TRANSLATIONS OF BIPOLAR VALUED MULTI FUZZY SUBSEMIGROUPS OF A SEMIGROUP

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Abstract

In this paper, some translation theorems of bipolar valued multi fuzzy subsemigroups of a semigroup is studied and prove some results on these.

Key Words: Bipolar valued fuzzy subset, bipolar valued multi fuzzy subset, bipolar valued multi fuzzy subsemigroup and translations.

Introduction

In 1965, Zadeh [10] introduced the notion of a fuzzy subset of a set, fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. Since then it has become a vigorous area of research in different domains, there have been a number of generalizations of this fundamental concept such as intuitionistic fuzzy sets, interval-valued fuzzy sets, soft sets etc. Lee [6] introduced the notion of bipolar-valued fuzzy sets. Bipolar-valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [−1, 1]. In a bipolar-valued fuzzy subset, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree (0, 1] indicates that elements somewhat satisfy the property and the membership degree [−1, 0) indicates that elements somewhat satisfy the implicit counter property. Bipolar valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [7]. Multi fuzzy sets was introduced by Sabu Sebastian, T.V.Ramakrishnan [8]. Bipolar valued fuzzy subgroups of a group, homomorphism in bipolar valued fuzzy subgroups of a group and bipolar valued fuzzy normal subgroups of a group was introduced by M.S.Anitha et al.[1, 2, 3]. Bipolar valued multi fuzzy subgroups of a group have defined and introduced by Santhi.V.K and G.Shyamala[9]. The papers were useful for developing the research paper. Indira.R and K.Arjunan [5] defined about using function in bipolar valued multi fuzzy subsemigroups of a semigroup. In this paper, some translation theorems are stated and proved. These theorems will be useful to further research.

1. Preliminaries

Definition 1.1[6]: A bipolar valued fuzzy set (BVFS) A in X is defined as an object of the form A = {< x, A+(x), A−(x) >/ x ∈ X}, where A+(x) : X → [0, 1] and A−(x) : X → [−1, 0]. The positive membership degree A+(x) denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set A and the negative membership degree A−(x) denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set A.

Example 1.2: A = {< a, 0.8, −0.4 >, < b, 0.4, −0.9 >, < c, 0.2, −0.7 >} is a bipolar valued fuzzy subset of X= {a, b, c}.

Definition 1.3[9]: A bipolar valued multi fuzzy set (BVMFS) A in X is defined as an object of the form A = {< x, A1+(x), A2+(x), ..., Ai+(x), A1−(x), A2−(x), ..., An−(x) >/ x ∈ X}, where A−i : X → [0, 1] and Ai− : X → [−1, 0] for all i = 1, 2, ..., n. The positive membership degrees Ai+(x) denote the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set A and the negative membership degrees Ai−(x) denote the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set A. It is denoted as A = (A1−, A2−, ..., Ai−, A1+, A2+, ..., Ai+, An+).

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Example 1.4: A = {< a, 0.4, 0.6, 0.4, −0.3, −0.5, −0.5 >, < b, 0.2, 0.6, 0.6, −0.7, −0.4, −0.8 >, < c, 0.4, 0.3, 0.7, −0.4, −0.6, −0.5 >} is a bipolar valued multi fuzzy subset of X = {a, b, c}.

Definition 1.5[5]: Let S be a semigroup. A bipolar valued multi fuzzy subset A = {A_1^+, A_2^+, ..., A_n^+, A_1^−, A_2^−, ..., A_n^−} of S is said to be a bipolar valued multi fuzzy subsemigroup of S (BVMFSSG) if the following conditions are satisfied

(i) A_i^−(xy) ≥ min{A_i^+(x), A_i^−(y)}

(ii) A_i(α) ≤ max{A_i^+(x), A_i^−(y)} for all x, y in S and for all i.

Example 1.6: Let S = {1, 2, 3} be a semigroup with respect to the ordinary multiplication. Then A = {1, 2, 3} and B = {1, 2, 3} are two bipolar valued multi fuzzy subsemigroups of S, then their intersection A ∩ B is a bipolar valued multi fuzzy subsemigroup of S.

Definition 1.7: Let A = {A_1^+, A_2^+, ..., A_n^+, A_1^−, A_2^−, ..., A_n^−} be a bipolar valued multi fuzzy subset of X. Then the following translations are defined as

(i) ?A(x) = {?A_1^+(x), ?A_2^+(x), ..., ?A_n^+(x), ?A_1^−(x), ?A_2^−(x), ..., ?A_n^−(x)}

(ii) !(x) = {A_1^+(x), A_2^+(x), ..., A_n^+(x), A_1^−(x), A_2^−(x), ..., A_n^−(x)}

(iii) P_α(x) = {P_α(A_1^+(x)), P_α(A_2^+(x)), ..., P_α(A_n^+(x)), P_α(A_1^−(x)), P_α(A_2^−(x)), ..., P_α(A_n^−(x))}

(iv) G_α(x) = {G_α(A_1^+(x)), G_α(A_2^+(x)), ..., G_α(A_n^+(x)), G_α(A_1^−(x)), G_α(A_2^−(x)), ..., G_α(A_n^−(x))}

Theorem 1.8: If A = {A_1^+, A_2^+, ..., A_n^+, A_1^−, A_2^−, ..., A_n^−} is a bipolar valued multi fuzzy subsemigroup of a semigroup S, then Q_α(x) is a bipolar valued multi fuzzy subset of X.

2. SOME THEOREMS

Theorem 2.1: If A = {A_1^+, A_2^+, ..., A_n^+, A_1^−, A_2^−, ..., A_n^−} is a bipolar valued multi fuzzy subsemigroup of a semigroup S, then ?A(x) is a bipolar valued multi fuzzy subsemigroup of S.

Proof: For every x, y in S, for all i, ?A_i^+(xy) = min{½, A_i^+(xy)} ≥ min{½, min{A_i^+(x), A_i^+(y)}} = min{½, A_i^+(x)} for all x and y in S. Hence ?A_i^+(xy) = max{½, A_i^+(xy)} = max{½, A_i^+(x)} for all x and y in S. Therefore ?A_i^+(xy) ≥ min{½, A_i^+(x), A_i^+(y)} for all x and y in S. Hence ?A_i^+(xy) is a bipolar valued multi fuzzy subsemigroup of S.

Theorem 2.2: If A = {A_1^+, A_2^+, ..., A_n^+, A_1^−, A_2^−, ..., A_n^−} is a bipolar valued multi fuzzy subsemigroup of a semigroup S, then !(x) is a bipolar valued multi fuzzy subsemigroup of S.

Proof: For every x, y in S, for all i, !(A_i^+(xy)) = max{½, A_i^+(xy)} ≥ max{½, min{A_i^+(x), A_i^+(y)}} = min{½, A_i^+(x)} for all x and y in S. Therefore !(A_i^+(xy)) ≤ max{½, A_i^+(x)} for all x and y in S. Hence !(A_i^+(xy)) is a bipolar valued multi fuzzy subsemigroup of S.

Theorem 2.3: If A = {A_1^+, A_2^+, ..., A_n^+, A_1^−, A_2^−, ..., A_n^−} is a bipolar valued multi fuzzy subsemigroup of a semigroup S, then Q_α(x) is a bipolar valued multi fuzzy subsemigroup of S.

Proof: For every x, y in S, for all i, Q_α(x) = min{A_i^+(x), A_i^−(y)} = min{A_i^+(x), A_i^−(y)} = min{A_i^+(x), A_i^−(y)} for all x and y in S. Therefore Q_α(x) is a bipolar valued multi fuzzy subsemigroup of S.
Theorem 2.4: If $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ is a bipolar valued multi fuzzy subsemigroup of a semigroup $S$, then $P_n \beta(A)$ is a bipolar valued multi fuzzy subsemigroup of $S$.

Proof: For every $x$, $y$ in $S$, $a_i$ in $[0, 1]$ and for all $i$, $P_n \beta(A_i)(xy) = \max \{ a_i, A_i(x), A_i(y) \} \geq \max \{ a_i, \min \{ A_i^+(x), A_i^-(y) \} \} = \min \{ a_i, A_i^-(x) \}, \max \{ a_i, A_i^+(y) \} \} = \min \{ P_n \beta(A_i)(x), P_n \beta(A_i)(y) \}$. Therefore $P_n \beta(A_i)(xy) \geq \min \{ P_n \beta(A_i)(x), P_n \beta(A_i)(y) \}$, for all $x$, $y$ in $S$. For every $x$, $y$ in $S$, $\beta_i$ in $[-1, 0]$ and for all $i$, $P_n \beta(A_i)(xy) = \min \{ \beta_i, A_i(x), A_i(y) \} \geq \min \{ \beta_i, \max \{ A_i^+(x), A_i^-(y) \} \} = \max \{ \beta_i, A_i^+(x) \}, \min \{ \beta_i, A_i^-(y) \} \} = \max \{ P_n \beta(A_i)(x), P_n \beta(A_i)(y) \}$. Therefore $P_n \beta(A)(xy) \leq \max \{ P_n \beta(A)(x), P_n \beta(A)(y) \}$, for all $x$, $y$ in $S$. Hence $P_n \beta(A)$ is a bipolar valued multi fuzzy subsemigroup of $S$.

Theorem 2.5: If $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ is a bipolar valued multi fuzzy subsemigroup of a semigroup $S$, then $G_n \beta(A)$ is a bipolar valued multi fuzzy subsemigroup of $S$.

Proof: For every $x$, $y$ in $S$, $a_i$ in $[0, 1]$ and for all $i$, $G_n \beta(A_i)(xy) = \alpha, \beta_i \geq a_i ( \min \{ A_i^+(x), A_i^-(y) \}) \} = \min \{ a_i, A_i^-(x) \}, \max \{ a_i, A_i^+(y) \} \} = \min \{ G_n \beta(A_i)(x), G_n \beta(A_i)(y) \}$, for all $x$, $y$ in $S$. For every $x$, $y$ in $S$, $\beta_i$ in $[-1, 0]$ and for all $i$, $G_n \beta(A_i)(xy) = \beta_i, A_i(x), A_i(y) \} \geq \min \{ \beta_i, \max \{ A_i^+(x), A_i^-(y) \} \} = \max \{ \beta_i, A_i^+(x) \}, \min \{ \beta_i, A_i^-(y) \} \} = \max \{ G_n \beta(A_i)(x), G_n \beta(A_i)(y) \}$. Therefore $G_n \beta(A)(xy) \leq \max \{ G_n \beta(A)(x), G_n \beta(A)(y) \}$, for all $x$, $y$ in $S$. Hence $G_n \beta(A)$ is a bipolar valued multi fuzzy subsemigroup of $S$.

Theorem 2.6: If $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ and $B = \langle B_1^+, B_2^+, \ldots, B_n^+, B_1^-, B_2^-, \ldots, B_n^- \rangle$ are bipolar valued multi fuzzy subsemigroups of a semigroup $S$, then !$(A \cap B) = !((A \cap B)$ is also a bipolar valued multi fuzzy subsemigroup of $S$.

Proof: The proof follows from the Theorems 1.8 and 2.2.

Theorem 2.7: If $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ and $B = \langle B_1^+, B_2^+, \ldots, B_n^+, B_1^-, B_2^-, \ldots, B_n^- \rangle$ are bipolar valued multi fuzzy subsemigroups of a semigroup $S$, then $(A \cap B) = !((A \cap B)$ is also a bipolar valued multi fuzzy subsemigroup of $S$.

Proof: The proof follows from the Theorems 1.8 and 2.1.

Theorem 2.8: If $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ is a bipolar valued multi fuzzy subsemigroup of a semigroup $S$, then !$(\{A\}(\cap B)) = \{A\}(\cap B)$ is also a bipolar valued multi fuzzy subsemigroup of $S$.

Proof: For every $x$ in $S$ and for all $i$, $\{A_i\}(x) = \min \{ \beta_i, A_i(x) \} \leq \beta_i$ and !$(\{A\}(\cap B)) = \{A\}(\cap B)$! is !$(\{A\}(\cap B)$ is also a bipolar valued multi fuzzy subsemigroup of $S$.

Hence $\{A\}(\cap B)$ is also a bipolar valued multi fuzzy subsemigroup of $S$.

Theorem 2.9: If $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ and $B = \langle B_1^+, B_2^+, \ldots, B_n^+, B_1^-, B_2^-, \ldots, B_n^- \rangle$ are bipolar valued multi fuzzy subsemigroups of a semigroup $S$, then $Q_n \beta(A \cap B) = Q_n \beta(A) \cap Q_n \beta(B)$ is also a bipolar valued multi fuzzy subsemigroup of $S$.

Proof: The proof follows from the Theorems 1.8 and 2.4.

Theorem 2.10: If $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ and $B = \langle B_1^+, B_2^+, \ldots, B_n^+, B_1^-, B_2^-, \ldots, B_n^- \rangle$ are bipolar valued multi fuzzy subsemigroups of a semigroup $S$, then $Q_n \beta(A \cap B) = Q_n \beta(A) \cap Q_n \beta(B)$ is also a bipolar valued multi fuzzy subsemigroup of $S$.

Proof: The proof follows from the Theorems 1.8 and 2.3.

Theorem 2.11: If $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ is a bipolar valued multi fuzzy subsemigroup of a semigroup $S$, then $P_n \beta(Q_n \beta(A)) = Q_n \beta(P_n \beta(A)) = \{ \alpha_1, \alpha_2, \ldots, \alpha_n, \beta_1, \beta_2, \ldots, \beta_n \}$ is also a bipolar valued multi fuzzy subsemigroup of $S$.

Proof: The proof follows from the Theorems 2.3 and 2.4.

Theorem 2.12: If $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ and $B = \langle B_1^+, B_2^+, \ldots, B_n^+, B_1^-, B_2^-, \ldots, B_n^- \rangle$ are bipolar valued multi fuzzy subsemigroups of a semigroup $S$, then $G_n \beta(A \cap B) = G_n \beta(A) \cap G_n \beta(B)$ is also a bipolar valued multi fuzzy subsemigroup of $S$.

Proof: The proof follows from the Theorems 1.8 and 2.5.
REFERENCES


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