

# COMPUTATION OF MULTIPLICATIVE BANHATTI-SOMBOR INDICES OF CERTAIN BENZENOID SYSTEMS

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## ABSTRACT:

**Recently,** a novel class of degree based topological indices was introduced, the so called Banhatti-Sombor indices. In this paper, we introduce the multiplicative first Banhatti-Sombor index, multiplicative first reduced Banhatti-Sombor index, multiplicative first  $\delta$ -Banhatti-Sombor index of a graph and compute exact formulas for some nanostructures.

**Keywords:** multiplicative first Banhatti-Sombor index, multiplicative first reduced Banhatti-Sombor index, multiplicative first  $\delta$ -Banhatti-Sombor index, nanostructure.

**Mathematics Subject Classification:** 05C05, 05C07, 05C90.

## 1. INTRODUCTION

Let  $G$  be a finite, simple, connected graph with vertex set  $V(G)$  and edge set  $E(G)$ . Let  $d_G(u)$  be the degree of a vertex  $u$  in a graph  $G$ . For undefined terms and notations, we refer [1].

Chemical Graph Theory is branch of Mathematical Chemistry which has an important effect on the development of Chemical Sciences. A molecular graph is a graph such that its vertices correspond to the atoms and the edges to the bonds. Topological indices are useful for establishing correlation between the structure of a molecular compound and its physicochemical properties. Numerous topological indices [2] have been considered in Theoretical Chemistry and have found some applications, especially in QSPR/QSAR research, see [3, 4].

In [5], Kulli introduced the first Banhatti-Sombor index, first reduced Banhatti-Sombor index, first  $\delta$ -Banhatti-Sombor index and they are defined as

$$BSO_1(G) = \sum_{uv \in E(G)} \left[ \frac{1}{d_G(u)^2} + \frac{1}{d_G(v)^2} \right]^{\frac{1}{2}},$$

$$RBSO_1(G) = \sum_{uv \in E(G)} \left[ \frac{1}{(d_G(u)-1)^2} + \frac{1}{(d_G(v)-1)^2} \right]^{\frac{1}{2}}, \text{ if } \delta(G) \geq 2$$

$$\delta BSO_1(G) = \sum_{uv \in E(G)} \left[ \frac{1}{(d_G(u)-\delta(G)+1)^2} + \frac{1}{(d_G(v)-\delta(G)+1)^2} \right]^{\frac{1}{2}},$$

where  $\delta(G)$  is the minimum degree among the vertices of  $G$ .

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Recently, some Sombor indices were studied, for example, in [6, 7, 8, 9, 10, 11, 12, 13, 14].

Inspired by work on Sombor indices, we put forward the multiplicative first Banhatti-Sombor index, multiplicative first reduced Banhatti-Sombor index and multiplicative first  $\delta$ -Banhatti-Sombor index of a graph and they are defined as

$$BSO_1 II(G) = \prod_{uv \in E(G)} \left[ \frac{1}{d_G(u)^2} + \frac{1}{d_G(v)^2} \right]^{\frac{1}{2}},$$

$$RBSO_1 II(G) = \prod_{uv \in E(G)} \left[ \frac{1}{(d_G(u)-1)^2} + \frac{1}{(d_G(v)-1)^2} \right]^{\frac{1}{2}}, \text{ if } \delta(G) \geq 2$$

$$\delta BSO_1 II(G) = \prod_{uv \in E(G)} \left[ \frac{1}{(d_G(u)-\delta(G)+1)^2} + \frac{1}{(d_G(v)-\delta(G)+1)^2} \right]^{\frac{1}{2}}$$

where  $\delta(G)$  is the minimum degree among the vertices of  $G$ .

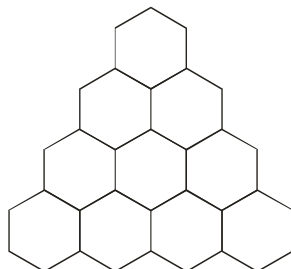
In this paper, we compute the multiplicative first Banhatti-Sombor index, multiplicative first reduced Banhatti-Sombor index, multiplicative first  $\delta$ -Banhatti-Sombor index of some families of benzenoid systems.

## 2. OBSERVATIONS

- (1) If  $\delta(G) = 1$ , then  $\delta BSO_1 II(G)$  is the multiplicative first Banhatti-Sombor index  $BSO_1 II(G)$ .
- (2) If  $\delta(G) = 2$ , then  $\delta BSO_1 II(G)$  is the multiplicative first reduced Banhatti-Sombor index  $RBSO_1 II(G)$ .

## 3. TRIANGULAR BENZENOIDS

In this section, we consider a family of triangular benzenoids. This family of benzenoids is denoted by  $T_p$ , where  $p$  is the number of hexagons in the base graph. Clearly  $T_p$  has  $\frac{1}{2}p(p+1)$  hexagons. The graph of  $T_4$  is shown in Figure 1.



**Figure-1:** The graph of  $T_4$

Let  $G$  be the graph of a triangular benzenoid  $T_p$ . The graph  $G$  has  $p^2 + 4p + 1$  vertices and  $\frac{3p(p+3)}{2}$  edges. From Figure 1, we see that the vertices of  $G$  are either of degree 2 or 3. Therefore  $\delta(G) = 2$ . By calculation, we obtain that  $G$  has three types of edges based on degrees of end vertices of each edge as follows:

$$E_1 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}, \quad |E_1| = 6.$$

$$E_2 = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}, \quad |E_2| = 6p - 6.$$

$$E_3 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}, \quad |E_3| = \frac{3p(p-1)}{2}.$$

In the following theorem, we compute the multiplicative first Banhatti-Sombor index of  $T_p$ .

**Theorem 1:** The multiplicative first Banhatti-Sombor index of a triangular benzenoid  $T_p$  is given by

$$BSO_1 II(T_p) = \left(\frac{1}{2}\right)^{\frac{6}{2}} \times \left(\frac{13}{36}\right)^{\frac{1}{2}(6p-6)} \times \left(\frac{2}{9}\right)^{\frac{3}{4}p(p-1)}.$$

**Proof:** By using definition and cardinalities of the edge partition of  $T_p$ , we have

$$\begin{aligned} BSO_1 II(T_p) &= \left(\frac{1}{2^2} + \frac{1}{2^2}\right)^{\frac{6}{2}} \times \left(\frac{1}{2^2} + \frac{1}{3^2}\right)^{\frac{1}{2}(6p-6)} \times \left(\frac{1}{3^2} + \frac{1}{3^2}\right)^{\frac{3}{4}p(p-1)} \\ &= \left(\frac{1}{2}\right)^{\frac{6}{2}} \times \left(\frac{13}{36}\right)^{\frac{1}{2}(6p-6)} \times \left(\frac{2}{9}\right)^{\frac{3}{4}p(p-1)}. \end{aligned}$$

In the following theorem, we determine the multiplicative first reduced Banhatti-Sombor index of  $T_p$ .

**Theorem 2:** The multiplicative first reduced Banhatti-Sombor index of a triangular benzenoid  $T_p$  is given by

$$RBSO_1 II(T_p) = (2)^{\frac{6}{2}} \times \left(\frac{5}{4}\right)^{\frac{1}{2}(6p-6)} \times \left(\frac{1}{2}\right)^{\frac{3}{4}p(p-1)}.$$

**Proof:** From definition and by using cardinalities of the edge partition of  $T_p$ , we obtain

$$\begin{aligned} RBSO_1 II(T_p) &= \left(\frac{1}{1^2} + \frac{1}{1^2}\right)^{\frac{6}{2}} \times \left(\frac{1}{1^2} + \frac{1}{2^2}\right)^{\frac{1}{2}(6p-6)} \times \left(\frac{1}{2^2} + \frac{1}{2^2}\right)^{\frac{3}{4}p(p-1)} \\ &= (2)^{\frac{6}{2}} \times \left(\frac{5}{4}\right)^{\frac{1}{2}(6p-6)} \times \left(\frac{1}{2}\right)^{\frac{3}{4}p(p-1)}. \end{aligned}$$

In the following theorem, we determine the multiplicative first  $\delta$ -Banhatti-Sombor index of  $T_p$ .

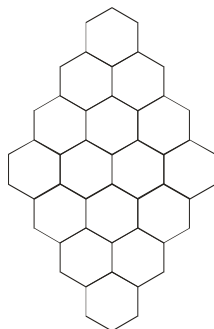
**Theorem 3:** The multiplicative first  $\delta$ -Banhatti-Sombor index of a triangular benzenoid  $T_p$  is given by

$$\delta BSO_1 II(T_p) = (2)^{\frac{6}{2}} \times \left(\frac{5}{4}\right)^{\frac{1}{2}(6p-6)} \times \left(\frac{1}{2}\right)^{\frac{3}{4}p(p-1)}.$$

**Proof:** By observation (2) and Theorem 2, the result follows.

#### 4. BENZENOID RHOMBUS

In this section, we consider a family of benzenoid rhombus. This family of benzenoids is denoted by  $R_p$ . The benzenoid rhombus  $R_p$  is obtained from two copies of a triangular benzenoid  $T_p$  by identifying hexagons in one of their base rows. The graph of  $R_4$  is depicted in Figure 2.



**Figure-2:** The graph of  $R_4$

Let  $G$  be the graph of a benzenoid rhombus  $R_p$ . The graph  $G$  has  $2p^2 + 4p$  vertices and  $3p^2 + 4p - 1$  edges. From Figure 2, it is easy to see that the vertices of  $R_p$  are either of degree 2 or 3. Thus  $\delta(R_p) = 2$ . By calculation, we obtain that  $G$  has three types of edges based on degrees of end vertices of each edge as follows:

$$\begin{aligned} E_1 &= \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}, & |E_1| &= 6. \\ E_2 &= \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}, & |E_2| &= 8p - 8. \\ E_3 &= \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}, & |E_3| &= 3p^2 - 4p + 1. \end{aligned}$$

In the following theorem, we compute the multiplicative first Banhatti-Sombor index of  $R_p$ .

**Theorem 4:** The multiplicative first Banhatti-Sombor index of a benzenoid rhombus  $R_p$  is given by

$$BSO_1 II(R_p) = \left(\frac{1}{2}\right)^{\frac{6}{2}} \times \left(\frac{13}{36}\right)^{\frac{1}{2}(8p-8)} \times \left(\frac{2}{9}\right)^{\frac{1}{2}(3p-4p+1)}.$$

**Proof:** From definition and by cardinalities of the edge partition of  $R_p$ , we deduce

$$\begin{aligned} BSO_1 II(R_p) &= \left(\frac{1}{2^2} + \frac{1}{2^2}\right)^{\frac{6}{2}} \times \left(\frac{1}{2^2} + \frac{1}{3^2}\right)^{\frac{1}{2}(8p-8)} \times \left(\frac{1}{3^2} + \frac{1}{3^2}\right)^{\frac{1}{2}(3p^2-4p+1)} \\ &= \left(\frac{1}{2}\right)^{\frac{6}{2}} \times \left(\frac{13}{36}\right)^{\frac{1}{2}(8p-8)} \times \left(\frac{2}{9}\right)^{\frac{1}{2}(3p-4p+1)}. \end{aligned}$$

In the following theorem, we determine the multiplicative first reduced Banhatti-Sombor index of  $R_p$ .

**Theorem 5:** The multiplicative first reduced Banhatti-Sombor index of a benzenoid rhombus  $R_p$  is given by

$$RBSO_1 II(R_p) = (2)^{\frac{6}{2}} \times \left(\frac{5}{4}\right)^{\frac{1}{2}(8p-8)} \times \left(\frac{1}{2}\right)^{\frac{1}{2}(3p^2-4p+1)}.$$

**Proof:** By using definition and by cardinalities of the edge partition of  $R_p$ , we derive

$$\begin{aligned} RBSO_1 II(R_p) &= \left(\frac{1}{1^2} + \frac{1}{1^2}\right)^{\frac{6}{2}} \times \left(\frac{1}{1^2} + \frac{1}{2^2}\right)^{\frac{1}{2}(8p-8)} \times \left(\frac{1}{2^2} + \frac{1}{2^2}\right)^{\frac{1}{2}(3p^2-4p+1)} \\ &= (2)^{\frac{6}{2}} \times \left(\frac{5}{4}\right)^{\frac{1}{2}(8p-8)} \times \left(\frac{1}{2}\right)^{\frac{1}{2}(3p^2-4p+1)}. \end{aligned}$$

In the following next theorem, we compute the multiplicative first  $\delta$ -Banhatti-Sombor index of  $R_p$ .

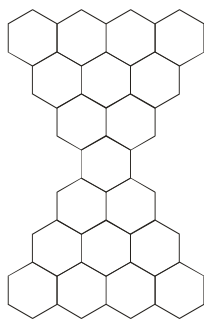
**Theorem 6:** The multiplicative first  $\delta$ -Banhatti-Sombor index of  $R_p$  is given by

$$\delta BSO_1 II(R_p) = (2)^{\frac{6}{2}} \times \left(\frac{5}{4}\right)^{\frac{1}{2}(8p-8)} \times \left(\frac{1}{2}\right)^{\frac{1}{2}(3p^2-4p+1)}.$$

**Proof:** By observation (2) and Theorem 5, the result follows.

## 5. BENZENOID HOURGLASS

In this section, we consider a family of benzenoid hourglass, which is denoted by  $X_p$ . This family is obtained from two copies of a triangular benzenoid  $T_p$  by overlapping hexagons. The graph of benzenoid hourglass is presented in Figure 3.



**Figure-3:** The graph of benzenoid hourglass

Let  $G$  be the graph of a benzenoid hourglass  $X_p$ . This graph  $G$  has  $2(p^2 + 4p - 2)$  vertices and  $3p^2 + 9p - 4$  edges. From Figure 3, we see that the vertices of  $X_p$  are either of degree 2 or 3. Thus  $\delta(X_p) = 2$ .

By algebraic method, we find that  $G$  has three types of edges based on degrees of end vertices of each as follows:

$$\begin{aligned} E_1 &= \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}, & |E_1| &= 8. \\ E_2 &= \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}, & |E_2| &= 12p - 16. \\ E_3 &= \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}, & |E_3| &= 3p^2 - 3p + 4. \end{aligned}$$

In the following theorem, we compute the multiplicative first Banhatti-Sombor index of  $X_p$ .

**Theorem 7:** The multiplicative first Banhatti-Sombor index of a benzenoid hourglass  $X_p$  is given by

$$BSO_1 II(X_p) = \left(\frac{1}{2}\right)^{\frac{8}{2}} \times \left(\frac{13}{36}\right)^{\frac{1}{2}(12p-16)} \times \left(\frac{2}{9}\right)^{\frac{1}{2}(3p^2-3p+4)}.$$

**Proof:** From definition and by cardinalities of the edge partition of  $X_p$ , we obtain

$$\begin{aligned} BSO_1 II(X_p) &= \left(\frac{1}{2^2} + \frac{1}{2^2}\right)^{\frac{8}{2}} \times \left(\frac{1}{2^2} + \frac{1}{3^2}\right)^{\frac{1}{2}(12p-16)} \times \left(\frac{1}{3^2} + \frac{1}{3^2}\right)^{\frac{1}{2}(3p^2-3p+4)} \\ &= \left(\frac{1}{2}\right)^{\frac{8}{2}} \times \left(\frac{13}{36}\right)^{\frac{1}{2}(12p-16)} \times \left(\frac{2}{9}\right)^{\frac{1}{2}(3p^2-3p+4)}. \end{aligned}$$

In the following theorem, we compute the multiplicative first reduced Banhatti-Sombor index of  $X_p$ .

**Theorem 8:** The multiplicative first reduced Banhatti-Sombor index of a benzenoid hourglass is given by

$$RBSO_1 II(X_p) = (2)^{\frac{8}{2}} \times \left(\frac{5}{4}\right)^{\frac{1}{2}(12p-16)} \times \left(\frac{1}{2}\right)^{\frac{1}{2}(3p^2-3p+1)}.$$

**Proof:** From definition and by cardinalities of the edge partition of  $X_p$ , we deduce

$$\begin{aligned} RBSO_1 II(X_p) &= \left(\frac{1}{1^2} + \frac{1}{1^2}\right)^{\frac{8}{2}} \times \left(\frac{1}{1^2} + \frac{1}{2^2}\right)^{\frac{1}{2}(12p-16)} \times \left(\frac{1}{2^2} + \frac{1}{2^2}\right)^{\frac{1}{2}(3p^2-3p+1)} \\ &= (2)^{\frac{8}{2}} \times \left(\frac{5}{4}\right)^{\frac{1}{2}(12p-16)} \times \left(\frac{1}{2}\right)^{\frac{1}{2}(3p^2-3p+1)}. \end{aligned}$$

In the next theorem, we determine the multiplicative first  $\delta$ -Banhatti-Sombor index of  $X_p$ .

**Theorem 9:** The multiplicative first  $\delta$ -Banhatti-Sombor index of a benzenoid hourglass  $X_p$  is given by

$$\delta BSO_1 II(X_p) = (2)^{\frac{8}{2}} \times \left(\frac{5}{4}\right)^{\frac{1}{2}(12p-16)} \times \left(\frac{1}{2}\right)^{\frac{1}{2}(3p^2-3p+1)}.$$

**Proof:** By observation (2) and from Theorem 8, we obtain the desired result.

## 6. JAGGED RECTANGLE BENZENOID SYSTEMS

In this section, we focus in the molecular graph structure of a jagged rectangle benzenoid system. This system is denoted by  $B_{m,n}$  for all  $m, n \in \mathbb{N}$ . Three chemical graphs of a jagged rectangle benzenoid system are presented in Figure 4.

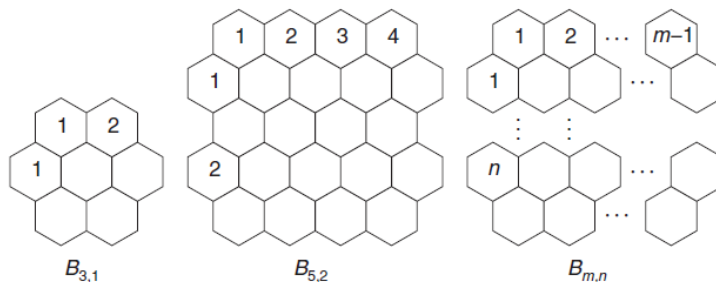


Figure-4

Let  $G$  be the graph of a jagged rectangle benzenoid system  $B_{m,n}$ . By calculation, we obtain that  $G$  has  $4mn + 4m + 2n - 2$  vertices and  $6mn + 5m + n - 4$  edges. From Figure 4, it is easy to see that the vertices of  $G$  are either of degree 2 or 3. Thus  $\delta(G)=2$ . By calculation, we obtain that the edge set of  $B_{m,n}$  can be divided into three partitions as follows:

$$\begin{aligned} E_1 &= \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}, & |E_1| &= 2n + 4. \\ E_2 &= \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}, & |E_2| &= 4m + 4n - 4. \\ E_3 &= \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}, & |E_3| &= 6mn + m - 5n - 4. \end{aligned}$$

In the following theorem, we determine the multiplicative first Banhatti-Sombor index of  $B_{m,n}$ .

**Theorem 10:** The multiplicative first Banhatti-Sombor inverse index of  $B_{m,n}$  is given by

$$BSO_1 II(B_{m,n}) = \left(\frac{1}{2}\right)^{\frac{1}{2}(2n+4)} \times \left(\frac{13}{36}\right)^{\frac{1}{2}(4m+4n-4)} \times \left(\frac{2}{9}\right)^{\frac{1}{2}(6mn+m-n-4)}.$$

**Proof:** From definition and by cardinalities of the edge partition of  $B_{m,n}$ , we obtain

$$\begin{aligned} BSO_1 II(B_{m,n}) &= \left(\frac{1}{2^2} + \frac{1}{2^2}\right)^{\frac{1}{2}(2n+4)} \times \left(\frac{1}{2^2} + \frac{1}{3^2}\right)^{\frac{1}{2}(4m+4n-4)} \times \left(\frac{1}{3^2} + \frac{1}{3^2}\right)^{\frac{1}{2}(6mn+m-n-4)} \\ &= \left(\frac{1}{2}\right)^{\frac{1}{2}(2n+4)} \times \left(\frac{13}{36}\right)^{\frac{1}{2}(4m+4n-4)} \times \left(\frac{2}{9}\right)^{\frac{1}{2}(6mn+m-n-4)}. \end{aligned}$$

In the following theorem, we compute the multiplicative first reduced Banhatti-Sombor index of  $B_{m,n}$ .

**Theorem 11:** The multiplicative first reduced Banhatti-Sombor index of  $B_{m,n}$  is given by

$$RBSO_1 II(B_{m,n}) = (2)^{\frac{1}{2}(2n+4)} \times \left(\frac{5}{4}\right)^{\frac{1}{2}(4m+4n-4)} \times \left(\frac{1}{2}\right)^{\frac{1}{2}(6mn+m-5n-4)}.$$

**Proof:** From definition and by cardinalities of the edge partition of  $B_{m,n}$ , we obtain

$$\begin{aligned} RBSO_1 II(B_{m,n}) &= \left(\frac{1}{1^2} + \frac{1}{1^2}\right)^{\frac{1}{2}(2n+4)} \times \left(\frac{1}{1^2} + \frac{1}{2^2}\right)^{\frac{1}{2}(4m+4n-4)} \times \left(\frac{1}{2^2} + \frac{1}{2^2}\right)^{\frac{1}{2}(6mn+m-5n-4)} \\ &= (2)^{\frac{1}{2}(2n+4)} \times \left(\frac{5}{4}\right)^{\frac{1}{2}(4m+4n-4)} \times \left(\frac{1}{2}\right)^{\frac{1}{2}(6mn+m-5n-4)}. \end{aligned}$$

In the following next theorem, we determine the multiplicative first  $\delta$ -Banhatti-Sombor index of  $B_{m,n}$ .

**Theorem 12:** The multiplicative first  $\delta$ -Banhatti-Sombor index of  $B_{m,n}$  is given by

$$\delta BSO_1 II(B_{m,n}) = (2)^{\frac{1}{2}(2n+4)} \times \left(\frac{5}{4}\right)^{\frac{1}{2}(4m+4n-4)} \times \left(\frac{1}{2}\right)^{\frac{1}{2}(6mn+m-5n-4)}.$$

**Proof:** From observation (2) and Theorem 11, we get the desired result.

## 7. CONCLUSION

In this study, we have introduced the multiplicative first Banhatti-Sombor index, multiplicative first reduced Banhatti-Sombor index, multiplicative first  $\delta$ -Banhatti-Sombor index of a graph and have computed exact formulas for certain benzenoid systems.

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