more about graph indices one can refer [4]. Some different graph indices can be found in [5, 6, 7, 8, 9, 10, 11].

In [12], Kulli introduced the multiplicative first and second status indices, defined as

$$S_1II(G) = \prod_{uv\in E(G)} \left[\sigma(u)+\sigma(v)\right], \qquad S_2II(G) = \prod_{uv\in E(G)} \sigma(u)\sigma(v).$$

We now propose the following multiplicative status indices.

The multiplicative vertex status index of a graph G is defined as $S_vII(G) = \prod_{u\in V(G)} \sigma(u)^2$.

The multiplicative total status index of a graph G is defined as $T_sII(G) = \prod_{u\in V(G)} \sigma(u)$.

The multiplicative modified vertex status index of a graph G is defined as ${}^mS_vII(G) = \prod_{u\in V(G)} \frac{1}{\sigma(u)^2}$.

The multiplicative status inverse is defined as $SI\ II(G) = \prod_{u\in V(G)} \frac{1}{\sigma(u)}$.

The multiplicative status zeroth order index of a graph G is defined as $SZII(G) = \prod_{u\in V(G)} \frac{1}{\sqrt{\sigma(u)}}$.

The multiplicative F-status index of a graph G is defined as $FSII(G) = \prod_{u\in V(G)} \sigma(u)^3$.

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The general multiplicative vertex status index of G is defined as $S_v^a II(G) = \prod_{u \in V(G)} \sigma(u)^a$, where a is a real number.

Some of the research work on status indices can be found in [13, 14, 15, 16, 17, 18]. Recently, some multiplicative indices were studied, for example, in [19, 20, 21, 22, 23, 24. 25, 26, 27, 28, 29, 30, 31, 32, 33, 34].

In this paper, the multiplicative vertex status index, multiplicative modified vertex status index, multiplicative F-status index, general multiplicative vertex status index of some standard graphs are computed.

2. RESULTS FOR COMPLETE GRAPHS

Theorem 1: If K_n is a complete graph with n vertices, then the general multiplicative vertex status index of K_n is

$$S_v^a II(K_n) = (n-1)^{an}. \tag{1}$$

Proof: Let K_n be a complete graph with n vertices. Then $\sigma(u) = n - 1$ for every vertex u of K_n. Therefore

$$S_v^a II(K_n) = \prod_{u \in V(K_n)} \sigma(u)^a = (n-1)^{an}.$$

We obtain the following results by using Theorem 1.

Corollary 1.1: If K_n is a complete graph with n vertices, then

(i) $S_v II(K_n) = (n-1)^{2n}$. (ii) $T_s II(K_n) = (n-1)^n$.

(iii) ${}^m S_v II(K_n) = \left(\dfrac{1}{(n-1)^2}\right)^n$. (iv) $SI\ II(K_n) = \left(\dfrac{1}{n-1}\right)^n$.

(v) $SZII(K_n) = \left(\dfrac{1}{n-1}\right)^{\frac{n}{2}}$. (vi) $FSII(K_n) = (n-1)^{3n}$.

Proof: Put a = 2, 1, –2, –1, –½, 3 in equation (1), we get the desired results.

3. RESULTS FOR CYCLES

Theorem 2: Let C_n be a cycle with n vertices. Then the general multiplicative vertex status index of C_n is

$$S_v^a II(C_n) = \left(\frac{n^2}{4}\right)^{an}, \quad \text{for } n \text{ is even,} \tag{2}$$

$$= \left(\frac{n^2-1}{4}\right)^{an}, \quad \text{for } n \text{ is odd.} \tag{3}$$

Proof: Let C_n be a cycle with n vertices.

Case-1: Suppose n is even. For every vertex u in C_n, $\sigma(u) = \dfrac{n^2}{4}$. Thus

$$S_v^a II(C_n) = \prod_{u \in V(C_n)} \sigma(u)^a = \left(\frac{n^2}{4}\right)^{an}.$$

Case-2: Suppose n is odd. Then $\sigma(u) = \dfrac{n^2-1}{4}$ for every vertex u in C_n. Thus

$$S_v^a II(C_n) = \prod_{u \in V(C_n)} \sigma(u)^a = \left(\frac{n^2-1}{4}\right)^{an}.$$

We establish the following results by Theorem 2.

Corollary 2.1: Let C_n be a cycle with n vertices. Then

(i) $S_vII(C_n) = \left(\frac{n^2}{4}\right)^{2n}$, if n is even,

$= \left(\frac{n^2-1}{4}\right)^{2n}$, if n is odd.

(ii) $T_vII(C_n) = \left(\frac{n^2}{4}\right)^{n}$, if n is even,

$= \left(\frac{n^2-1}{4}\right)^{n}$, if n is odd.

(iii) ${}^mS_vII(C_n) = \left(\frac{n^2}{4}\right)^{-2n}$, if n is even,

$= \left(\frac{n^2-1}{4}\right)^{-2n}$, If n is odd.

(iv) $SI\ II(C_n) = \left(\frac{n^2}{4}\right)^{-n}$, if n is even,

$= \left(\frac{n^2-1}{4}\right)^{-n}$, if n is odd.

(v) $SZII(C_n) = \left(\frac{n^2}{4}\right)^{-\frac{n}{2}}$, if n is even,

$= \left(\frac{n^2-1}{4}\right)^{-\frac{n}{2}}$, if n is odd.

(vi) $FSII(C_n) = \left(\frac{n^2}{4}\right)^{3n}$, if n is even,

$= \left(\frac{n^2-1}{4}\right)^{3n}$, if n is odd.

Proof: Put a = 2, 1, –2, –1, –½, 3 in equations (2) and (3), we get the desired results.

4. RESULTS FOR COMPLETE BIPARTITE GRAPHS

Let $K_{p,q}$ be a complete bipartite graph. Then it has $p+q$ vertices and pq edges. In $K_{p,q}$, there are two types of status vertices as given in Table 1.

$\sigma(u) \setminus u \in E(K_{p,q})$	$p + 2(q-1)$	$q + 2(p-1)$
Number of vertices	q	p

Table-1: Status vertex partition of $K_{p,q}$

Theorem 3: The general multiplicative vertex status index of $K_{p,q}$ is

$$S_v^aII(K_{p,q}) = [p+2(q-1)]^{aq} \times [q+2(p-1)]^{ap}. \quad (4)$$

Proof: Let $K_{p,q}$ be a complete bipartite graph. By definition, we have

$$S_v^aII(K_{p,q}) = \prod_{u \in V(K_{p,q})} \sigma(u)^a.$$

By using Table 1, we deduce

$$S_v^aII(K_{p,q}) = [p+2(q-1)]^{aq} \times [q+2(p-1)]^{ap}.$$

From Theorem 3, we establish the following results.

Corollary 3.1: Let $K_{p,q}$ be a complete bipartite graph. Then

(i) $S_v II\left(K_{p,q}\right)=\left[p+2(q-1)\right]^{2q}\times\left[q+2(p-1)\right]^{2p}$.

(ii) $T_s II\left(K_{p,q}\right)=\left[p+2(q-1)\right]^{q}\times\left[q+2(p-1)\right]^{p}$.

(iii) ${}^{m}S_v II\left(K_{p,q}\right)=\frac{1}{\left[p+2(q-1)\right]^{2q}}\times\frac{1}{\left[q+2(p-1)\right]^{2p}}$.

(iv) $SIII\left(K_{p,q}\right)=\frac{1}{\left[p+2(q-1)\right]^{q}}\times\frac{1}{\left[q+2(p-1)\right]^{p}}$.

(v) $SZII\left(K_{p,q}\right)=\frac{1}{\left[p+2(q-1)\right]^{\frac{q}{2}}}\times\frac{1}{\left[q+2(p-1)\right]^{\frac{p}{2}}}$.

(vi) $FSII\left(K_{p,q}\right)=\left[p+2(q-1)\right]^{3q}\times\left[q+2(p-1)\right]^{3p}$.

Proof: Put a = 2, 1, –2, –1, –½, 3 in equation (4), we obtain the desired results.

5. RESULTS FOR WHEEL GRAPHS

A wheel graph, denoted by W_n, is the join of C_n and K_1. A wheel graph W_4 is shown in Figure 1.

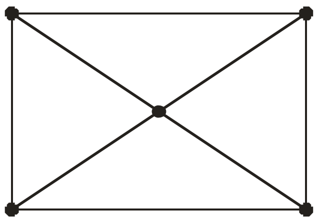

Figure-1: Wheel graph W_4

Clearly, a wheel graph W_n has n+1 vertices and $2n$ edges. This graph has two types of status vertices as given in Table 2.

$\sigma(u) \setminus u \in V(W_n)$	n	$2n-3$
Number of vertices	1	n

Table-2: Status vertex partition of W_n

Theorem 4: The general multiplicative vertex status index of a wheel graph W_n is

$$S_v^a II\left(W_n\right)=n^a\times(2n-3)^{an}. \tag{5}$$

Proof: By definition and by using Table 2, we deduce

$$S_v^a II\left(W_n\right)=\prod_{u\in V(W_n)}\sigma(u)^a=\left(n^a\right)^1\times\left[(2n-3)^a\right]^n=n^a\times(2n-3)^{an}$$

From Theorem 4, we obtain the following results.

Corollary 4.1: Let W_n be a wheel graph with n+1 vertices and $2n$ edges. Then

(i) $S_v II\left(W_n\right)=n^2(2n-3)^{2n}$.

(ii) $T_S II\left(W_n\right)=n(2n-3)^{n}$.

(iii) ${}^{m}S_v II\left(W_n\right)=\frac{1}{n^2(2n-3)^{2n}}$.

(iv) $SIII\left(W_n\right)=\frac{1}{n(2n-3)^{n}}$.

(v) $SZII\left(W_n\right)=\frac{1}{\sqrt{n(2n-3)^{n}}}$.

(vi) $FSII\left(W_n\right)=n^3(2n-3)^{3n}$.

Proof: Put a = 2, 1, –2, –1, –½, 3 in equation (5), we get the desired results.

6. RESULTS FOR FRIENDSHIP GRAPHS

A friendship graph, denoted by F_n, is the graph obtained by taking $n \geq 2$ copies of C_3 with vertex in common. The friendship graph F_4 is shown in Figure 2.

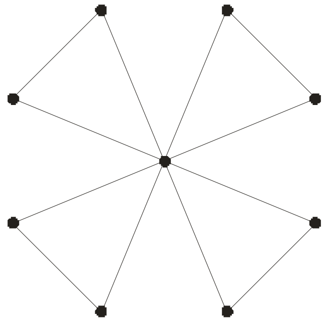

Figure-2: Friendship graph F_4

Clearly, a friendship graph F_n has $2n + 1$ vertices and $3n$ edges. This graph has two types of status vertices as given in Table 3.

$\sigma(u) \setminus u \in V(F_n)$	$2n$	$4n-3$
Number of vertices	1	2n

Table-3: Status vertex partition of F_n

Theorem 5: The general multiplicative vertex status index of a friendship graph F_n is

$$S_v^a II(F_n) = (2n)^a \times (4n-3)^{2an}. \tag{6}$$

Proof: By definition and using Table 3, we derive

$$S_v^a II(F_n) = \prod_{u \in V(F_n)} \sigma(u)^a = \left((2n)^a\right)^1 \times \left[(4n-3)^a\right]^{2n} = (2n)^a \times (4n-3)^{2an}.$$

We establish the following results from Theorem 5.

Corollary 5.1: Let F_n be a friendship graph with $2n+1$ vertices and $3n$ edges. Then

(i) $S_v II(F_n) = 4n^2 (4n-3)^{4n}$.

(ii) $T_S II(F_n) = 2n(4n-3)^{2n}$.

(iii) ${}^m S_v II(F_n) = \dfrac{1}{4n^2 (4n-3)^{4n}}$.

(iv) $SI\ II(F_n) = \dfrac{1}{2n(4n-3)^{2n}}$.

(v) $SZII(F_n) = \dfrac{1}{(2n-3)^n \sqrt{2n}}$.

(vi) $FSII(F_n) = 8n^3 (4n-3)^{6n}$.

Proof: Put a = 2, 1, –2, –1, –½, 3 in equation (6), we obtain the required results.

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