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MAGNETOHYDRODYNAMIC FLOW IN THE PRESENCE THERMAL RADIATION OF AN OPTICALLY THIN GRAY FLUID OVER A MOVING PLATE

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ABSTRACT

The magnetohydrodynamic unsteady free convection flow over a moving porous vertical plate of an optically thin gray fluid in the presence of thermal radiation when the magnetic Reynolds number is not negligible so that the induced magnetic field is taken into account. The influence of numerus parameters on the process characteristics is studied.

Keywords: *radiation, magnetohydrodynamics.*

INTRODUCTION

Many processes in engineering areas occur at high temperature in the precence of thermal radiation on the flow. Plasma physics, gas turbines, the various propulsion devices for aircraft, missiles, satellites and space vehicles, flow through a porous medium in the presence of thermal radiation and glass production are some examples of such engineering areas.

Seddeek *et al.* [1] studied the effects of thermal radiation and thermal diffusivity on heat transfer over a stretching surface with variable heat flux. The unsteady magnetohydrodynamic flow in the presence of thermal radiation has been analyzed by Perdikis and Rapti [2]. Also, Perdikis and Rapti [3] studied the magnetohydrodynamic flow and thermal radiation of a rarefied gas. Chaudhary *et al.* [4] presented a computational modeling of partial slip effects on magnetohydrodynamic boundary layer flow past an exponential stretching surface in presence of thermal radiation. Ibrahim *et al.*[5] investigated the influence of Joule heating and heat source on radiative magnetohydrodynamic flow over a stretching porous sheet with power-law heat flux. In all these studies the fluid was assumed to be optically thick and using the Rosseland approximation for thermal radiation.

England and Emery [6] studied thermal radiation effects of an optically thin fluid bounded by a stationary vertical plate. The magnetohydrodynamic free convection flow with radiative heat transfer in a rotating and optically thin fluid was investigated by Bestman and Adiepong [7] when the magnetic Reynolds number is negligible. The first order chemical reaction effects on a parabolic flow past an infinite vertical plate with variable temperature and mass diffusion in the presence of thermal radiation for an optically thin fluid has been analyzed by Muthucumaraswamy and Sivakumar [8]. The thermal radiation for an optically thin fluid and the effects on magnetohydrodynamic flow past a linearly accelerated inclined plate in a porous medium with variable temperature was investigated by Endalew and Nayak [9]

In this paper we study the magnetohydrodynamic unsteady free convection flow over a moving porous vertical plate of an optically thin gray fluid in the presence of thermal radiation when the magnetic Reynolds number is not negligible so that the induced magnetic field is taken into account.

ANALYSIS

We consider the unsteady two-dimensional free convection flow of an incompressible, viscous, electrically conducting fluid, optically thin gray fluid in the presence thermal radiation, over an infinite electrically non-conducting and moving vertical porous plate. The origin of the coordinate system is on the plate with the x' -axis along the plate and the y' –axis normal to it. It is assumed that the applied magnetic field is uniform and perpendicular to the plate, so that in the region of the plate the magnetic field to be of the form $\overrightarrow{H'} = (H'_{x'}, H_0^*, 0)$ where H_0^* is the externally applied magnetic field normal to the plate.

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The equations governing the problem are:

<u>Continuity equation</u>

$$\frac{\partial \upsilon'}{\partial y'} = 0 \tag{1}$$

<u>Equation of motion</u>

$$\frac{\partial u'}{\partial t'} + \upsilon' \frac{\partial u'}{\partial y'} = \frac{dU'}{dt'} + \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty) + \frac{\mu}{\rho} H_0^* \frac{\partial H'_{x'}}{\partial y'} \tag{2}$$

<u>Equation of magnetic field</u>

$$\frac{\partial H'_{x'}}{\partial t'} + \upsilon' \frac{\partial H'_{x'}}{\partial y'} = H_0^* \frac{\partial u'}{\partial y'} + \frac{1}{\sigma\mu} \frac{\partial^2 H'_{x'}}{\partial y'^2} \tag{3}$$

<u>Energy equation</u>

$$\frac{\partial T'}{\partial t'} + \upsilon' \frac{\partial T'}{\partial y'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y'} \tag{4}$$

where u', υ' are the components of the velocity parallel and perpendicular to the plate, t' is the time, $U'(t')$ the free stream velocity, ν the kinematic viscosity, g the acceleration due to gravity, β the coefficient of volume expansion, T' the fluid temperature, T'_∞ the fluid temperature at infinity, μ the magnetic permeability, ρ the fluid density, σ the electrical conductivity, k the thermal conductivity, c_p the specific heat at constant pressure and q_r the radiative heat flux.

The boundary conditions are:

$$\begin{aligned} &u' = U_1, \quad \upsilon' = -\upsilon_0, \quad T = T'_w, \quad H'_{x'} = 0, \quad at \quad y' = 0 \\ &u' \to U' = U_2(1 + \varepsilon e^{i\omega' t'}), \quad T' \to T'_\infty, \quad H'_{x'} \to 0, \quad as \quad y' \to \infty \end{aligned} \tag{5}$$

where U_1 is the velocity of the plate, υ_0 the constant suction velocity and the negative sign indicates that it is towards the plate, T'_w the temperature at the plate, U_2 the mean free stream velocity, ω' the frequency of vibration of the fluid and ε $(\varepsilon < 1)$ a constant quantity.

From equation (1) we get

$$\upsilon' = -\upsilon_0 \tag{6}$$

In the case of an optically thin gray fluid the local radiant absorption is expressed as [6]-[9]

$$-\frac{\partial q_r}{\partial y'} = 4a\sigma^*(T'^4_\infty - T'^4) \tag{7}$$

where a is the absorption coefficient and σ^* the Stefan-Boltzman constant.

We assume that the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T'^4 in a Taylor series about T'_∞ and neglecting higher-order terms, thus

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \quad . \tag{8}$$

Using the transformations

$$y = \frac{y'\upsilon_0}{\nu}$$

$$t = \frac{t'\upsilon_0^2}{4\nu}$$

$$u = \frac{u'}{U_2}$$

$$m = \frac{U_1}{U_2}$$

$$U = \frac{U'}{U_2}$$

$$T = \frac{T' - T'_\infty}{T'_w - T'_\infty}$$

$$\omega = \frac{4\nu\omega'}{\upsilon_0^2} \tag{9}$$

$$H = \left(\frac{\mu}{\rho}\right)^{1/2} \frac{H'_{x'}}{U_2}$$

$$M = \left(\frac{\mu}{\rho}\right)^{1/2} \frac{H_0^*}{\upsilon_0} \quad \text{(magnetic parameter)}$$

$$P_m = \nu\sigma\mu \quad \text{(magnetic Prandtl number)}$$

$$G = \frac{\nu g \beta (T'_w - T'_\infty)}{U_2 \upsilon_0^2} \quad \text{(Grashof number)}$$

$$P = \frac{\rho \nu c_p}{k} \quad \text{(Prandtl number)}$$

$$S = \frac{64 a \sigma^* T_\infty'^3 \nu}{\rho \upsilon_0^2 c_p} \quad \text{(radiation parameter),}$$

Equations (2), (3) and (4) through (6), (7), (8) and (9) become

$$\frac{\partial u}{\partial t} - 4\frac{\partial u}{\partial y} = \frac{dU}{dt} + 4GT + 4\frac{\partial^2 u}{\partial y^2} + 4M\frac{\partial H}{\partial y} \tag{10}$$

$$P\frac{\partial T}{\partial t} - 4P\frac{\partial T}{\partial y} = 4\frac{\partial^2 T}{\partial y^2} - PST \tag{11}$$

$$\frac{\partial H}{\partial t} - 4\frac{\partial H}{\partial y} = 4M\frac{\partial u}{\partial y} + \frac{4}{P_m}\frac{\partial^2 H}{\partial y^2} \quad . \tag{12}$$

The corresponding boundary condition are

$$\begin{aligned} &u = m, \quad T = 1, \quad H = 0, \quad at \quad y = 0 \\ &u \to 1 + \varepsilon e^{i\omega t}, \quad T \to 0, \quad H \to 0 \quad as \quad y \to \infty \quad . \end{aligned} \tag{13}$$

In order to solve the system of the differential equations (10)-(12), we assume that

$$u(y,t) = u_0(y) + \varepsilon e^{i\omega t} u_1(y) + \ldots \tag{14}$$

$$T(y,t) = T_0(y) + \varepsilon e^{i\omega t} T_1(y) + \ldots \tag{15}$$

$$H(y,t) = H_0(y) + \varepsilon e^{i\omega t} H_1(y) + \ldots \tag{16}$$

On substituting equations (14)-(16) into equations (10)-(12), we get the following system of differential equations

$$\frac{d^2 u_0}{dy^2} + \frac{du_0}{dy} + M\frac{dH_0}{dy} + GT_0 = 0 \tag{17}$$

$$\frac{d^2 T_0}{dy^2} + P\frac{dT_0}{dy} - \frac{PS}{4}T_0 = 0 \tag{18}$$

$$\frac{d^2 H_0}{dy^2} + P_m\frac{dH_0}{dy} + MP_m\frac{du_0}{dy} = 0 \tag{19}$$

$$\frac{d^2 u_1}{dy^2} + \frac{du_1}{dy} - \frac{i\omega}{4}u_1 = -\frac{i\omega}{4} - M\frac{dH_1}{dy} - GT_1 \tag{20}$$

$$\frac{d^2T_1}{dy^2}+P\frac{dT_1}{dy}-\frac{i\omega}{4}PT_1-\frac{PS}{4}T_1=0 \tag{21}$$

$$\frac{d^2H_1}{dy^2}+P_m\frac{dH_1}{dy}-\frac{i\omega}{4}P_mH_1=-MP_m\frac{du_1}{dy} \tag{22}$$

The corresponding boundary conditions (13) are

$$\begin{aligned} &u_0=m,\quad u_1=0,\quad T_0=1,\quad T_1=0,\quad H_0=0,\quad H_1=0 \quad at \ \ y=0 \\ &u_0\to 1,\quad u_1\to 1,\quad T_0\to 0,\quad T_1\to 0,\quad H_0\to 0,\quad H_1\to 0 \quad as \ \ y\to\infty \ . \end{aligned} \tag{23}$$

In order to solve the system of the differential equations (17)-(22), we put

$$\begin{aligned} u_1(y)&=u_{11}(y)+iu_{12}(y) \\ T_1(y)&=T_{11}(y)+iT_{12}(y) \\ H_1(y)&=H_{11}(y)+iH_{12}(y) \end{aligned} \tag{24}$$

and equating the terms which are independent of i and the coefficients of i, we get

$$\frac{d^2u_0}{dy^2}+\frac{du_0}{dy}+M\frac{dH_0}{dy}+GT_0=0 \tag{25}$$

$$\frac{d^2T_0}{dy^2}+P\frac{dT_0}{dy}-\frac{PS}{4}T_0=0 \tag{26}$$

$$\frac{d^2H_0}{dy^2}+P_m\frac{dH_0}{dy}+MP_m\frac{du_0}{dy}=0 \tag{27}$$

$$\frac{d^2u_{11}}{dy^2}+\frac{du_{11}}{dy}+\frac{\omega}{4}u_{12}=-M\frac{dH_{11}}{dy}-GT_{11} \tag{28}$$

$$\frac{d^2u_{12}}{dy^2}+\frac{du_{12}}{dy}-\frac{\omega}{4}u_{11}=-\frac{\omega}{4}-M\frac{dH_{12}}{dy}-GT_{12} \tag{29}$$

$$\frac{d^2T_{11}}{dy^2}+P\frac{dT_{11}}{dy}-\frac{PS}{4}T_{11}+\frac{\omega}{4}PT_{12}=0 \tag{30}$$

$$\frac{d^2T_{12}}{dy^2}+P\frac{dT_{12}}{dy}-\frac{PS}{4}T_{12}-\frac{\omega}{4}PT_{11}=0 \tag{31}$$

$$\frac{d^2H_{11}}{dy^2}+P_m\frac{dH_{11}}{dy}+\frac{\omega}{4}P_mH_{12}=-MP_m\frac{du_{11}}{dy} \tag{32}$$

$$\frac{d^2H_{12}}{dy^2}+P_m\frac{dH_{12}}{dy}-\frac{\omega}{4}P_mH_{11}=-MP_m\frac{du_{12}}{dy} \quad . \tag{33}$$

The corresponding boundary conditions (23) give

$$\left.\begin{aligned} &u_0=m,\quad u_{11}=0,\quad u_{12}=0,\\ &T_0=1,\quad T_{11}=0,\quad T_{12}=0,\\ &H_0=0,\quad H_{11}=0\quad H_{12}=0 \end{aligned}\right\} \quad at \ \ y=0 \tag{34}$$

$$\left.\begin{aligned} &u_0\to 1,\quad u_{11}\to 1,\quad u_{12}\to 0,\\ &T_0\to 0,\quad T_{11}\to 0,\quad T_{12}\to 0,\\ &H_0\to 0,\quad H_{11}\to 0\quad H_{12}\to 0 \end{aligned}\right\} \quad as \ \ y\to\infty \quad .$$

The solutions for the real part of the velocity, temperature and magnetic field are given respectively by the expressions

$$u(y,t)=u_0+\varepsilon(u_{11}\cos\omega t-u_{12}\sin\omega t) \tag{35}$$

$$T(y,t)=T_0+\varepsilon(T_{11}\cos\omega t-T_{12}\sin\omega t) \tag{36}$$

$$H(y,t)=H_0+\varepsilon(H_{11}\cos\omega t-H_{12}\sin\omega t) \quad . \tag{37}$$

When $\omega t = \dfrac{\pi}{2}$ the above expressions become

$$u(y) = u_0 - \varepsilon u_{12} \tag{38}$$

$$T(y) = T_0 - \varepsilon T_{12} \tag{39}$$

$$H(y) = H_0 - \varepsilon H_{12} \tag{40}$$

where $u_0,\ T_0,\ H_0, u_{12},\ T_{12},\ H_{12}$ derive from the numerical solution of the system of the differential equations (25)-(33), under the boundary conditions (34), by using shooting method.

RESULTS AND DISCUSSION

In order to understand the physical situation of the problem we have computed the numerical values of the velocity, temperature for different values of physical parameters. The obtained numerical values are illustrated in Figs 1-4.

Fig.1 demonstrates the effect the radiation parameter S on the velocity $u(y)$, when $M = 0.2$, $P = 2$, $P_m = 0.001$, $G = 3$, $\varepsilon = 0.1$, $m = 0.4$. It is observed that the velocity decreases with the decrease of the radiation parameter S.

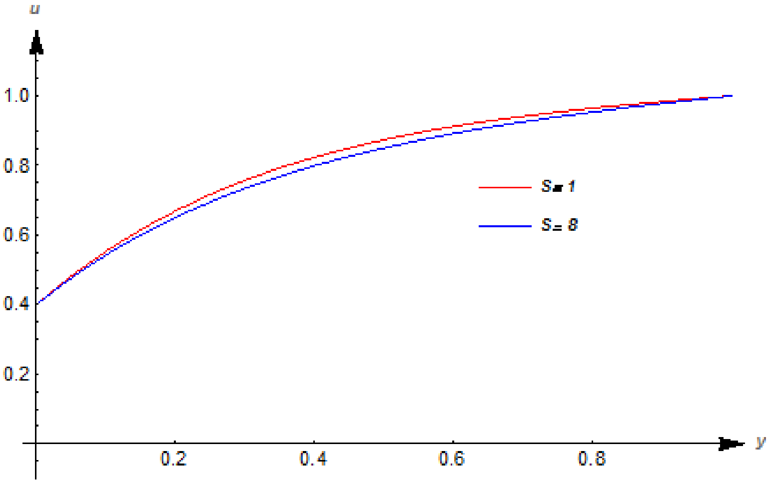

Fig.-1: Velocity profiles for different values of radiation parameter S

The effect of the Grashof number G on the velocity $u(y)$, is shown in Fig. 2, when $M = 0.2$, $P = 2$, $P_m = 0.001$, $S = 1$, $\varepsilon = 0.1$, $m = 0.4$. It is noticed that when the Grashof number G increases the velocity also increases.

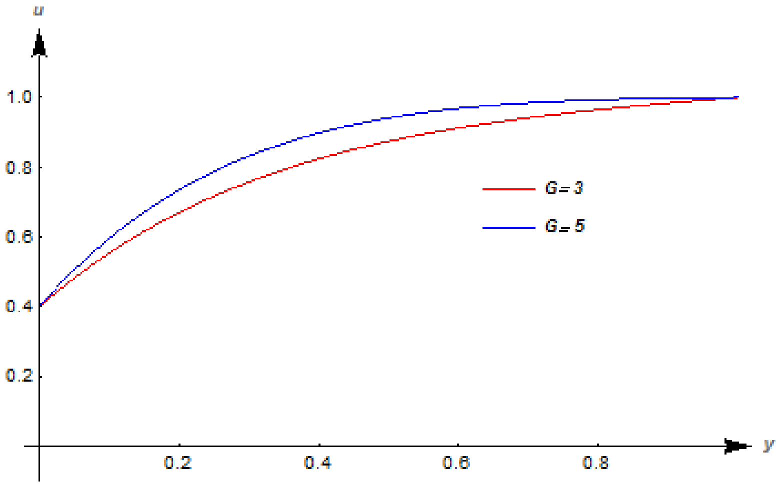

Fig.-2: Velocity profiles for different values of Grashof number G

Fig. 3 shows the effect of the magnetic parameter M on the velocity $u(y)$, when $P = 2$, $G = 3$, $P_m = 0.001$, $S = 1$, $\varepsilon = 0.1$, $m = 0.4$. It is observed that the velocity decrease with the increase of the magnetic parameter M.

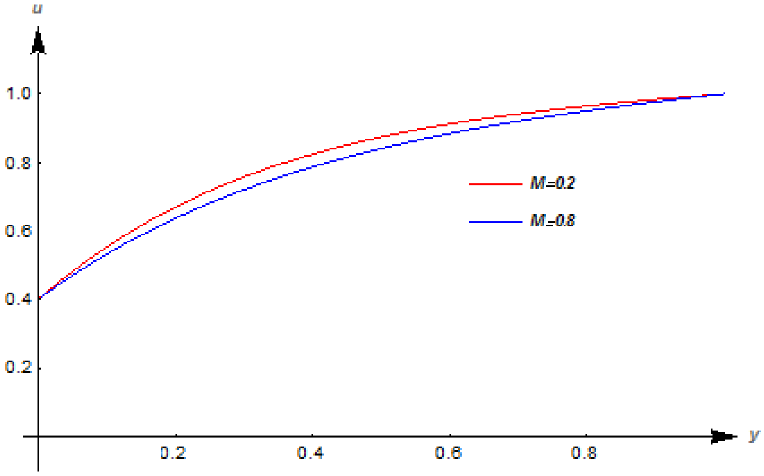

Fig.-3: Velocity profiles for different values of magnetic parameter M

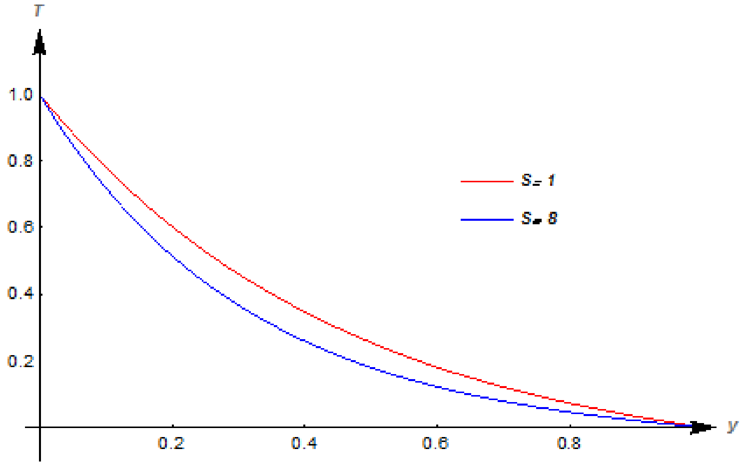

Fig.-4: Temperature profiles for different values of radiation parameter S

Fig.4 demonstrates the effect the radiation parameter S on the temperature $T(y)$, when $P=2$, $\varepsilon=0.1$. It is observed that the temperature decreases with the decrease of the radiation parameter S.

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