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Almost *v*-closed mappings

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ABSTRACT

The aim of the paper is to study basic properties of v-closed mappings and interrelations with other mappings.

Keywords: *v-closed sets, v-continuity, v-irresolute, v-closed mappings and almost v-closed mappings.*

Ams: *54C10, 54C08, 54C05.*

1. INTRODUCTION:

Mappings plays an important role in the study of modern mathematics, especially in Topology and Functional Analysis. Closed mappings are one such mapping which are studied for different types of closed sets by various mathematicians for the past many years. In this paper we tried to study a new variety of closed maps called almost *v*-closed maps. Throughout the paper X, Y means a topological spaces (X,τ) and (Y,σ) unless otherwise mentioned without any separation axioms defined in it.

2. PRELIMINARIES:

Definition 2.1: $A\subset X$ is called
(i) pre-open if $A\subseteq (cl\ A)^o$ and pre-closed if $cl\{(A^o)\} \subseteq A$;
(ii) semi-open if $A\subseteq cl\{(A^o)\}$ and semi-closed if $(cl\ A)^o\subseteq A$;
(iii)semipre-open[β-open] if $A\subseteq cl\{((cl\ A)^o)\}$ and semipre-closed[β-closed] if $(cl\{(A^o)\})^o\subseteq A$;
(iv) α-open if $A\subseteq (cl\{(A^o)\})^o$ and α-closed if $cl\{((cl\ A)^o)\}\subseteq A$;
(v) regular open if $A = (cl\ A)^o$ and regular closed if $A = cl\{(A^o)\}$
(vi) *v*-open if there exists a regular open set U such that $U\subseteq A\subseteq cl\ U$.
(vii)rα-closed if there exists a regular closed set U such that $\alpha\ (U)o \subseteq A\subseteq U$.
(viii)g-closed[resp: rg-closed] if $cl\ A\subseteq U$ whenever $A\subseteq U$ and U is open[resp: regular open].

Note 1: From the above definition we have the following implication diagram.

al.rα.closed set → al.*v*-closed set
↑ ↓
al.r. closed set →al.s. closed set → al. β. closed set
↓
al.closed set → al. α. closed set
↓
al.p. closed set

Definition 2.2: A function f: X→ Y is said to be

1. Continuous [resp: pre-continuous; semi-continuous; β-continuous; α-continuous; nearly-continuous; *v*-continuous; rα-continuous] if the inverse image of every open set is open [resp:pre-open; semi-open; β-open; α-open; regular-open; *v*-open; rα-open]

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2. Irresolute [resp:pre-irresolute; β-irresolute; α-irresolute; nearly-irresolute; *v*-irresolute; rα-irresolute] if the inverse image of every semi-open[resp:pre-open; β-open; α-open; regular-open; *v*-open; rα-open] set is semi-open[resp:pre-open; β-open; α-open; regular-open; *v*-open; rα-open]

3. closed[resp:pre-closed; semi-closed; β-closed; α-closed; nearly-closed; rα-closed] if the image of every closed set is closed[resp:pre-closed; semi-closed; β-closed; α-closed; regular-closed; rα-closed] almost closed[resp:almost pre-closed; almost semi-closed; almost β-closed; almost α-closed; almost nearly-closed; almost rα-closed] if the image of every regular closed set is closed[resp:pre-closed; semi-closed; β-closed; α-closed; regular-closed; rα-closed]

4. g-continuous [resp: rg-continuous] if the inverse image of every open set is g-open [resp: rg-open]

Definition 2.3: X is said to be $T_{1/2}$[r-$T_{1/2}$] if every [regular-] generalized closed set is [regular-] closed 3.
Almost *v*-closed mappings:

Definition 3.1: A function *f:* X→ Y is said to be almost *v*-closed if image of every regular closed set in X is *v*-closed in Y

Theorem 3.1:
(i) Every almost-r-closed map is almost *v*-closed but not conversely.
(ii) Every almost-r-closed map is almost rα-closed but not conversely.
(iii) Every almost rα-closed map is almost *v*-closed but not conversely.
(iv) Every almost *v*-closed map is almost semi-closed but not conversely.
(v) Every almost *v*-closed map is almost β-closed but not conversely.
(vi) Every almost-r-closed map is almost closed but not conversely.
(vii) Every almost-r-closed map is almost semi-closed but not conversely.

Proof: (i) *f* is almost-r-closed ⇒ image of every regular closed set is r-closed ⇒ image of every regular closed set is *v*-closed [since every r-closed set is *v*-closed] ⇒ *f* is almost *v*-closed.

Similarly we can prove the remaining parts using definition 2.1 and Note 1.

Example 1: Let X = Y = {a, b, c}; τ = σ = {ϕ, {a}, {b}, {a, b}, X}. Let *f:* X→ Y is identity map. Then *f* is almost *v*-closed.

Example 2: Let X = Y = {a, b, c}; τ = σ = {ϕ, {a}, {b}, {a, b}, X}. Let *f:* X→ Y be the map defined as *f*(a) = b; *f*(b) = c and *f*(c) = a is not almost *v*-closed.

Example 3: Let X = Y = {a, b, c}; τ = {ϕ, {a}, {b, c}, X} and σ = {ϕ, {a}, {b}, {a, b}, X} and let *f:* X→ Y is identity map. Then *f* is almost *v*-closed}, almost semi-closed, almost β-closed and almost rα-closed but not almost-closed, almost-r-closed, almost-pre-closed and almost α-closed.

Example 4: Let X = Y = {a, b, c}; τ = {ϕ, {a}, {b, c}, X} and σ = {ϕ, {a, c}, X} and let *f:* X→ Y is identity map. Then *f* is almost β-closed but not almost *v*-closed}, almost semi-closed, almost rα-closed, almost-closed, almost-r-closed, almost-pre-closed and almost α-closed.

Example 5: Let X = Y = {a, b, c}; τ = {ϕ, {a}, {b, c}, X} and σ = {ϕ, {a, c}, X} and let *f:* X→ Y be the map defined as *f*(a) = c; *f*(b) = b and *f*(c) = a. Then *f* is almost β-closed and almost-pre-closed but not almost *v*-closed}, almost semi-closed, almost rα-closed, almost-closed, almost-r-closed and almost α-closed.

Example 6: Let X = Y = {a, b, c}; τ = {ϕ, {a}, {b, c}, X} and σ = {ϕ, {b}, {a, b}, {b, c}, X} and let *f:* X→ Y be the map defined as *f* (a) = c; *f* (b) = a and *f*(c) = c. Then *f* is almost α-closed and almost-closed but not almost *v*-closed}.

Theorem 3.2:
(i) If RαO(Y) = RO(Y), then *f* is almost rα-closed iff *f* is almost-r-closed.
(ii) If RαO(Y) = *v*O(Y), then *f* is almost rα-closed iff *f* is almost *v*-closed.
(iii) If *v*O(Y) = RO(Y), then *f* is almost-r-closed iff *f* is almost *v*-closed.
(iv) If *v*O(Y) = αO(Y), then *f* is almost α-closed iff *f* is almost *v*-closed.

Corollary 3.1:
(i) Every almost rα-closed map is almost semi-closed and hence almost β-closed but not conversely

(ii)Every almost-r-closed map is almost β-closed but not conversely

Note 2:
(i) almost closed map and almost *v*-closed map are independent to each other
(ii) almost α-closed map and almost *v*-closed map are independent to each other
(iii) almost pre-closed map and almost *v*-closed map are independent to each other

Example 7: In Example 2 above, *f* is almost closed; almost pre-closed and almost α-closed but not almost *v*-closed.

Example 8: *f* as in Example 3 is almost *v*-closed but not almost closed; almost pre-closed and almost α-closed.

Example 9: *f* as in Example 6 is almost closed; almost pre-closed and almost α-closed but not almost *v*-closed.

Example 10: *f* as in Example 5 is almost β-closed but not almost *v*-closed

Note 3: We have the following implication diagram among the closed maps.

al.rα.c-map → **al.*v.c*-map**
↑ ↓
al.r.c-map → al.s.c-map → al. β.c-map
↓
al.cpen map → al. α.c-map
↓
al.p.c-map

Theorem 3.3:
(i) If *f* is almost closed and *g* is *v*-closed then *g*•*f* is almost *v*-closed
(ii) If *f* is almost closed and *g* is r-closed then *g*•*f* is almost *v*-closed
(iii)If *f* and *g* are almost-r-closed then *g*•*f* is almost *v*-closed
(iv) If *f* is almost-r-closed and *g* is almost *v*-closed then *g*•*f* is almost *v*-closed

Proof: (i) Let A be regular closed set in X ⇒ *f* (A) is closed in Y ⇒ *g* (*f* (A)) is *v*-closed in Z ⇒ *g*•*f*(A) is *v*-closed in Z ⇒ *g*• *f* is almost *v*-closed.

Similarly we can prove the remaining parts and so omitted.

Corollary 3.2:
(i) If *f* is almost closed and *g* is *v*-closed[r-closed] then *g*•*f* is almost semi-closed and hence almost β-closed

(ii) If *f* and *g* are almost-r-closed then *g*•*f* is almost semi-closed and hence almost β-closed.

(iii) If *f* is almost-r-closed and *g* is almost *v*-closed then *g*•*f* is almost semi-closed and hence almost β-closed

Theorem 3.4: If *f:* X→ Y is almost *v*-closed, then $v(cl\{f(A)\})\subset f(cl\{A\})$

Proof: Let A⊂ X and *f:* X→ Y is *v*-closed gives $f(cl\{A\})$ is *v*-closed in Y and $f(A) \subset f(cl\{A\})$ which in turn gives

$v(cl\{f(A)\}) \subset vcl\{(f(cl\{A\}))\}$ (1)

Since $f(cl\{A\})$ is *v*-closed in Y, $vcl\{(f(cl\{A\}))\} = f(cl\{A\})$ (2)

combaining (1) and (2) we have $v(cl\{f(A)\}) \subset (f(cl\{A\}))$ for every subset A of X.

Remark 1: converse is not true in general.

Corollary 3.3: If *f:* X→ Y is almost r-closed, then $v(cl\{f(A)\}) \subset f(cl\{A\})$

Theorem 3.5: If *f:* X→ Y is almost *v*-closed and A⊂ X is r-closed, then *f* (A) is τ_v-closed in Y.

Proof: Let A⊂ X and *f:* X→ Y is almost *v*-closed implies $v(cl\{f(A)\}) \subset f(cl\{A\})$ which in turn implies $v(cl\{f(A)\}) \subset f(A)$, since $f(A) = f(cl\{A\})$. But $f(A) \subset v(cl\{f(A)\})$. Combaining we get $f(A) = v(cl\{f(A)\})$. Therefore $f(A)$ is τ_v-closed in Y.

Corollary 3.4: If f: X→ Y is almost r-closed, then f (A) is τ_v-closed in Y if A is r-closed set in X.

Theorem 3.6: If v(cl{A}) = r(cl{A}) for every A⊂ Y, then the following are equivalent:

(i) f: X→ Y is v-closed map
(ii) v(cl{f(A)}) ⊂ f(cl{A})

Proof: (i) ⇒ (ii) follows from theorem 3.4

(ii) ⇒ (i) Let A be any r-closed set in X, then f(A) = f(cl{A})⊃ v(cl{f(A)}) by hypothesis. We have f(A) ⊂ v(cl{f(A)}). Combaining we get f(A) = v(cl{f(A)}) = r(cl{f(A)})[by given condition] which implies f(A) is r-closed and hence v-closed. Thus f is v-closed.

Theorem 3.7: f: X→ Y is almost v-closed iff for each subset S of Y and each r-closed set U containing f^{-1}(S), there is a v-closed set V of Y such that S⊂ V and f^{-1}(V) ⊂ U.

Proof: Assume f is almost v-closed, S⊂ Y and U an r-closed set of X containing f^{-1}(S), then f(X - U) is v-closed in Y and V = Y - f(X - U) is v-closed in Y. f^{-1}(S) ⊂ U implies S⊂ V and f^{-1}(V) = X - f^{-1}(f(X - U)) ⊂ X - (X - U) = U.

Conversely let F be r-closed in X, then f^{-1}(f(F^c))⊂ F^c. By hypothesis, exists V∈ v O(Y) such that f(F^c)⊂ V and f^{-1}(V) ⊂ F^c and so f⊂ (f^{-1}(V))^c. Hence V^c⊂f(F) ⊂ f[(f^{-1}(V))^c]⊂ V^c implies f(F) ⊂ V^c, which implies f(F) = V^c. Thus f(F) is v-closed in Y and therefore f is v-closed.

Remark 2: composition of two almost v-closed maps is not almost v-closed in general

Theorem 3.8: Let X, Y, Z be topological spaces and every v-closed set is r-closed in Y, then the composition of two almost v-closed maps is almost v-closed.

Proof: Let A be regular closed in X ⇒ f (A) is v-closed in Y ⇒ f(A) is r-closed in Y[by assumption] ⇒ g(f(A)) is v-closed in Z ⇒ g•f(A) is v-closed in Z ⇒ g•f is almost v-closed.

Theorem 3.9: If f: X→ Y is almost g-closed; g:Y→ Z is v-closed[r-closed] and Y is $T_{1/2}$[r-$T_{1/2}$], then g•f is almost v-closed.

Proof:(i) Let A be regular closed in X ⇒ f(A) is g-closed in Y ⇒ f(A) is closed in Y[since Y is $T_{1/2}$] ⇒ g(f(A)) is v-closed in Z ⇒ g•f(A) is v-closed in Z ⇒ g•f is almost v-closed.

(ii) Since every g-closed set is rg-closed, this part follows from the above case.

Corollary 3.5: If f: X→ Y is almost g-closed; g: Y→ Z is v-closed[r-closed] and Y is $T_{1/2}${r-$T_{1/2}$}, then g•f is almost semi-closed and hence almost β-closed.

Corollary 3.6: If f: X→ Y is almost g-closed; g: Y→ Z is almost v-closed[almost r-closed] and Y is r-$T_{1/2}$, then g•f is almost semi-closed and hence almost β-closed.

Proof: Since every g-closed set is rg-closed, the proof follows from theorem 3.9.

Theorem 3.10: If f: X→ Y is almost rg-closed; g: Y→ Z is v-closed[r-closed] and Y is r-$T_{1/2}$, then g•f is almost v-closed.

Proof: Let A be regular closed in X ⇒ f(A) is rg-closed in Y ⇒ f(A) is r-closed in Y[since Y is r-$T_{1/2}$] ⇒ f(A) is closed in Y[since every r-closed set is closed] ⇒ g(f(A)) is v-closed in Z ⇒ g•f(A) is v-closed in Z ⇒ g•f is almost v-closed.

Theorem 3.11: If f: X→ Y is almost rg-closed; g: Y→ Z is almost v-closed [almost r-closed] and Y is r-$T_{1/2}$, then g•f is almost v-closed.

Corollary 3.7: If f: X→ Y is almost rg-closed; g: Y→ Z is v-closed[r-closed] and Y is r-$T_{1/2}$, then g•f is almost semi-closed and hence almost β-closed.

Corollary 3.8: If f: X→ Y is almost rg-closed; g: Y→ Z is almost v-closed[almost r-closed] and Y is r-$T_{1/2}$, then g•f is almost semi-closed and hence almost β-closed.

Theorem 3.12: If *f:* X→ Y; *g*:Y→ Z be two mappings such that *g•f* is *v*-closed[r-closed]. Then the following are true
(i) If *f* is continuous [r-continuous] and surjective, then *g* is *v*-closed
(ii) If *f* is g-continuous, surjective and X is $T_{1/2}$, then *g* is *v*-closed
(iii)If *f* is g-continuous [rg-continuous], surjective and X is r-$T_{1/2}$, then *g* is *v*-closed

Proof: (i) Let A be regular closed in Y ⇒ f^{-1}(A) is closed in X ⇒ *g•f*(f^{-1}(A)) is *v*-closed in Z ⇒ *g*(A) is *v*-closed in Z ⇒ *g* is almost *v*-closed.

Similarly we can prove the remaining parts and so omitted.

Corollary 3.9: If *f:* X→ Y; *g*: Y→ Z be two mappings such that *g•f* is *v*-closed[r-closed]. Then the following are true

(i) If *f* is continuous[r-continuous] and surjective, then *g* is semi-closed and hence β-closed
(ii) If *f* is g-continuous, surjective and X is $T_{1/2}$, then *g* is semi-closed and hence β-closed
(iii)If *f* is g-continuous [rg-continuous], surjective and X is r-$T_{1/2}$, then *g* is semi-closed and hence β-closed

Theorem 3.13: If X is *v*-regular, *f:* X→ Y is r-closed, nearly-continuous, *v*-closed surjection and A^- = A for every *v*-closed set in Y, then Y is *v*-regular.

Proof: Let $p\in U\in vO(Y)$, there exists a point $x\in X$ such that $f(x) = p$ by surjection. Since *X* is *v*-regular and *f* is nearly-continuous there exists $V\in RC(X)$ such that $x\in V\subset V^-\subset f^{-1}(U)$ which implies

$$p\in f(V)\subset f(V^-) \subset U \quad (1)$$

for *f* is *v*-closed, $f(V^-)\subset U$ is *v*-closed. By hypothesis $(f(V^-))^- = f(V^-)$ and

$$(f(V^-))^- = (f(V)^- \quad (2)$$

Combaining (1) and (2) $p\in f(V)\subset(f(V))^- \subset U$ and $f(V)$ is r-closed. Hence Y is *v*-regular.

Corollary 3.10: If X is *v*-regular, *f:* X→ Y is r-closed, nearly-continuous, *v*-closed surjection and A^- = A for every r-closed set in Y, then Y is *v*-regular.

Theorem 3.14: If *f:* X→ Y is almost *v*-closed [almost-r-closed] and A is regular closed set of X, then $f_A:(X, \tau_A) \to (Y, \sigma)$ is *v*-closed.

Proof: Let F be r-closed set in A. Then *f* = A∩ E for some r-closed set E of X and so F is r-closed in X which implies *f*(A) is *v*-closed in Y. But $f(F) = f_A(F)$ and therefore f_A is *v*-closed.

Corollary 3.11: If *f:* X→ Y is almost *v*-closed [almost-r-closed] and A is regular closed set of X, then $f_A:(X, \tau_A) \to (Y, \sigma)$ is semi-closed and hence β-closed.

Theorem 3.15: If *f:* X→ Y is almost *v*-closed [almost-r-closed], X is $T_{1/2}$ and A is g-closed set of X, then $f_A:(X, \tau_A) \to (Y, \sigma)$ is almost *v*-closed.

Corollary 3.12: If *f:* X→ Y is almost *v*-closed [almost-r-closed], X is $T_{1/2}$ and A is g-closed set of X, then $f_A:(X, \tau_A) \to (Y, \sigma)$ is almost semi-closed and hence almost β-closed.

Theorem 3.16: If $f_i: X_i \to Y_i$ be almost *v*-closed [almost-r-closed] for i = 1, 2. Let $f: X_1\times X_2 \to Y_1\times Y_2$ be defined as follows: $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$. Then $f: X_1\times X_2 \to Y_1\times Y_2$ is almost *v*-closed.

Proof: Let $U_1\times U_2\subset X_1\times X_2$ where U_i is regular closed in X_i for i = 1, 2. Then $f(U_1\times U_2) = f_1(U_1) \times f_2(U_2)$ a *v*-closed set in $Y_1\times Y_2$. Thus $f(U_1\times U_2)$ is *v*-closed and hence *f* is almost *v*-closed.

Corollary 3.13: If $f_i: X_i \to Y_i$ be almost *v*-closed [almost r-closed] for i = 1, 2. Let $f: X_1\times X_2 \to Y_1\times Y_2$ be defined as follows: $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$. Then $f: X_1\times X_2 \to Y_1\times Y_2$ is almost semi-closed and hence almost β-closed.

Theorem 3.17: Let $h: X \to X_1\times X_2$ be almost *v*-closed [almost-r-closed]. Let $f_i: X \to X_i$ be defined as $h(x) = (x_1, x_2)$ and $f_i(x) = x_i$. Then $f_i: X \to X_i$ is almost *v*-closed for i = 1, 2.

Corollary 3.14: Let h: X → $X_1 \times X_2$ be almost *v*-closed [almost-r-closed]. Let f_i: X → X_i be defined as $h(x) = (x_1, x_2)$ and $f_i(x) = x_i$. Then f_i: X → X_i is almost semi-closed and hence almost β-closed for i = 1, 2.

CONCLUSION:

We studied some properties and interrelations of almost *v*-closed mappings.

REFERENCES:

[1] Asit Kumar Sen and P. Bhattcharya, On preclosed mappings, Bull. Cal. Math. Soc., 85(1993)409-412.

[2.] S. Balasubramanian, C. Sandhya & P.A.S. Vyjayanthi, *v*-open sets and *v*-mappings, I.J.M.S.E.A., Vol.4 No. II(2010)395-405.

[3] S. Balasubramanian & P.A.S. Vyjayanthi, *v*-open mappings, Scientia Magna Vol 6. No. 4(2010) 118 - 124.

[4] S. Balasubramanian, C. Sandhya & P.A.S. Vyjayanthi, note on *v*-continuity, Bull. Kerala Math Association-Vol. 6, No.2, (2010 December) 21-28.

[5] S. Balasubramanian & P.A.S. Vyjayanthi, *v*-closed mappings, Jr. Advanced Research in Pure Mathematics, Vol. 3. No. 1(2011)135 - 143.

[6] Di Maio. G., and Noiri.T, I.J.P.A.M., 18(3) (1987) 226-233.

[7] J. Dontchev, Mem.Fac.Sci.Kochi Univ.ser. A., Math., 16(1995), 35-48.

[8] W. Dunham, $T_{1/2}$ Spaces, Kyungpook Math. J.17 (1977), 161-169.

[9] P. E. Long and L. L. Herington, Basic Properties of Regular Closed Functions, Rend. Cir. Mat. Palermo, 27(1978), 20-28.

[10] S. R. Malghan, Generalized closed maps, J. Karnatak Univ. Sci., 27(1982), 82 -88.

[11] A. S. Mashour, et.al., On pre-continuity and weak pre-continuity, Math. Phys. Soc. Egypt 53(1982), 47-53.

[12] A. S. Mashour, I. A. Hasanein and S. N. El. Deep, α-continuous and α-open mappings, Acta Math. Hungar., 41(1983), 213-218.

[13] T. Noiri, A generalization of closed mappings, Atti. Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur., 54 (1973), 412-415.

[14] T. Noiri, Almost α*g*-closed functions and separation axioms, Actaq Math. Hungar. 82(3) (1999), 193-205.

[15] N. Palaniappan, Studies on Regular-Generalized Closed Sets and Maps in Topological Spaces, Ph. D Thesis, Alagappa University, Karikudi, (1995).

[16] Shantha Bose., Semi open sets and semicontinuity, Bull. Cal. Math. Soc.,

[17] A. K. Sen and P.Bhattacharyya, On preclosed mappings, Bull. Cal. Math. Soc., 85(1993), 409-412.

[18] M. K. Singal and A. R. Singal, Almost continuous mappings, Yokohoma Math. J.13 (1973), 27-31.

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