

**INTUITIONISTIC FUZZY STRONGLY α GENERALIZED STAR CLOSED SETS
IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES**

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ABSTRACT

Our aim is to develop new concepts of intuitionistic fuzzy sets called as intuitionistic fuzzy strongly α generalized star closed set in intuitionistic fuzzy topological spaces. In this paper, we have investigated the relationship between the intuitionistic fuzzy strongly α generalized star closed set and the already existing intuitionistic closed sets. We also have analyzed their properties and their some interesting theorems.

Key Word: IFG*C set, IFS α G*C set, IFS α G*O set.

I. INTRODUCTION

Several mathematicians have produced their works in Fuzzy sets. After Zadeh [19] introducing the concept of fuzzy sets, Atanassov [2] extended the concept to intuitionistic fuzzy sets. Later Coker [5] developed the notion of intuitionistic fuzzy topological space. Various concepts of generalised closed sets have been considered by many authors using the concept of intuitionistic fuzzy sets.

II. PRELIMINARIES

Definition 2.1: [2] Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$ where the functions $\mu_A: X \rightarrow [0,1]$ and $\nu_A: X \rightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. The set of all intuitionistic fuzzy set in X is denoted by $IFS(X)$.

Definition 2.2: [2] Let $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$ be an IF set on X , then the complement A^C may be defined as $A^C = \{ \langle x, \nu_A(x), \mu_A(x) \rangle : x \in X \}$
For any two IFS sets A and B , $(A \cup B)^C = A^C \cap B^C$; $(A \cap B)^C = A^C \cup B^C$

Definition 2.3: [2] For any two IFS sets $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle : x \in X \}$. we may have

1. $A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x) \forall x \in X$
2. $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle$
3. $A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle$

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Definition 2.4: [3] An intuitionistic fuzzy topology (IFT in short) on X is a family of IFSs in X satisfying the following axioms.

1. $\tilde{0}, \tilde{1} \in \tau$
2. $A \cup B \in \tau$ for any $A, B \in \tau$
3. $\cap A_i \in \tau$ for any family $\{A_i / i \in J\} \subseteq \tau$.

In this case, the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X . The complement (A^c) of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X . In this paper, Intuitionistic fuzzy interior is denoted by int and Intuitionistic fuzzy closure is denoted by cl .

Definition 2.5: [3] Let (X, τ) be an IFTS and $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle : x \in X\}$ be an IFS in X . Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by $int(A) = \cup \{O / O \text{ is an IFOS in } X \text{ and } O \subseteq A\}$ and $cl(A) = \cap \{W / W \text{ is an IFCS in } X \text{ and } A \subseteq W\}$.

Proposition 2.1: [3] For any IFS A in (X, τ) , we have

- (1) $int(\tilde{0}) = \tilde{0}$ and $cl(\tilde{0}) = \tilde{0}$,
- (2) $int(\tilde{1}) = \tilde{1}$ and $cl(\tilde{1}) = \tilde{1}$,
- (3) $(int(A))^c = cl(A^c)$,
- (4) $(cl(A))^c = int(A^c)$

Definition 2.6: An intuitionistic fuzzy set W of an intuitionistic fuzzy topological space (X, τ) is called as

1. an intuitionistic fuzzy semi open set (IFSO) [7] if $W \subseteq cl(int(W))$ and Intuitionistic fuzzy semi closed set if $int(cl(W)) \subseteq W$.
2. an Intuitionistic fuzzy pre-open set (IFPO) [7] if $W \subseteq int(cl(W))$ and Intuitionistic fuzzy pre-closed set (IFPC) if $cl(int(W)) \subseteq W$.
3. an Intuitionistic fuzzy α -open set (IF α O) [8] if $W \subseteq int(cl(int(W)))$ and an Intuitionistic fuzzy α -closed set (IF α C) if $cl(int(cl(W))) \subseteq W$.
4. an Intuitionistic fuzzy semi pre-open set (IFSPO) (Intuitionistic fuzzy β -open set) [18] if $W \subseteq cl(int(cl(W)))$ and Intuitionistic fuzzy semi pre-closed set (IFSPC) if $int(cl(int(W))) \subseteq W$.
5. an Intuitionistic fuzzy semi generalized closed set (simply IFGSC) [14] if $Scl(W) \subseteq O$, whenever $W \subseteq O$ and O is IFO in X .
6. an Intuitionistic fuzzy generalized star closed set (simply IFG^{*}C) [6] if $cl(W) \subseteq O$ whenever $W \subseteq O$ and O is IFGO in X .
7. an Intuitionistic fuzzy strongly generalized star closed set (simply IF strongly G^{*}C) [9] if $cl(int(W)) \subseteq O$ whenever $W \subseteq O$ and O is IFGO in X .
8. an Intuitionistic fuzzy regular weakly generalized closed set (briefly IFRWGC) [12] if $cl(int(W)) \subseteq O$, whenever $W \subseteq O$ and O is IFO in (X, τ_1) .
9. an Intuitionistic fuzzy generalized pre closed (briefly IFGPC) [11] if $Pcl(W) \subseteq O$ whenever $W \subseteq O$ and O is IFO in (X, τ_1) .
10. an Intuitionistic fuzzy weakly generalized closed set (briefly IFWGC) [10] if $cl(int(W)) \subseteq O$, whenever $W \subseteq O$ and O is IFSO in X .
11. an Intuitionistic fuzzy generalized star pre closed set (briefly IFG^{*}PC) if $Pcl(W) \subseteq O$, whenever $W \subseteq O$ and O is IFGO in X .
12. an Intuitionistic fuzzy generalized pre semi closed (IFGPSC) set [13] if $Pcl(W) \subseteq O$ whenever $W \subseteq O$ and O is IFSO in X .
13. an Intuitionistic fuzzy semi weakly generalized closed (IFSWG^{*}C) set [156] if $cl(int(W)) \subseteq O$ whenever $W \subseteq O$ and O is IFSO in X .
14. an Intuitionistic fuzzy generalized semi regular closed (IFGSRC) set [1] if $Scl(W) \subseteq O$ whenever $W \subseteq O$ and O is IFRO in X .
15. an Intuitionistic fuzzy generalized pre regular closed set (IFGPRC) [17] if $Pcl(W) \subseteq O$ whenever $W \subseteq O$ and O is IFRO in X .

III. INTUITIONISTIC FUZZY STRONGLY α GENERALIZED STAR CLOSED SETS (IFS α G^{*}C SETS)

In this section, we introduce the concept of Intuitionistic fuzzy strongly alpha generalized star closed sets (IFS α G^{*}C) in intuitionistic topological space.

Definition 3.1: An IFS W of an intuitionistic fuzzy topological space (X, τ) is called an intuitionistic fuzzy strongly alpha generalized star closed set (IFS α G^{*}C) if $acl(W) \subseteq O$, whenever $W \subseteq O$ and O is IFG^{*}O.

Example 3.1: Let $X = \{a, b\}$ and $\tau = \{\tilde{0}, A, B, \tilde{1}\}$ where $A = \{< a, 0.2, 0.7 >, < b, 0.2, 0.7 >\}$, $B = \{< a, 0.3, 0.5 >, < b, 0.2, 0.5 >\}$. Then (X, τ) is an IFTS. Consider the IFS, $W = \{< a, 0.6, 0.4 >, < b, 0.7, 0.5 >\}$. Let O be any IFG*O set such that $W \subseteq O$. Then $\alpha cl(W) \subseteq W$. Hence W is IFS α G*C set.

Definition 3.2: An IFS W of an intuitionistic fuzzy topological space (X, τ) is called an intuitionistic fuzzy strongly α generalized star open set (IFS α G*O) if the complement of W is IFS α G*C.

Theorem 3.1: Every IFC is IFS α G*C set in X . The converse need not be true.

Proof: Suppose W is IFC. Now, $cl(int(cl(W))) \subseteq cl(W) = W$. Let O be any IFG*O set containing W . Then $\alpha cl(W) \subseteq O$. Thus W is IFS α G*C set.

Example 3.2: Let $X = \{a, b\}$ and $\tau = \{\tilde{0}, A, B, \tilde{1}\}$ where $A = \{< a, 0.2, 0.7 >, < b, 0.2, 0.7 >\}$, $B = \{< a, 0.3, 0.5 >, < b, 0.2, 0.5 >\}$. Then (X, τ) is an IFTS. Consider the IFS, $W = \{< a, 0.6, 0.2 >, < b, 0.7, 0.2 >\}$. Let O be any IFG*O set such that $W \subseteq O$. Then $\alpha cl(W) \subseteq O$. Hence W is IFS α G*C set. But W is not IFC.

Theorem 3.2: Every IFS α G*C is a IFGSC set but not conversely.

Proof: Let W be any IFS α G*C set and O be any IFO set such that $W \subseteq O$. Since every IFO is IFG*O, O is an IFG*O set. Therefore, $\alpha cl(W) \subseteq O$ which implies $Scl(W) \subseteq O$. Hence W is IFGSC.

Example 3.3: Let $X = \{a, b\}$ and $\tau = \{\tilde{0}, A, B, \tilde{1}\}$ where $A = \{< a, 0.3, 0.6 >, < b, 0.2, 0.7 >\}$, $B = \{< a, 0.2, 0.5 >, < b, 0.2, 0.5 >\}$. Then (X, τ) is an IFTS. Consider the IFS, $W = \{< a, 0.3, 0.6 >, < b, 0.2, 0.7 >\}$. Now, $Scl(W) \subseteq U$ whenever $W \subseteq U$ and U is IFO. Hence W is IFGSC set. Let $O = A = \{< a, 0.3, 0.6 >, < b, 0.2, 0.7 >\}$. Then O is IFG*O and $W \subseteq O$. But $\alpha cl(W) \not\subseteq O$. Hence W is not IFS α G*C.

Theorem 3.3: Every IFS α G*C is a IFGPC set but not conversely.

Proof: W be any IFS α G*C set and O be any IFO set such that $W \subseteq O$. Since every IFO is IFG*O, O is an IFG*O set. Therefore, $\alpha cl(W) \subseteq O$. Now, $Pcl(W) \subseteq \alpha cl(W) \Rightarrow Pcl(W) \subseteq O$. Thus W is IFGPC.

Example 3.4: Let $X = \{a, b\}$ and $\tau = \{\tilde{0}, A, B, \tilde{1}\}$ where $A = \{< a, 0.3, 0.6 >, < b, 0.3, 0.5 >\}$, $B = \{< a, 0.5, 0.3 >, < b, 0.5, 0.3 >\}$. Then (X, τ) is an IFTS. Consider the IFS, $W = \{< a, 0.5, 0.4 >, < b, 0.5, 0.4 >\}$. Now, $Pcl(W) \subseteq U$ whenever $W \subseteq U$ and U is IFO. Hence W is IFGPC set. Let $O = B = \{< a, 0.5, 0.3 >, < b, 0.5, 0.3 >\}$. Then O is IFG*O and $W \subseteq O$. But $\alpha cl(W) \not\subseteq O$. Hence W is not IFS α G*C.

Theorem 3.5: Every IFS α G*C is a IFWGC set but not conversely.

Proof: Let W be any IFS α G*C set and O be any IFO set such that $W \subseteq O$. Since every IFO is IFG*O, O is an IFG*O set. Therefore, $\alpha cl(W) \subseteq O$ which implies $cl(int(W)) \subseteq O$. Hence W is IFWGC.

Example 3.5: Let $X = \{a, b\}$ and $\tau = \{\tilde{0}, A, B, \tilde{1}\}$ where $A = \{< a, 0.2, 0.7 >, < b, 0.3, 0.7 >\}$, $B = \{< a, 0.6, 0.2 >, < b, 0.6, 0.3 >\}$. Then (X, τ) is an IFTS. Consider the IFS, $W = \{< a, 0.6, 0.4 >, < b, 0.5, 0.4 >\}$. Now, $cl(int(W)) \subseteq B^c \subseteq U$ whenever $W \subseteq U$ and U is IFO. Hence W is IFWGC set. Let $O = B = \{< a, 0.6, 0.6 >, < b, 0.6, 0.3 >\}$. Then O is IFG*O and $W \subseteq O$. But $\alpha cl(W) \not\subseteq O$. Hence W is not IFS α G*C.

Remark 3.1: IF Strongly G*C and IFS α G*C are independent.

Example 3.6: Let $X = \{a, b\}$ and $\tau = \{\tilde{0}, A, B, \tilde{1}\}$ where $A = \{< a, 0.2, 0.7 >, < b, 0.2, 0.7 >\}$, $B = \{< a, 0.6, 0.3 >, < b, 0.6, 0.3 >\}$. Then (X, τ) is an IFTS. Consider the IFS, $W = \{< a, 0.5, 0.3 >, < b, 0.4, 0.2 >\}$. Let O be any IFGO such that $W \subseteq O$. Then $cl(int(W)) = B^c \subseteq O$. Hence W is IF Strongly G*C. Let $T = \{< a, 0.6, 0.3 >, < b, 0.6, 0.3 >\}$. Then T is an IFG*O set. Also $W \subseteq T$. Now, $\alpha cl(W) \not\subseteq T$. Hence W is not IFS α G*C.

Example 3.7: Let $X = \{a, b\}$ and $\tau = \{\tilde{0}, A, B, \tilde{1}\}$ where $A = \{< a, 0.4, 0.3 >, < b, 0.4, 0.3 >\}$, $B = \{< a, 0.1, 0.7 >, < b, 0.1, 0.7 >\}$. Then (X, τ) is an IFTS. Consider the IFS, $W = \{< a, 0.6, 0.3 >, < b, 0.4, 0.2 >\}$. Let O be any IFG*O. Then $\alpha cl(W) \subseteq O$. Hence W is IFS α G*C. Let $T = \{< a, 0.6, 0.2 >, < b, 0.6, 0.2 >\}$. Then T is an IFGO set. Also $W \subseteq T$, $cl(int(W)) = B^c \not\subseteq T$. Hence W is not IF Strongly G*C.

Remark 3.2: IFSWGC and IFS α G*C are independent.

Example 3.8: Let $X = \{a, b\}$ and $\tau = \{\tilde{0}, A, B, \tilde{1}\}$ where $A = \{\langle a, 0.3, 0.6 \rangle, \langle b, 0.2, 0.7 \rangle\}$, $B = \{\langle a, 0.5, 0.3 \rangle, \langle b, 0.6, 0.3 \rangle\}$. Then (X, τ) is an IFTS. Consider the IFS, $W = \{\langle a, 0.5, 0.5 \rangle, \langle b, 0.3, 0.4 \rangle\}$. Let O be any IFSO such that $W \subseteq O$. Then $cl(int(W)) = B^c \subseteq O$. Hence W is IFSWGC. Let $S = \{\langle a, 0.5, 0.3 \rangle, \langle b, 0.6, 0.3 \rangle\}$. Then S is an IFG*O set. Also $W \subseteq S$, $\alpha cl(W) \not\subseteq S$. Hence W is not IFS α G*C.

Example 3.9: Let $X = \{a, b\}$ and $\tau = \{\tilde{0}, A, B, \tilde{1}\}$ where $A = \{\langle a, 0.3, 0.3 \rangle, \langle b, 0.4, 0.3 \rangle\}$, $B = \{\langle a, 0.1, 0.7 \rangle, \langle b, 0.2, 0.6 \rangle\}$. Then (X, τ) is an IFTS. Consider the IFS, $W = \{\langle a, 0.5, 0.3 \rangle, \langle b, 0.4, 0.3 \rangle\}$. Let O be any IFG*O. Then $\alpha cl(W) \subseteq O$. Hence W is IFS α G*C. Let $T = \{\langle a, 0.6, 0.2 \rangle, \langle b, 0.6, 0.2 \rangle\}$. Then T is an IFSO set. Also $W \subseteq T$, $cl(int(W)) = B^c \not\subseteq T$. Hence W is not IFSWGC.

Remark 3.3: IFGPSC and IFS α G*C are independent.

Example 3.10: Let $X = \{a, b\}$ and $\tau = \{\tilde{0}, A, B, \tilde{1}\}$ where $A = \{\langle a, 0.2, 0.5 \rangle, \langle b, 0.2, 0.7 \rangle\}$, $B = \{\langle a, 0.4, 0.3 \rangle, \langle b, 0.6, 0.3 \rangle\}$. Then (X, τ) is an IFTS. Consider the IFS, $W = \{\langle a, 0.4, 0.4 \rangle, \langle b, 0.3, 0.4 \rangle\}$. Let O be any IFSO. Then $Pcl(W) \subseteq O$. Hence W is IFGPSC. Let $T = \{\langle a, 0.4, 0.3 \rangle, \langle b, 0.6, 0.3 \rangle\}$. Then T is an IFG*O set. Also $W \subseteq T$, $\alpha cl(W) = A^c \not\subseteq T$. Hence W is not IFS α G*C.

Example 3.11: Let $X = \{a, b\}$ and $\tau = \{\tilde{0}, A, B, \tilde{1}\}$ where $A = \{\langle a, 0.2, 0.7 \rangle, \langle b, 0.2, 0.6 \rangle\}$, $B = \{\langle a, 0.5, 0.4 \rangle, \langle b, 0.4, 0.4 \rangle\}$. Then (X, τ) is an IFTS. Consider the IFS, $W = \{\langle a, 0.6, 0.4 \rangle, \langle b, 0.4, 0.4 \rangle\}$. Let O be any IFG*O. Then $\alpha cl(W) \subseteq O$. Hence W is IFS α G*C. Let $T = W$, then T is an IFSO set. Also $W \subseteq T$, $Pcl(W) \not\subseteq T$. Hence W is not IFGPSC.

Theorem 3.5: Every IFS α G*C is a IFGPRC set but not conversely.

Proof: Let W be any IFS α G*C set and O be any IFRO set such that $W \subseteq O$. Since every IFRO is IFO and every IFO is IFG*O, O is an IFG*O set. Therefore, $\alpha cl(W) \subseteq O$. Since $Pcl(W) \subseteq \alpha cl(W)$, we have $Pcl(W) \subseteq O$. Hence W is IFGPRC.

Example 3.12: Let $X = \{a, b\}$ and $\tau = \{\tilde{0}, A, B, \tilde{1}\}$ where $A = \{\langle a, 0.2, 0.6 \rangle, \langle b, 0.2, 0.5 \rangle\}$, $B = \{\langle a, 0.4, 0.2 \rangle, \langle b, 0.5, 0.3 \rangle\}$. Then (X, τ) is an IFTS. Consider the IFS, $W = \{\langle a, 0.4, 0.4 \rangle, \langle b, 0.2, 0.3 \rangle\}$. Let O be any IFRO such that $W \subseteq O$. Then $Pcl(W) \subseteq O$. Hence W is IFGPRC. Let $T = \{\langle a, 0.4, 0.2 \rangle, \langle b, 0.5, 0.3 \rangle\}$. Then T is an IFG*O set. Also $W \subseteq T$, $\alpha cl(W) = A^c \not\subseteq T$. Hence W is not IFS α G*C.

Remark 3.4: IFG*PC and IFS α G*C are independent.

Example 3.13: Let $X = \{a, b\}$ and $\tau = \{\tilde{0}, A, B, \tilde{1}\}$ where $A = \{\langle a, 0.2, 0.8 \rangle, \langle b, 0.3, 0.7 \rangle\}$, $B = \{\langle a, 0.7, 0.3 \rangle, \langle b, 0.6, 0.4 \rangle\}$. Then (X, τ) is an IFTS. Consider the IFS, $W = \{\langle a, 0.6, 0.4 \rangle, \langle b, 0.5, 0.5 \rangle\}$. Let O be any IFGO such that $W \subseteq O$. Then $Pcl(W) \subseteq O$. Hence W is IFG*PC. Let $T = \{\langle a, 0.7, 0.3 \rangle, \langle b, 0.6, 0.4 \rangle\}$. Then T is an IFG*O set. Also $W \subseteq T$, $\alpha cl(W) \not\subseteq T$. Hence W is not IFS α G*C.

Example 3.14: Let $X = \{a, b\}$ and $\tau = \{\tilde{0}, A, B, \tilde{1}\}$ where $A = \{\langle a, 0.6, 0.4 \rangle, \langle b, 0.7, 0.3 \rangle\}$, $B = \{\langle a, 0.2, 0.8 \rangle, \langle b, 0.1, 0.9 \rangle\}$. Then (X, τ) is an IFTS. Consider the IFS, $W = \{\langle a, 0.7, 0.3 \rangle, \langle b, 0.8, 0.2 \rangle\}$. Let O be any IFG*O. Then $\alpha cl(W) \subseteq O$. Hence W is IFS α G*C. Let $T = \{\langle a, 0.8, 0.2 \rangle, \langle b, 0.8, 0.2 \rangle\}$. Then T is an IFGO set. Also $W \subseteq T$, $Pcl(W) = W \cup cl(int(W)) \not\subseteq T$. Hence W is not IFG*PC.

Theorem 3.6: Every IFS α G*C is a IFGSRC set but not conversely.

Proof: Let W be any IFS α G*C set and O be any IFRO set such that $W \subseteq O$. Since every IFRO is IFO and every IFO is IFG*O, O is an IFG*O set. Therefore, $\alpha cl(W) \subseteq O \Rightarrow Scl(W) \subseteq O$. Hence W is IFGSRC.

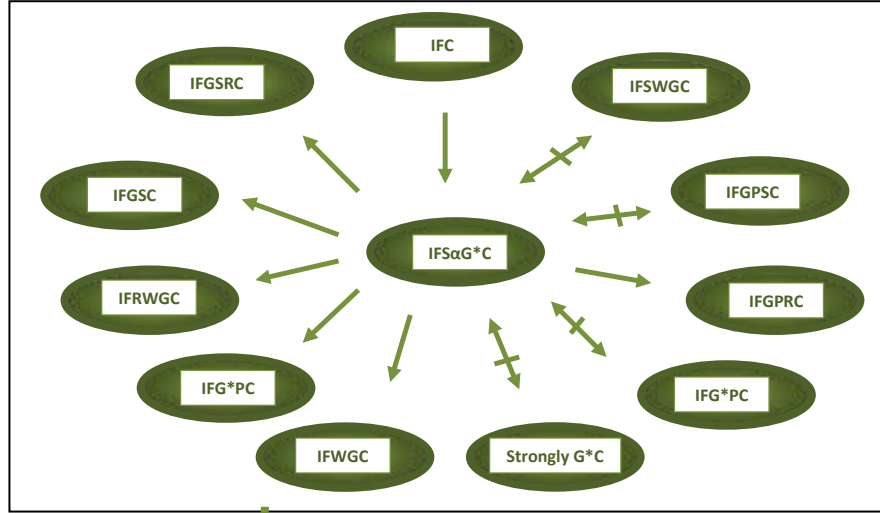
Example 3.15: Let $X = \{a, b\}$ and $\tau = \{\tilde{0}, A, B, \tilde{1}\}$ where $A = \{\langle a, 0.2, 0.5 \rangle, \langle b, 0.3, 0.6 \rangle\}$, $B = \{\langle a, 0.5, 0.1 \rangle, \langle b, 0.5, 0.2 \rangle\}$. Then (X, τ) is an IFTS. Consider the IFS, $W = \{\langle a, 0.4, 0.3 \rangle, \langle b, 0.5, 0.4 \rangle\}$. Let O be any IFRO. Then $Scl(W) \subseteq O$. Hence W is IFGSRC. Let $T = \{\langle a, 0.5, 0.1 \rangle, \langle b, 0.5, 0.2 \rangle\}$. Then T is an IFG*O set. Also $W \subseteq T$, $\alpha cl(W) \not\subseteq P$. Hence W is not IFS α G*C.

Theorem 3.7: Every IFS α G*C is a IFRWGC set but not conversely.

Proof: Let W be any IFS α G*C set and O be any IFRO set such that $W \subseteq O$. Since every IFRO is IFO and every IFO is IFG*O, O is an IFG*O set. Therefore, $\alpha cl(W) \subseteq O \Rightarrow cl(int(W)) \subseteq O$. Hence W is IFRWGC.

Example 3.16: Let $X = \{a, b\}$ and $\tau = \{\tilde{0}, A, B, \tilde{1}\}$ where $A = \{< a, 0.2, 0.6 >, < b, 0.1, 0.7 >\}$, $B = \{< a, 0.4, 0.3 >, < b, 0.5, 0.3 >\}$, then (X, τ) is an IFTS. Consider the IFS, $W = \{< a, 0.3, 0.4 >, < b, 0.4, 0.3 >\}$. Let O be any IFRO, then $cl(int(W)) \subseteq O$. Hence W is IFRWGC. Let $P = \{< a, 0.4, 0.3 >, < b, 0.5, 0.3 >\}$, then P is an IFG*O set, Also $W \subseteq P$, $acl(W) \not\subseteq P$. Hence W is not IFS α G*C.

The following diagram shows the relationships of IFS α G*C sets with some other sets discussed in this section.



where $A \xrightarrow{\text{solid green arrow}} B$ (resp. $A \xleftrightarrow{\text{double-headed green arrow}} B$) represents A implies B (resp. A and B are independent).

IV. SOME BASIC PROPERTIES OF INTOITIONISTIC FUZZY STRONGLY α GENERALIZED STAR CLOSED SETS

Theorem 3.1: An IFS A is IFS α G*C iff $acl(A) - A$ containing no non zero IFG*C set.

Proof: Suppose that F is a non zero IFG*C set such that $F \subseteq acl(A) - A$. Then $F \subseteq acl(A) \cap A^c$. Then $F \subseteq acl(A)$ and $F \subseteq A^c$ that implies $A \subseteq F^c$. Here F^c is IFG*O and A is IFS α G*C. We have $acl(A) \subseteq F^c$, $F \subseteq acl(A) \cap (acl(A))^c = \tilde{0}$ implies $acl(A) - A$ contains no non-zero IFG*C set.

Sufficiency: Let $A \subseteq O$ and O is IFG*O. Suppose that $acl(A)$ is not contained in O . Then $acl(A) \cap O^c$ is a non-zero IFG*C set of $acl(A) - A$ which is contradiction. Therefore $acl(A) \subseteq O$ and hence A is IFS α G*C.

Theorem 3.2: If B is IFS α G*C set and $B \subseteq A \subseteq acl(B)$ then A is IFS α G*C.

Proof: Let B be IFS α G*C and O be any IFG*O set such that $A \subseteq O$. Then $B \subseteq O$ which implies $acl(A) \subseteq acl(B) \subseteq O$. Hence A is IFS α G*C.

Theorem 3.3: A is any IFS α G*O iff $B \subseteq aint(A)$ where B is IFG*C & $B \subseteq A$.

Proof: Let A be any IFS α G*O set. Let B be IFG*C and $B \subseteq A$. Then $A^c \subseteq B^c$ which implies $acl(A^c) \subseteq B^c$, since A^c is IFS α G*C and B^c is IFG*O. Therefore, we have $B \subseteq aint(A)$. Conversely assume that $B \subseteq aint(A)$ whenever B is IFG*C and $B \subseteq A$. Let O be any IFG*O. Then O^c is IFG*C. Therefore by assumption, $O^c \subseteq aint(A)$ which implies $acl(A^c) \subseteq O$. Hence A is IFS α G*O.

Theorem 3.4: If $aint(A) \subseteq B \subseteq A$ and A is IFS α G*O, then B is IFS α G*O.

Proof: $aint(A) \subseteq B \subseteq A$ implies $A^c \subseteq B^c \subseteq acl(A)^c$. Since A is IFS α G*O, A^c is IFS α G*C. Therefore by Theorem 3.2, B^c is IFS α G*C. Hence B is IFS α G*O.

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