International Journal of Mathematical Archive-12(5), 2021, 17-22
Available online through www.ijma.info ISSN 2229 – 5046

FIXED POINT THEOREMS FOR MODIFIED F-CONTRACTION MAPPINGS

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(Received On: 09-04-21; Revised & Accepted On: 09-05-21)

ABSTRACT

*In this paper, we introduce the notion of Modified F-Contraction condition in the non-Archimedean fuzzy metric space, which is a fuzzy version of F-contraction condition of the Hardy-Rozers type in the non-Archimedean fuzzy metric space. Our result generalizes many results of fixed-point theory in the existing literature, specially the results of Wardowski [D.Wardowski, Fixed points of a new type of contractive in complete metric space, Fixed point Theory Appl. 2012,2012,94.], Cosentino and Vetro [V. Cosentino and P. Vetro, Fixed point Result for F-contractive mappings of Hardy-Rogers Type, Filomat (**28**)(2014), 715-722] and Piri and Kuman [H. Piri and P. Kuman. Some fixed point theorems concerning F-contraction in complete metric spaces. Fixed point theory Appl. 2014, 21(2014)]. An illustrative example is given to validate our main theorem.*

Keywords and Phrases: *Banach contraction principle, Fixed point, F-Contraction condition, Fuzzy metric space, non-Archimedean fuzzy metric space.*

Ams Mathematics Subject Classification: *47H10, 54H25, 54A40, 54E35.*

1. INTRODUCTION AND PRELIMINARIES

In 1922, the Banach Contraction Principle [1] was introduced to prove a fixed-point result in complete metric space. This theorem has many applications in various fields of science and many branches of mathematics. Besides, most of the authors introduced many works related to fixed point theory in different spaces. Historically, in 1975 Kramosil and Michalek [10] used the notion of fuzzy metric and compared it with that of statistical metric space and proved them equivalent. In this line, George and Veeramani [4] defined a Housdorff topology on the fuzzy metric space and proved some known results of metric spaces including Baire theorem for fuzzy metric spaces. Extending the above two concepts, Gregory and Sapena [5] gave fixed point theorems for complete fuzzy metric spaces in the sense of GV and also in KM fuzzy metric spaces. This was complete in Grabiec's [6] sense, *i.e.*, fuzzy version of Cauchy sequence with completeness. Developing in the same line of F-contraction, various contractive conditions are used to prove the fixed-point theorem in fuzzy metric spaces, some of them can be seen in [13], [14], [15], [16], [17], [8] etc. At the same time many authors extend the results of Banach contraction principal using many relaxed contractive mappings.

Following fixed point theorem was proved for contractive condition in metric space:

1.1 Theorem ([3]): Let (X, d) be a complete metric space and let $T: X \to X$ be a self mapping. Assume $d(Tx, Ty) < d(x, y)$ holds for all $x, y \in X$ with $x \neq y$. Then T has a unique fixed point in X.

In 2012, Wardowski [20] introduced a new type of contraction called F- contraction and generalized the Banach contraction principle in the following way:

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1.2 Definition ([20]): Let *(X, d)* be a metric space. A self-mapping *T* on *X* is called an *F- contraction* if there exist $\tau > 0$ and F$\in \mathcal{F}$ such that

$$\tau + F(d(Tx, Ty)) \leq F(d(x, y)) \tag{1.1}$$

for all x, y ∈ X with d(Tx, Ty) > 0 where $\mathcal{F}$ is the family of all functions F: $R_+ \to R$ satisfying the following conditions:

(F_1): F is strictly increasing: x < y ⇨ F(x) < F(y)

(F_2): For each sequence $\{\alpha_n\} \in R_+$ of positive numbers $\lim_{n\to\infty} \alpha_n = 0$ if and only if $\lim_{n\to\infty} F(\alpha_n) = -\infty$

(F_3): There exists k ∈ (0,1) such that $\lim_{\alpha\to 0+} \alpha^k F(\alpha_n) = 0$.

1.3 Remark: Obviously, if T satisfy inequality (1) and F$\in \mathcal{F}$ is an increasing function, then T is contractive,
i.e. $d(Tx, Ty) < d(x, y)$ $\forall x, y \in X$ and $x \neq y$ so, T is continuous.

1.4 Example: Following are some examples of *F- contraction* in metric spaces:

(a) Let F: $R_+ \to R$ be a strictly increasing function in the family $\mathcal{F}$ with F(α) = ln α, then $\lim_{n\to\infty} F(\alpha_n) = -\infty$ and $\tau > 0$ is given by 0 < d(Tx, Ty) ≤ $\exp(-\tau)$ d(x, y), for all $x, y \in X$ and x≠ y.

(b) Let F: $R_+ \to R$ be a strictly increasing function in the family $\mathcal{F}$ with F(α) = α + ln α, then $\lim_{n\to\infty} F(\alpha_n) = -\infty$ and $\tau > 0$ is given by d(Tx, Ty). $\exp[d(Tx,Ty) - d(x,y)] \leq \exp(-\tau)$ d(x, y), for all $x, y \in X$ and x≠ y.

1.5 Remark: From above Examples, it is clear that *the F-contraction has reduced in the Banach contraction.* Therefore, the Banach contraction condition is a particular case of F-contraction. Thus F-contraction is a generalized form of Banach contraction. More examples can be seen in [Wardowski, D.]

1.6 Theorem: ([20]): Let *(X, d)* be a complete metric space and let *T: X →X* be an F-contraction. Then *T* has a unique fixed-point $z \in X$ and for every $x \in X$ the sequence $\{T_x^n\}_{n\in N}$ converges to *z*.

Cosentino and Vetro [2] introduced the notion of Hardy-Rogers type ([7]) F-contraction in the following way to generalized the result of Wadrowski:

1.7 Definition ([2]): Let *(X, d)* be a metric space. A self-mapping *T* on *X* is called an *F-contraction of Hardy-Rogers-type* if there exist $\tau > 0$ and function F $\in \mathcal{F}$ satisfying:

$$\tau + F(d(Tx, Ty)) \leq F[\alpha d(x, y)+\beta d(x, Tx)+\gamma d(y, Ty)+\delta d(x, Ty)+Ld(y, Tx)]$$

for all $x, y \in X$ with $d(Tx, Ty) > 0$, where $\alpha, \beta, \gamma, \delta, L \geq 0$ $\gamma \neq 1$ and $\alpha + \beta + \gamma + 2\delta = 1$, where $\mathcal{F}$ is the family of all functions which satisfy (F1), (F2) and (F3).

1.8 Theorem ([2]): Let *(X, d)* be a complete metric space and let *T* be a self mapping on *X*. Assume that *T* is an F-contraction of the Hardy-Rogers type where $\gamma \neq 1$. Then *T* has a fixed point. Moreover, if $\alpha + \delta + L \leq 1$, then fixed point of *T* is unique.

Before we move to non-Archimedean fuzzy metric space and F-contraction defined in it, we give the definitions of fuzzy metric space and t-norms existing in the literature:

1.9 Definition ([18]):- A binary operation $*$:[0, 1]× [0, 1]→[0, 1] is called a *continuous triangular norm* (t-norm) if the following conditions satisfy:

TN-1 $*$ is commutative and associative.

TN-2 $*$ is continuous.

TN-3 $*(a,1) = a$ for every $a \in [0,1]$.

TN-4 $*(a, b) \leq *(c, d)$ whenever $a \leq c$, $b \leq d$ and $a, b, c, d \in [0,1]$.

1.10 Definition ([4]): A fuzzy metric space is an ordered triple *(X, d, ∗)* such that *X* is a nonempty set, $*$ is a continuous t-norm and *M* is a fuzzy set on $X^2 \times (0,\infty)$ satisfying the following conditions:

Fd-1: $d(x, y, t) > 0$ (non-negativity)

Fd-2: $d(x, y, t)=1$ *iff* $x = y$ (antisymmetric)

Fd-3: $d(x, y, t) = d(y, x, t)$ (symmetric)

Fd-4: $d(x, z, t+s) \geq d(x, y, t) * d(y, z, s)$ (triangular inequality)

Fd-5: $d(x, y, *) : (0,\infty) \to [0, 1]$ is continuous, for all $x, y, z \in X$ and $s, t > 0$.

1.11 Remark: If, in the Definition 1.10, the triangular inequality Fd-4 is replaced by the following condition:

(NA): $d(x, z, \max\{t, s\}) \geq d(x,y,t) * d(y,z,s)$ $\forall x,y,z \in X$, $s,t > 0$.

Or, equivalently,

$$d(x, z, t) \geq d(x, y, t) * d(y, z, t) \tag{1.2}$$

then the triple *(X, d, ∗)* is called a *non-Archimedean fuzzy metric space* [9].
Mihet [11], [12] and Vetro [19] gave some fixed point theorems for the non-Archimedean fuzzy metric spaces.

1.12 Definition: Let *(X, d, ∗)* be a fuzzy metric space. It is called *d-metric* if every d-Cauchy sequence is convergent.
Now, we are ready to introduce our *modified F-contraction* mapping condition in the non-Archimedean fuzzy metric space in the following way:
Let *(X, d, ∗)* be a fuzzy metric space, then the self mapping *T: X → X* is called the *modified F-contraction* if there exist $\tau > 0$ and F∈ $\mathcal{F}$ such that

$$\tau + F(d(Tx, Ty, t) \leq F[\alpha d(x, y, t)+\beta d(x, Tx, t)+\gamma d(y, Ty, t) + \delta d(x, Ty, t)+ L\, d(y, Ty, t)] \tag{1. 3}$$

holds for any x, y ∈ X, s, t > 0 with $d(Tx, Ty, t) > 0$, where $\alpha, \beta, \gamma, \delta, L \geq 0$ $\gamma \neq 1$ $\alpha+\beta+\gamma+2\delta = 1$, $\delta < \frac{1}{2}$, $\gamma < 1$ and $\alpha + \delta + L \leq 1$.

2. MAIN RESULTS

Below we give our main result for the non-Archimedean fuzzy metric space satisfying the *modified F-contraction* condition.

2.1 Theorem: Let *(X, d, ∗)* be a non-Archimedean fuzzy metric space and let *T: X → X* be a modified F-contraction mapping as defined in equation (3). Then T has a unique fixed point z, where F: $R_+ \to R$ is an increasing function. Moreover, the sequence $\{T_x^n\}$ converges to z, for some x ∈ X and n ∈ N.

Proof: Let $x_0 \in X$ be an arbitrary point and construct a sequence $\{x_n\} \in X$, n ∈ N by

$$x_1 = Tx_0, \quad x_2 = Tx_1 \Longrightarrow x_2 = T(Tx_0) = T^2_{x0}$$

continuing this process, we get $x_n = T^n_{x0}$ *i.e.,* $Tx_n = x_{n+1}$

Trivially, if we take some $x_n = x_{n+1}$, then x_{n+1} is a fixed point of T, which completes the proof.

So, let us take $x_n \neq x_{n+1}$ $\forall n \in N$, Denote by $d_n = d(x_n, x_{n+1}, t)$, the fuzzy-metric for t-norm t > 0. Then by hypothesis and monotony of F∈ $\mathcal{F}$, we have $\forall n \in N$

$$\begin{aligned}
\tau + F(d_n) &= \tau + F(x_n, x_{n+1}, t)) \\
&= \tau + F(Tx_{n-1}, Tx_n, t)) \quad \forall n \in N \\
&\leq F[\alpha d(x_{n-1}, x_n, t)+\beta d(x_{n-1}, Tx_{n-1}, t)+ \gamma d(x_n, Tx_n, t)+ \delta d(x_{n-1}, Tx_{n-1}, t)+ Ld(x_n, Tx_{n+1}, t)] \\
&= F[\alpha d(x_{n-1}, x_n, t)+\beta d(x_{n-1}, x_n, t)+ \gamma\, d(x_n, x_{n+1}, t)+ \delta d(x_{n-1}, x_n, t)+ Ld(x_n, x_n, t)] \\
&= F\ \alpha d_{n-1} + \beta d_{n-1} + \gamma\, d_n + \delta\ d(x_{n-1}, x_{n+1}, t)+ 0,\ as\ d(x_n, x_n, t)=0
\end{aligned}$$

Since $d_n = d(x_n, x_{n+1}, t)$ and $d_{n-1} = d(x_n, x_{n-1}, t)$ so we have

$$\begin{aligned}
d_n + d_{n-1} &= d(x_n, x_{n+1}, t) + d(x_n, x_{n-1}, t) = d(x_{n-1}, x_{n+1}, t) \\
&\leq F[\alpha d_{n-1} + \beta d_{n-1} + \gamma\, d_n + \delta\, (d_n + d_{n-1})] \\
&= F[(\alpha + \beta + \delta)\, d_{n-1} + (\gamma + \delta)\, (d_n)],
\end{aligned} \tag{2.1}$$

Therefore, we have

$$F(d_n) \leq F[(\alpha + \beta + \delta)\, d_{n-1} + (\gamma + \delta)\, (d_n)] - \tau$$

But $\tau > 0$, so

$$F(d_n) < F[(\alpha + \beta + \delta)\, d_{n-1} + (\gamma + \delta)\, (d_n)] \tag{2.2}$$

Thus from monotonicity of F, we have

$$\begin{aligned}
& d_n < (\alpha + \beta + \delta)\, d_{n-1} + (\gamma + \delta)\, (d_n) \\
& \Rightarrow (1-\gamma-\delta)\, d_n < (\alpha + \beta + \delta)\, d_{n-1} \\
& \Rightarrow d_n < \frac{(\alpha + \beta + \delta)}{(1-\gamma-\delta)}\ d_{n-1} \\
& i.e.,\ d_n < d_{n-1}
\end{aligned}$$

$for\ all\ n \in N$ and $\gamma \neq 1, \alpha + \beta + \gamma + 2\delta = 1 \Rightarrow (\alpha + \beta + \delta) = (1 - \gamma - \delta)$

Thus the sequence $\{d_n\}$ is a strictly decreasing, so there exist a positive real number, say d, such that

$$\lim_{n\to\infty} d_n = d \Rightarrow \lim_{n\to\infty} (x_n, x_{n+1}, t) = d, \text{ for } t > 0.$$

Suppose that $d > 0$, since F is an increasing function there exist $\lim_{x\to d} F(x) = F(d+0)$,

so from inequality (2.2) $F(d+0) \leq F(d+0) - \tau$

which is contradiction, therefore $\lim_{n\to\infty} d_n = 0 \Rightarrow \lim_{n\to\infty} (x_n, x_{n+1}, t) = 0$.

Now we want to show that $\{x_n\}$ is a Cauchy sequence.

Suppose to the contrary, we assume that $\{x_n\}$ is not a Cauchy sequence, then there exist $\epsilon > 0$ and $t > 0$ and a sequence n(k), m(k) $\in$ N and n(k) >m(k)> k and

$$d(x_{n(k)}, x_{m(k)},t_0) > \epsilon, \quad d(x_{n(k)-1}, x_{m(k)},t_0) \le \epsilon \ \ \forall \, k \in N$$

Then we have, $\epsilon < d(x_{n(k)}, x_{m(k)},t_0) \le d(x_{n(k)}, x_{n(k)-1},t_0) * d(x_{n(k)-1}, x_{m(k)},t_0) = d(x_{n(k)-1}, x_{n(k)},t_0) * \epsilon$

So, from above inequality $\lim_{n\to\infty} d(x_{n(k)}, x_{m(k)},t_0) = \epsilon,$

since $d(x_{n(k)}, x_{m(k)}, t_0) > \epsilon > 0$, by hypothesis and monotony of F we have,

$$\begin{aligned}\tau + F(d(x_{n(k)}, x_{m(k)}, t_0)) &\le F[\alpha d(x_{n(k)-1}, x_{m(k)-1}, t_0) + \beta d(x_{n(k)-1}, Tx_{n(k)-1}, t_0) + \gamma d(x_{m(k)-1}, Tx_{m(k)-1}, t_0) \\ &\quad + \delta d(x_{n(k)-1}, Tx_{m(k)-1}, t_0) + Ld(Tx_{n(k)-1}, x_{m(k)-1}, t_0)] \\ &= F[\alpha d(x_{n(k)-1}, x_{m(k)-1}, t_0) + \beta d(x_{n(k)-1}, x_{n(k)}, t_0) + \gamma d(x_{m(k)-1}, x_{m(k)}, t_0) \\ &\quad + \delta d(x_{n(k)-1}, x_{m(k)}, t_0) + Ld(x_{n(k)}, x_{m(k)-1}, t_0)], \text{ as equation (2.1)} \\ &\le F[\alpha\{ d(x_{n(k)}, x_{m(k)}, t_0) + d_{n(k)-1}+ d_{m(k)-1}\} + \beta d_{n(k)-1} + \gamma d_{m(k)-1} + \delta\{d(x_{n(k)}, x_{m(k)}, t_0) + d_{n(k)-1}\} \\ &\quad + L\{d(x_{n(k)}, x_{m(k)}, t_0) + d_{mk)-1}\}] \\ &= F[(\alpha+\delta+L) d(x_{n(k)}, x_{m(k)}, t_0) + (\alpha +\beta + \delta) d_{n(k)-1} + (\alpha+\gamma+L) d_{m(k)-1}].\end{aligned}$$

Since $\alpha+\delta+L \le 1$ & $\alpha + \beta + \gamma + 2\delta = 1$ so $\alpha +\beta + \delta = 1-\gamma - \delta$.

Also $\gamma < 1$ and $\delta < \frac{1}{2}$ $\Rightarrow \alpha +\beta + \delta < 1$.

So, we have

$$\tau + F(d(x_{n(k)}, x_{m(k)}, t_0)) \le F [d(x_{n(k)}, x_{m(k)}, t_0) + d_{m(k)-1} +(\alpha +\gamma+ L) d_{m(k)-1}]$$

Taking limit as n→ ∞ in above, we have

$$\tau + F(\varepsilon+0) \le F(\varepsilon+0),$$

which is contradiction. Thus$\{x_n\}$ is a Cauchy sequence in X. Further, since (X, d, $*$) is a complete fuzzy metric space, there exist z $\in$ X such that $\lim_{n\to\infty} x_n = z$.

If there exist a sequence $\{n_{(k)}\}$, k $\in$ N of natural numbers such that $x_{n(k)+1} = Tx_{n(k)} =Tz$, then $\lim_{n\to\infty} x_{n(k)+1} = z$. Which implies Tz = z. Hence z is a fixed point of T.

Finally, the continuity of T yields

$$d(Tz, z, t) = \lim_{n\to\infty} (Tx_n, x_n, t) = \lim_{n\to\infty} (x_{n+1}, x_n, t) =1.$$

Thus, z is a fixed point of *(X, d, $*$)*.

Now we show that T has a unique fixed point. Suppose z_1, z_2 are two fixed points of T, Indeed $z_1, z_2 \in X$ and $Tz_1= z_1\neq z_2= Tz_2$ Then we get, t > 0

$$\begin{aligned}\tau + F(d(z_1, z_2, t)) &= \tau + F(d(Tz_1, Tz_2, t)) \\ &\le F[\alpha d(z_1, z_2 , t)+\beta d(z_1, Tz_1, t)+ \gamma d(z_2,Tz_2, t)+ \delta d(z_1, Tz_2, t)+ Ld(z_2, Tz_1, t)] \\ &= F[\alpha d(z_1, z_2 , t)+\beta d(z_1, z_1, t)+ \gamma d(z_2, z_2, t)+ \delta d(z_1, z_2, t)+ Ld(z_2, z_1, t)]. \quad [\text{using } d(z_1, z_1, t) = d(z_2, z_2, t) = 0] \\ &\le F[(\alpha+\delta+L).d(z_1, z_2, t)], \quad [\text{as, } (\alpha+\delta+L) \le 1] \\ &\le F[d(z_1, z_2, t)]\end{aligned}$$

which is contradiction. Therefore T has a unique fixed point in *X*. This completes the proof.

Following is an example of our Main Theorem 2.1

2.2 Example: Let *X= [0,1)*, define the t-norm by *a$*$b = max{a, b}*, then

$$d(x, y, t) = \frac{1}{\{1-\min(x,y)\}} \quad \text{when } x\neq y$$

$$d(x, y, t) = 1, \quad \text{when } x = y$$

for all t > 0, let F: [0,1)→ R such that $F(x) = \frac{1}{1-x}$ $\forall \in$ [0,1), define T: X →X, by $T(x) = \frac{5}{2} x^2$ $\forall$ x $\in$X,

It is clear that *(X, d, $*$)* be a non-Archimedean fuzzy metric space.

(F1): F is a strictly increasing sequence, as $x_1 < x_2$ implies $F(x_1) = (1-x_1)^{-1} < (1-x_2)^{-1} = F(x_2)$.

(F2): For each sequence $\{\alpha_n\}$ of positive numbers, $\lim_{n\to\infty} \alpha_n = 0$ iff $\lim_{n\to\infty} [1/(1-\alpha_n)]^{-1} = 0$.

(F3):There exist k $\in$ [0,1): $\lim_{\alpha\to 0+} \alpha^k F(\alpha) = \lim_{\alpha\to 0+} \alpha^k (1+\alpha+ \alpha^2+\ldots..) = 0 + 0 +\ldots. = 0$.

Thus, F satisfy conditions (F1), (F2), (F3). Further, From Remark-6 of Müzeyyen [13], the given function

$F(x) = \frac{1}{1-x}$ $\forall \in$ [0,1) is a F-contraction T.

Following cases arises in order to check that T reduces to F-contraction condition:

Case-1: Let x < y ∀ x, y ∈ [0,1), since $x^2 < x$, $y^2 < y$, then min (x, y, t) < min (Tx, Ty, t), so $\delta > 0$ such that

$$\tau + \frac{1}{[\min(Tx,Ty,t)-1]} \leq \frac{1}{[\min(x,y,t)-1]}$$

i.e., $$\tau + \frac{1-\min(Tx,Ty,t)}{\min(Tx,Ty,t)} \leq \frac{1-\min(x,y,t)}{\min(x,y,t)}$$

or, $$\tau + \frac{1-\min(Tx,Ty,t)}{[1+\min(Tx,Ty,t)-1]} \leq \frac{1-\min(x,y,t)}{[1+\min(x,y,t)-1]}$$

Then we get

$$\tau + \frac{1}{1-\frac{1}{1-\min(Tx,Ty,t)}} \leq \frac{1}{1-\frac{1}{1-\min(x,y,t)}}$$

i.e., $$\tau + \frac{1}{1-\mathrm{d}(Tx,Ty,\mathrm{t})} \leq \frac{1}{1-\mathrm{d}(x,y,\mathrm{t})}$$

$$\tau + \mathrm{F}(\mathrm{d}(\mathrm{Tx}, \mathrm{Ty}, \mathrm{t}) \leq \mathrm{F}(\mathrm{d}(\mathrm{x}, \mathrm{y}, \mathrm{t}).$$

Case-2: let x= 0, $0 < y < 1$ since $x^2 = 0$, $y^2 < y$. Then min {x, y} = y > y^2 = min {Tx, Ty}

Hence, $\mathrm{d}(\mathrm{Tx}, \mathrm{Ty}, \mathrm{t}) = \frac{1}{1-\min(Tx,Ty,t)} \leq \frac{1}{1-\min(x,y,t)} = \mathrm{d}(\mathrm{Tx}, \mathrm{Ty}, \mathrm{t})$

there exist $\tau > 0$, such that

$$\tau + \frac{1}{1-\min(Tx,Ty,t)} \leq \frac{1}{1-\min(x,y,t)}$$

that is $\tau + \mathrm{F}(\mathrm{d}(\mathrm{Tx}, \mathrm{Ty}, \mathrm{t}) \leq \mathrm{F}(\mathrm{d}(\mathrm{x}, \mathrm{y}, \mathrm{t})$

Therefore, T is an F-contraction, then all the conditions of above theorem (main results) hold. By actual calculation, x = 0 is the unique fixed point of T for F. This verifies our main Theorem.

2.3 Corollary: Let (X, d, ∗) be a non-Archimedean fuzzy metric space and let T be a self mapping on X, assume that there exists an increasing function F: $R_+ \to R$ and $\tau > 0$ such that

$$\tau + \mathrm{F}(\mathrm{d}(\mathrm{Tx}, \mathrm{Ty}, \mathrm{t}) \leq \mathrm{F}[\alpha \mathrm{d}(\mathrm{x}, \mathrm{y}, \mathrm{t}) + \beta \mathrm{d}(\mathrm{x}, \mathrm{Tx}, \mathrm{t}) + \gamma \mathrm{d}(\mathrm{y}, \mathrm{Ty}, \mathrm{t})]$$

∀ x, y ∈ X and $T_x \neq Ty$, $t > 0$ $\alpha + \beta + \gamma = 1$, $\alpha > 0$, Then T has a unique fixed point in X.

2.4 Corollary: Let (X, d, ∗) be a non-Archimedean fuzzy metric space and T:X→X, assume that there exist F: $R_+ \to R$ is an increasing function and $\tau > 0$ such that

$$\tau + \mathrm{F}(\mathrm{d}(\mathrm{Tx}, \mathrm{Ty}, \mathrm{t}) \leq \mathrm{F}[\alpha \mathrm{d}(\mathrm{x},\mathrm{y},\mathrm{t})] \ \forall\, x, y \in X \text{ and } Tx \neq Ty,\ t > 0 \ \ \alpha + \beta + \gamma = 1,\ \alpha > 0,$$

Then *T* has a unique fixed point in *X*.

CONCLUSION

In this paper, we introduce modified contraction types in non-Archimedean fuzzy metric space and presented new fixed-point results. Our results can be expended solutions to new problems can be produced in this way. Also, a new more general contraction can be achieved in F-contraction in other spaces.

COMPETING INTERESTS

The authors declare that they have no competing interests.

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Source of support: Nil, Conflict of interest: None Declared.