International Journal of Mathematical Archive-11(5), 2020, 13-16
Available online through www.ijma.info ISSN 2229 – 5046

ON NEW CLASS OF AXIOMS IN TOPOLOGICAL SPACES

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(Received On: 31-03-20; Revised & Accepted On: 13-04-20)

ABSTRACT

In this paper, we study some separation axioms namely, SG-T_o-space, SG -T_1 -space and SG -T_2-space and their properties. We also obtain some of their characterizations.

Key Words: *SG-T_O-Space, SG-T_1 –Space, SG-T_2-Space.*

2010 Mathematics Subject Classification: *54A05, 54D05, 54C10.*

1. INTRODUCTION

In the year 1987, [1] P.Bhattacharya and B.K.Lahiri introduced and studied SG-closed and SG-open sets respectively. In this paper we define and study the properties of a new topological axioms called SG-T_o-space, SG-T_1 – space, SG-T_2-space.

II. PRELIMINARIES

Throughout this paper space (X, τ) and (Y, σ) (or simply X and Y) always denote topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X, Cl(A), Int(A), A^c,P-Cl(A) and P-int(A) denote the Closure of A, Interior of A , Compliment of A, pre-closure of A and pre-interior of (A) in X respectively.

Definition 2.1: A subset A of a topological space (X, τ) is called

1. Semi Generalised Closed Set [1] if Scl(A) $\subseteq$U whenever A$\subseteq$U and U is Semi-open in X
2. A pre generalized pre regular ωeakly closed set (briefly pgprω-closed set) if pCl(A) [3] whenever A$\subseteq$U and U is rgα-open in (X, τ).
3. A subset A of a topological space (X, τ) is called pre generalized pre regular ωeakly open (briefly pgprω-open) [4] set in X if A^c is pgprω-closed in X.

Defintion 3: A map f : (X, τ) -» (Y, σ) is called

(i) SG-continuous map [2] if f^{-1} (v) is SG closed in (X,τ) for every closed V in (Y, σ).
(ii) SG-irresolute map [2] if f^{-1}(v) is SG closed in (X,τ) for every SG-closed V in (Y, σ).
(iii) SG-closed map [2] if f^{-1}(v) is SG closed in (X,τ) for every closed V in (Y, σ).
(iv) SG-open map [2] if f^{-1}(v) is SG closed in (X,τ) for every closed V in (Y, σ).

4. SEMI GENERALISED SPACE:

Definition 4.4.1: A topological space (X, τ) is called SG -T_o-space if for any pair of distinct points x, y of (X,τ) there exists an SG-open set G such that x ∈ G, y ∉ G or x ∉ G, y ∈ G.

Example 4.4.2: Let X = {a, b}, τ = {φ,{b}, X}. Then (X, τ) is SG-T_o- space, since for any pair of distinct points a, b of (X,τ) there exists an SG-T_o open set{b} such that a ∉{b}, b ∈{b}.

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Remark 4.4.3: Every SG-space is SG-T_o-space.

Theorem 4.4.4: Every subspace of a SG-T_o-space is SG-T_o-space.

Proof: Let (X,τ) be a SG -T_o-space and (Y,τ_y) be a subspace of (X,τ). Let Y_1 and Y_2 be two distinct points of (Y,τ_y). Since (Y,τ_y) is subspace of (X,τ),Y_1 and Y_2 are also distinct points of (X,τ). As (X,τ) is SG-T_o-space, there exists an SG-open set G such that $Y_1 \in G$, $Y_2 \notin G$. Then $Y \cap G$ is SG-open in (Y,τ_y) containing but Y_1 not Y_2. Hence (Y,τ_y) is SG-T_o-space.

Theorem 4.4.5: Let f: (X,τ) -» (Y, μ) be an injection, SG-irresolute map. If (Y,μ) is SG-T_o-space, then (X,τ) is SG-T_o-space.

Proof: Suppose (Y, μ) is SG-T_o-space. Let a and b be two distinct points in(X,τ).As f is an injection f(a) and f(b) are distinct points in (Y,μ). Since(Y,μ) is SG-T_o-space, there exists an SG-open set G in (Y,μ) such that $f(a) \in G$ and $f(b) \notin G$. As f is SG-irresolute, $f^{-1}(G)$ is SG-open set in (X,τ) such that $a \in f^{-1}(G)$ and $b \notin f^{-1}(G)$. Hence (X,τ) is SG-T_o-space.

Theorem 4.4.6: If (X,τ) is SG-T_o-space, T_{SG}-space and (Y, τ_y) is SG-closed subspace of (X,τ), then (Y,τ_y) is SG-T_o -Space.

Proof: Let (X,τ) be SG-T_o-space, T_{SG}-space and (Y,τ_y) is SG-closed subspace of (X,τ). Let a and b be two distinct points of Y. Since Y is subspace of (X,τ), a and b are distinct points of (X,τ). As (X,τ) is SG-T_o -space, there exists an SG-open set G such that $a \in G$ and $b \notin G$. Again since (X,τ) is TSG-space, G is open in (X,τ).Then $Y \cap G$is open. So $Y \cap G$ is SG-open such that $a \in Y \cap G$ and $b \notin Y \cap G$. Hence (Y,τ_y) is SG-T_o–space.

Theorem 4.4.7: Let f: (X,τ) -» (Y, μ) be bijective SG-open map from a SG-T_0 Space (X,τ) onto a topological space (Y,τ_y). If (X,τ) is T_{SG}-space, then (Y, μ) is SG-T_0 Space.

Proof: Let a and b be two distinct points of (Y,τ_y). Since f is bijective, there exist two distinct points e and d of (X,τ) such that f(c) = a and f(d) = b. As(X,τ) is SG-T_0 Space, there exists a SG-open set G such that $c \in G$ and $d \notin G$. Since (X,τ) is T_{SG}-space, G is open in (X,τ). Then f(G) is SG-open in (Y, μ), since f is SG-open, such that $a \in f(G)$ and $b \notin f(G)$. Hence (Y,τ_y) is SG-T_0-space.

Definition 4.4.8: A topological space (X,τ) is said to be SG-T_1-space if for any pair of distinct points a and b of (X,τ) there exist SG-open sets G and H such that $a \in G$, $b \notin G$ and $a \notin H$, $b \in H$.

Example 4.4.9: Let X = {a, b} and $\tau = \{\emptyset, \{a\}, X\}$. Then (X,τ) is a topological space. Here a and b are two distinct points of (X,τ), then there exist SG-open sets {a}, {b} such that $a \in \{a\}$, $b \notin \{a\}$ and $a \notin \{b\}$, $b \in \{b\}$.Therefore (X,τ) is SG-T_0 space.

Theorem 4.4.10: If (X,τ) is SG-T_1-space, then (X,τ) is SG-T_o-space.

Proof: Let (X,τ) be aSG-T_1-space. Let a and b be two distinct points of (X,τ). Since (X,τ) is SG-T_1-space, there exist SG-open sets G and H such that $a \in G$, $b \notin G$ and $a \notin H$, $b \in H$. Hence we have $a \in G$, $b \notin G$. Therefore (X,τ) is SG-T_o-space. The converse of the above theorem need not be true as seen from the following example.

Example 4.4.11: Let X = {a, b} and $\tau = \{\varphi, \{b\}, X\}$. Then (X,τ) is SG-T_o-space but not SG-T_1-space. For any two distinct points a, b of X and an SG-open set {b} such that $a \notin \{b\}$, $b \in \{b\}$ but then there is no SG-open set G with $a \in G$, $b \notin G$ for $a \neq b$.

Theorem 4.4.12: If f: (X,τ) -» (Y,τ_y) is a bijective SG-open map from a SG-T_1-space and T_{SG}-space (X,τ) on to a topological space (Y,τ_y), then (Y,τ_y) is SG-T_1-space.

Proof: Let (X,τ) be a SG-T_1-space and T_{SG}-space. Let a and b be two distinct points of (Y,τ_y). Since f is bijective there exist distinct points c and d of (X,τ) such that f(c) = a and f(d) = b. Since (X,τ) is SG-T_1-space there exist SG-open sets G and H such that $c \in G$, $d \notin G$ and $c \notin H$, $d \in H$. Since (X,τ) is T_{SG}-space, G and H are open sets in (X,τ) also f is SG-open f(G) and f(H) are SG-open sets such that $a = f(c) \in f(G)$, $b = f(d) \notin f(G)$ and $a= f(c) \notin f(H)$, $b = f(d) \in f(H)$. Hence (Y,τ_y) is SG-T_1-space.

Theorem 4.4.13: If (X,τ) is SG T_1 space and T_{SG}-space, Y is a subspace of (X,τ), then Y is SG T_1 space.

Proof: Let (X,τ) be a SG T_1 space and T_{SG}-space. Let Y be a subspace of (X,τ). Let a and b be two distract points of Y. Since $Y\subseteq X$, a and b are also distinct points of X. Since (X,τ) is SG-T_1-space, there exist SG-open sets G and H such that $a \in G$, $b \notin G$ and $a \notin H$, $b \in H$. Again since (X,τ) is T_{SG}-space, G and H are open sets in (X,τ), then $Y\cap G$ and $Y\cap H$ are open sets so SG-open sets of Y such that $a \in Y\cap G$, $b\notin Y\cap G$ and $a \notin Y\cap H$, $b \in Y\cap H$. Hence Y is SG T_1 space.

Theorem 4.4.14: If: (X,τ) -» (Y,τ_y) is injective SG-irresolute map from a topological space (X,τ) into SG-T_1-space (Y,τ_y), then (X,τ) is SG-T_1 - space.

Proof: Let a and b be two distinct points of (X,τ). Since f is injective, f(a) and f(b) are distinct points of (Y,τ_y). Since (Y,τ_y) is SG-T_1 space there exist SG-open sets G and H such that $f(a) \in G$, $f(b) \notin G$ and $f(a) \notin H$, $f(b) \in H$.Since f is SG- irresolute, $f^{-1}(G)$ and $f^{-1}(H)$ are SG-open sets in (X,τ) such that $a \in f^{-1}(G)$, $b \notin f^{-1}(G)$ and $a \notin f^{1}(H)$, $b\in f^{-1}(H)$. Hence (X,τ) is SG-T_1space.

Definition 4.4.15: A topological space (X,τ). is said to be SG-T_2- space(or T_{SG}-Hausdorff space) if for every pair of distinct points x, y of X there exist T_{SG}-open sets M and N such that $x\in N$, $y\in M$ and $N\cap M = \emptyset$.

Example 4.4.16: Let $X = \{a, b\}$, $\tau = \{\emptyset, \{a\}, \{b\}, X\}$. Then (X,τ) is topological space. Then (X,τ) is SG-T_2-space. T_{SG}-open sets are $\emptyset$, {a}, {b},and X. Let a and b be a pair of distinct points of X, then there exist T_{SG} - open sets {a} and {b} such that $a\in\{a\}$, $b\in\{b\}$ and $\{a\}\cap\{b\} = \emptyset$. Hence (X,τ) is SG-T_2-space.

Theorem 4.4.17: Every SG-T_2- space is SG T_1space.

Proof: Let (X,τ) be a SG-T_2- space. Let x and y be two distinct points in X. Since (X,τ) is SG-T_2- space, there exist disjoint T_{SG}-open sets U and V such that $x\in U$, and $y \in V$. This implies, $x \in U$, $y \notin U$ and $x \in V$, $y \notin V$. Hence (X,τ) is SG-T_2- space.

Theorem 4.4.18: If (X,τ) is SG-T_2-space, T_{SG}- space and (Y,τ_y) is subspace of (X,τ), then (Y,τ_y) is also SG-T_2-space.

Proof: Let (X,τ), be a SG-T_2 - space and let Y be a subset of X. Let x and y be any two distinct points in Y. Since $Y\subseteq X$, x and y are also distinct points of X. Since (X,τ) is SG-T_2 - space, there exist disjoint T_{SG}-open sets G and H which are also disjoint open sets, since (X,τ) is T_{SG} - space. So $G\cap Y$ and$H\cap Y$ are open sets and so T_{SG}- open sets in (Y,τ_y). Also $x\in G$, $x \in Y$implies $x\in G\cap V$ and $y\in H$ and $y\in Y$ this implies $y\in Y\cap H$, since $G\cap H = \emptyset$, we have $(Y\cap G)\cap(Y\cap H) = \emptyset$. Thus $G\cap Y$ and $H\cap Y$ are disjointT_{SG}-open sets in Y such that $x\in G\cap Y$, $y\in H\cap Y$ and $(Y\cap G)\cap(Y\cap H)= \emptyset$. Hence (Y,τ_y) is SG-T_2 - space.

Theorem 4.4.19: Let (X,τ), be a topological space. Then (X,τ), is SG-T_2- space if and only if the intersection of all T_{SG}-closed neighbourhood of each point of X is singleton.

Proof: Suppose (X,τ), is SG-T_{2}-space. Let x and y be any two distinct points of X. Since X is SG-T_2-space, there exist open sets G and H such that $x\in G$, $y\in H$ and $G\cap H = \emptyset$. Since $G\cap H = \emptyset$. implies $x\in G\subseteq X\text{-}H$. So X-H is T_{SG}-closed neighbourhood of x, which does not contain y. Thus y does not belong to the intersection of all T_{SG}-closed neighbourhood of x. Since y is arbitrary, the intersection of all T_{SG}-closed neighbourhoods of x is the singleton {x}.

Conversely, let (x) be the intersection of all T_{SG}-closed neighbourhoods of an arbitrary point $x\in X$. Let y be any point of Xdifferent from x. Since y does not belong to the intersection, there exists aT_{SG}-closed neighbourhood N of x such that $y\notin N$. Since N isT_{SG}-neighbourhood of x, there exists an T_{SG}-open set G such $x \in G\subseteq X$. Thus G and X - N are T_{SG}-open sets such that $x\subseteq G$, $y\in X\text{-}N$ and$G\cap(X - N) = \emptyset$. Hence (X,τ) is SG-T_2-space.

Theorem 4.4.20: Let f: (X,τ), -» (Y,τ_y) be a bijective SG-open map. If (X,τ) is SG-T_2- space and T_{SG} space, then (Y,τ_y) is also SG-T_2- space.

Proof: Let (X,τ), is SG-T_2- space and T_{SG}- space. Let y_1 and y_2 be two distinct points of Y. Since f is bijective map, there exist distinct points x_1 and x_2 of Xsuch that $f(x_i) = y_j$ and $f(x_2) = y_2$. Since (X,τ) is SG-T_2- space, there exist SG-open sets G and H such that $X_1\in G$, $X_2\in H$ and $G\cap H =\emptyset$. Since (X,τ) is T_{SG}- space, G and H are open sets, then f(G) and f(H) are SG- open sets of(Y,τ_y) , since f is pprw-open, such that $y_1 = f(x_1) \in f(G)$, $y_2 = f(x_2) \in f(H)$ and $f(G)\cap f(H) = \emptyset$. Therefore we have $f(G)\cap f(H) = f(G\cap H) =\emptyset$. Hence (Y,τ_y) is SGT_2-space.

Theorem 4.4.21: Let (X,τ) be a topological space and let (Y,τ_y) be a SG-T_2-space. Let f: (X,τ)—>(Y,τ_y) be an injective SG-irresolute map. Then (X,τ) is SG-T_2-space.

Proof: Let X_1 and X_2 be any two distinct points of X. Since f is injective, $x_1 \neq x_2$ implies $f(x_1) \neq f(x_2)$.Let $y_1 = f(x_1)$, $y_2 = f(x_2)$ so that $x_1 = f^{-1}(y_1)$, $x_2 = f^{-1}(y_2)$. Then $y_l, y_2 \in Y$ such that $y_1 \neq y_2$. Since (Y,τ_y) is SG-T_2-space there exist T_{SG}-open sets G and H such that $y_1 \in G$, $y_2 \in G$ and $G \cap H = \emptyset$. As f isT_{SG}-irresolute $f^{-1}(G)$ and $f^{-1}(H)$ are T_{SG}-open sets of (X,τ). Now $f^{-1}(G) \cap f^{-1}(H) = f^{-1}(G \cap H) = f^{-1}(\emptyset) = \emptyset$ and $y_1 \in G$ implies $f^{-}1(y_1) \in f^{-1}(G)$ implies$X_1 \in f^{-1}(G)$, $y_2 \in H$ implies $f^{-1}(y_2) \in f^{-1}(H)$ implies $x_2 \in f^{-1}(H)$.Thus for every pair of distinct points x_1, x_2 of X there exist disjoint T_{SG}-open sets $f^{-1}(G)$ and $f^{-1}(H)$ such that $X_1 \in f^{-1}(G)$, $x_2 \in f'^{-1}(H)$. Hence (X,τ) is SG-T_2-space.

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Source of support: Nil, Conflict of interest: None Declared.