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# ON SUPPORT NEIGHBOURLY IRREGULAR INTERVAL-VALUED FUZZY GRAPHS <br> K. PRIYADHARSHINI*1 ${ }^{* 1}$ AND N. R. SANTHI MAHESWARI ${ }^{\mathbf{2}}$ 

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#### Abstract

In this paper, support neighbourly irregular interval-valued fuzzy graphs and support totally neighbourly irregular interval-valued fuzzy graphs are defined. Comparative study between support neighbourly irregular interval-valued fuzzy graph and totally support neighbourly irregular interval-valued fuzzy graph is done. A necessary and sufficient condition under which they are equivalent is provided.


Keywords: support(2-degree) of a vertex in fuzzy graph, interval-valued fuzzy graph, support neighbourly irregular fuzzy graph, support neighbourly totally irregular fuzzy graph.

AMSsubjectclassification: 05C12, 03E72, $05 C 72$.

## 1. INTRODUCTION

In this paper, we consider only finite, simple, connected graphs. We denote the vertex set and the edge set of a graph $G$ by $V(G)$ and $E(G)$ respectively. The degree of a vertex $v$ is the number of edges incident at $v$, and it is denoted by $d(v)$. A graph $G$ is regular if all its vertices have the same degree. The notion of fuzzy sets was introduced by Zadeh as a way of representing uncertainity and vagueness [29]. The first definition of fuzzy graph was introduced by Haufmann in 1973. In 1975, A. Rosenfeld introduced the concept of fuzzy graphs [9]. The theory of graph is an extremely useful tool for solving combinatorial problems in different areas. Irregular fuzzy graphs play a central role in combinatorics and theoretical computer science.In 1975, Zadeh introduced the notion of interval-valued fuzzy sets as an extension of fuzzy set[30] in which the values of the membership degree are intervals of numbers instead of the numbers. In 2011, Akram and Dudek[1] defined interval-valued fuzzy graphs and give some operations on them.

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## 2. REVIEW OF LITERATURE

Nagoorgani and Radha introduced the concept of degree, total degree, regular fuzzy graphs in 2008 [6]. Nagoorgani and Latha introduced the concept of irregular fuzzy graphs, neighbourly irregular fuzzy graphs and highly irregular fuzzy graphs in 2012 [7]. N.R.Santhi Maheswari and C.Sekar introduced (2, k)-regular fuzzy graphs and totally (2, k)-regular fuzzy graphs, (r,2,k)-regular fuzzy graphs,(m, k)-regular fuzzy graphs and (r, m, k)-regular fuzzy graphs [10, 14, 15, 16]. N.R.Santhi Maheswari and C. Sekar introduced 2-neighbourly irregular fuzzy graphs and m-neighbourly irregular fuzzy graphs [21,13]. N.R.Santhi Maheswari and C.Sekar introduced an edge irregular fuzzy graphs, neighbourly edge irregular fuzzy graphs and strongly edge irregular fuzzy graph [17,11,18]. D.S.Cao, introduced 2-degree of vertex v is the the sum of the degrees of the vertices adjacent to v and it is denoted by $\mathrm{t}(\mathrm{v})$ [3]. A.Yu, M.Lu and F.Tian, introduced pseudo degree (average degree) of a vertex $v$ is ( $\mathrm{t}(\mathrm{v})$ ) / $\mathrm{d}(\mathrm{v})$, where $\mathrm{d}(\mathrm{v})$, is the number of edges incident at the vertex v [2]. N.R.Santhi Maheswari and C.Sekar introduced 2-degree of a vertex in fuzzy graphs, pseudo degree of a vertex in fuzzy graph and pseudo regular fuzzy graphs[12]. N.R Santhi Maheswari and M.Sutha introduced concept of pseudo irregular fuzzy graphs and highly pseudo irregular fuzzy graphs[19]. N.R.Santhi Maheswari and M.Rajeswari introduced the concept of strongly pseudo irregular fuzzy graphs [20]. N.R.Santhi Maheswari and V.Jeyapratha introduced the concept of neighbourly pseudo irregular fuzzy graphs[22]. N.R.Santhi Maheswari and K.Amutha introduced support neighbourly edge irregular graphs and 1-neighbourly edge irregular graphs, Pseudo Edge Regular and Pseudo Neighbourly edge irregular graphs [23,24,25]. J.Krishnaveni and N.R.Santhi Maheswari introduced support and total support of a vertex in fuzzy graphs, support neighbourly irregular fuzzy graphs and support neighbourly totally irregular fuzzy graphs[4].N.R.Santhi Maheswari and K.Priyadharshini introduced support highly irregular fuzzy graphs[26]. These ideas motivate us to introduce support neighbourly irregular interval-valued fuzzy graphs and support totally neighbourly irregular interval-valued fuzzy graphs and discussed some of its properties.

## 3. PRELIMINARIES

We present some known definitions and results for ready reference to go through the work presented in this paper. By graph, we mean a pair $G^{*}=(V, E)$, where $V$ is the set and $E$ is a relation on $V$. The elements of $V$ are vertices of $G^{*}$ and the elements of E are edges of $\mathrm{G}^{*}$.

Definition 3.1: 2-degree (support) of $v$ is defined as the sum of the degrees of the vertices adjacent to $v$ and it is denoted by $t(v)[3]$.

Definition 3.2: Average (pseudo) degree of $v$ is defined as $(t(v)) /(d(v))$, where $t(v)$ is the 2-degree of $v$ and $d(v)$ is the degree of $v$ and it is denoted by $d_{a}(v)$ [2].

Definition 3.3: A graph is called pseudo-regular if every vertex of $G$ has equal (pseudo) average-degree [2] .
Definition 3.4: A fuzzy graph $G:(\sigma, \mu)$ is a pair of functions $(\sigma, \mu)$, where $\sigma: V \rightarrow[0,1]$ is a fuzzy subset of a non-empty set $V$ and $\mu: V X V \rightarrow[0,1]$ is a symmetric fuzzy relation on $\sigma$ such that for all $u, v$ in $V$, the relation $\sigma(u v) \leq \sigma(u) \Lambda \sigma(v)$ is satisfied. A fuzzy graph $G$ is called complete fuzzy graph if the relation $\sigma(u v)=\sigma(u) \Lambda \sigma(v)$ is satisfied [6].

Definition 3.5: Let $G:(\sigma, \mu)$ be a fuzzy graph on $G^{*}(V, E)$. The degree of a vertex $u$ in $G$ is denoted by $d(u)$ and is defined as $d(u)=\sum^{\dagger} \mu(u v)$, for all $u v \in E[6]$.

Definition 3.6: Let $G$ : $(\sigma, \mu)$ be a fuzzy graph on $G^{*}(V, E)$. The total degree of a vertex $u$ in $G$ is denoted by $t d(u)$ and is defined as $t d(u)=d(u)+\sigma(u)$, for all $u \in V[6]$.

Definition 3.7: Let $G:(\sigma, \mu)$ be a fuzzy graph on $G^{*}(V, E)$. Then $G$ is said to be an irregular fuzzy graph, if there is a vertex which is adjacent to the vertices with distinct degrees[7].

Definition 3.8: Let $G$ : $(\sigma, \mu)$ be a fuzzy graph on $G^{*}(V, E)$. Then $G$ is said to be a totally irregular fuzzy graph if there is a vertex which is adjacent to the vertices with distinct total degrees[7].

Definition 3.9: Let $G:(\sigma, \mu)$ be a fuzzy graph on $G^{*}(V, E)$. Then $G$ is said to be a neighbourly irregular fuzzy graph if every two adjacent vertices of $G$ have distinct degrees[7].

Definition 3.10: Let $G$ : $(\sigma, \mu)$ be a fuzzy graph on $G^{*}(V, E)$. Then $G$ is said to be a neighbourly totally irregular fuzzy graph if every two adjacent vertices have distinct total degrees[7].

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Definition 3.11: Let $G:(\sigma, \mu)$ be a fuzzy graph on $G^{*}(V, E)$. Then $G$ is said to be a highly irregular fuzzy graph if every vertex of $G$ is adjacent to vertices with distinct degrees[7].

Definition 3.12: Let $G:(\sigma, \mu)$ be a fuzzy graph on $G^{*}(V, E)$. Then $G$ is said to be a highly totally irregular fuzzy graph if every vertex of $G$ is adjacent to vertices with distinct total degrees[7].

Definition 3.13: Let $G:(\sigma, \mu)$ be a fuzzy graph on $G^{*}(V, E)$. Then $G$ is said to be a regular fuzzy graph if all the vertices of $G$ have same degree[6].

Definition 3.14: Let $G:(\sigma, \mu)$ be a fuzzy graph on $G^{*}(V, E)$. Then $G$ is said to be a totally regular fuzzy graph if all the vertices of $G$ have same total degree[6].

Definition 3.15: Let $G:(\sigma, \mu)$ be a fuzzy graph on $G^{*}:(V, E)$. The support (2-degree) of a vertex $v$ in $G$ is defined as the sum of degrees of the vertices adjacent to $v$ and is denoted by $s(v)$. That is, $s(v)=\Sigma d G(u)$, where $d G(u)$ is the degree of the vertex $u$ which is adjacent with the vertex v[4].

Definition 3.16: Let $G:(\sigma, \mu)$ be a fuzzy graph on $G^{*}(V, E)$. The total support of a vertex $v$ in $G$ is denoted by $t s(v)$ and is defined as $t s(v)=s(v)+\sigma(v)$, for all $v \in V[4]$.

Definition 3.17: A graph $G$ is said to be a support neighbourly irregular fuzzy graph if every two adjacent vertices of $G$ have distinct supports[4].

Definition 3.18: A graph $G$ is said to be a support neighbourly totally irregular graph if every two adjacent vertices of $G$ have distinct total supports[4].

Definition 3.19: A graph $G$ is said to be a support highly irregular fuzzy graph if every vertex of $G$ is adjacent to the vertices having distinct supports[4].

Definition 3.20: A graph $G$ is said to be a support highly totally irregular graph if every vertex of $G$ is adjacent to the vertices having distinct total supports[26].

Definition 3.21: An interval-valued fuzzy graph with an underlying set $V$ is defined to be the pair $(A, B)$, where $A=\left(\mu_{A}^{-}, \mu_{A}^{+}\right)$is an interval-valued fuzzy set on $V$ such that $\mu_{A}^{-}(x) \leq \mu_{A}^{+}(x)$, for all $x \in V$ and $B=\left(\mu_{B}^{-}, \mu_{B}^{+}\right)$is an interval-valued fuzzy set on $E$ such that $\mu_{B}^{-}(x, y) \leq \min \left(\left(\mu_{A}^{-}(x), \mu_{A}^{-}(y)\right)\right)$ and $\mu_{B}^{+}(x, y) \leq \min \left(\left(\mu_{A}^{+}(x), \mu_{A}^{+}(y)\right)\right)$, for all edge $x y \in E$. Hence $A$ is called an interval-valued fuzzy vertex set on $V$ and $B$ is called an interval-valued fuzzy edge set on $E$.

Definition 3.22: Let $G:(A, B)$ be an interval-valued fuzzy graph. The negative degree of a vertex $u \in G$ is defined as $d_{G}^{-}(u)=\sum{ }^{\dagger} \mu_{B}^{-}(u, v)$, for $u v \in E$. The positive degree of a vertex $u \in G$ is defined as $d_{G}^{+}(u)=\Sigma \mu_{B}^{+}(u, v)$, for $u v \in E$ and $\mu_{B}^{+}(u v)=\mu_{B}^{-}(u v)=0$ if $u v$ not in $E$. The degree of a vertex $u$ is defined as $d_{G}(u)=\left(d_{G}^{-}(u), d_{G}^{+}(u)\right)$.

Definition 3.23: Let $G:(A, B)$ be an interval-valued fuzzy graph on $G^{*}(V, E)$. The total degree of a vertex $u \in V$ is denoted by $t d_{G}(u)$ and is defined as $t d_{G}(u)=\left(t d_{G}^{-}(u), t d_{G}^{+}(u)\right)$, where $t d_{G}^{-}(u)=\sum{ }^{\dagger}\left(\mu_{B}^{-}(u, v)+\left(\mu_{A}^{-}(u)\right)\right.$ and $t d_{G}^{+}(u)=\sum^{\dagger}\left(\mu_{B}^{+}(u, v)+\left(\mu_{A}^{+}(u)\right)\right.$.

Definition 3.24: Let $G:(A, B)$ be an interval-valued fuzzy graph on $G^{*}(V, E)$, where $A=\left(\mu_{A}^{-}, \mu_{A}^{+}\right)$and $B=\left(\mu_{B}^{-}, \mu_{B}^{+}\right)$ be two interval-valued fuzzy sets on a non-empty set $V$ and $E \subseteq V \times V$ respectively. Then $G$ is said to be regular interval-valued fuzzy graph if all the vertices of $G$ has same degree $\left(c_{1}, c_{2}\right)$.

Definition 3.25: Let $G:(A, B)$ be an interval-valued fuzzy graph on $G^{*}(V, E)$, then $G$ is said to be totally regular interval-valued fuzzy graph if all the vertices of $G$ has same total degree $\left(c_{1}, c_{2}\right)$.

## 4. SUPPORT IRREGULAR INTERVAL-VALUED FUZZY GRAPH

Definition 4.1: Let $G:(A, B)$ be an interval-valued fuzzy graph on $G^{*}(V, E)$. Then $G$ is said to be a support irregular interval-valued fuzzy graph if there exist at least a vertex which is adjacent to the vertices with distinct supports.

Example 4.2: Let $G:(A, B)$ be an interval-valued fuzzy graph on $G *(V, E)$.


Here, $s_{G}(u)=(0.6,0.8), s_{G}(v)=(0.7,0.9), s_{G}(w)=(0.5,0.7), s_{G}(x)=(0.5,0.7), s_{G}(y)=(0.7,0.9)$.
Therefore the graph is support irregular interval-valued fuzzy graph.
Definition 4.3: Let $G:(A, B)$ be an interval-valued fuzzy graph on $G^{*}(V, E)$. Then $G$ is said to be an totally support irregular if there exist at least a vertex which is adjacent to the vertices with distinct total supports.

Example 4.4: Let $G:(A, B)$ be an interval-valued fuzzy graph on $G^{*}(V, E)$.


Here, $t s_{G}(u)=(2.1,3.1), t s_{G}(v)=(2.2,3.2), t s_{G}(w)=(2.3,3.3), t s_{G}(x)=(2.4,3.4)$.
Therefore the graph $G$ is totally support irregular interval-valued fuzzy graph.
Remark 4.5: Every support irregular interval-valued fuzzy graph need not be support totally irregular interval-valued fuzzy graph.

Example 4.6: In fig.1, $t s_{G}(u)=(1.4,1.6)$ for all $u \in V$. Therefore the graph is $G$ is support irregular but not totally support irregular interval-valued fuzzy graph.

Remark 4.7: Every support totally irregular interval-valued fuzzy graph need not be support irregular interval-valued fuzzy graph.

Example 4.8: In fig.2, $s_{G}(u)=(1.8,2.7)$ for all $u \in V$. Therefore the graph is $G$ is support totally irregular but not support irregular interval-valued fuzzy graph.

Theorem 4.9: Let $G:(A, B)$ be an interval-valued fuzzy graph on $G^{*}(V, E)$ and $A$ is a constant function. Then the following conditions are equivalent (i) $G$ is support irregular interval-valued fuzzy graph (ii) $G$ is totally support irregular interval-valued fuzzy graph.

Proof: Assume that $A(u)=\left(\mu_{A}^{-}(u), \mu_{A}^{+}(u)\right)=\left(c_{1}, c_{2}\right)$, for all $u \in V$, where $c_{1}$ and $c_{2}$ are constant. Suppose $G$ is a support irregular interval-valued fuzzy graph. Then, there exist a vertex which is adjacent to the vertices with distinct support. Let $v_{1}$ and $v_{2}$ be the adjacent vertices of $v_{3}$ with distinct supports ( $l_{1}, l_{1}$ ) and ( $m_{1}, m_{2}$ ) respectively. Then $\left(l_{1}, l_{1}\right) \neq\left(m_{1}, m_{2}\right)$. Suppose $G$ is not a totally support neighbourly irregular interval-valued fuzzy graph. Then, every vertex of $G$ which is adjacent to the vertices with same total support $\Rightarrow t s_{G}\left(v_{1}\right)=t s_{G}\left(v_{2}\right) \Rightarrow d_{G}\left(v_{1}\right)+A\left(v_{1}\right)=$ $d_{G}\left(v_{2}\right)+A\left(v_{2}\right) \Rightarrow\left(l_{1}, l_{2}\right)+\left(c_{1}, c_{2}\right)=\left(m_{1}, m_{2}\right)+\left(c_{1}, c_{2}\right) \Rightarrow\left(l_{1}, l_{2}\right)=\left(m_{1}, m_{2}\right)$, which is a contradiction to $\left(l_{1}, l_{2}\right) \neq\left(m_{1}, m_{2}\right)$. Hence $G$ is totally support irregular interval-valued fuzzy graph. Thus $(i i) \Rightarrow$ (i) is proved. Hence (i) and (ii) are equivalent.

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Now, suppose $G$ is a support irregular interval-valued fuzzy graph. Then, at least one vertex of $G$ which is adjacent to the vertices with distinct total support. Let $v_{1}$ and $v_{2}$ be the adjacent vertices of $v_{3}$ with distinct total support.

Now
$t_{G}\left(v_{1}\right) \neq t_{G}\left(v_{2}\right) \Rightarrow d_{G}\left(v_{1}\right)+A\left(v_{1}\right) \neq d_{G}\left(v_{2}\right)+A\left(v_{2}\right) \Rightarrow d_{G}\left(v_{1}\right)+\left(c_{1}, c_{2}\right) \neq d_{G}\left(v_{2}\right)+\left(c_{1}, c_{2}\right) \Rightarrow d_{G}\left(v_{1}\right) \neq$ $d_{G}\left(v_{2}\right)$. Hence $G$ is support irregular interval-valued fuzzy graph. Thus (ii) $\Rightarrow(i)$ is proved. Hence ( $i$ ) and (ii) are equivalent.

## 5. SUPPORT NEIGHBOURLY IRREGULAR INTERVAL-VALUED FUZZY GRAPHS

In this section, we define support neighbourly irregular interval-valued fuzzy graph and totally support neighbourly irregular interval-valued fuzzy graph and discussed about its properties.

Definition 5.1: Let $G:(A, B)$ be an interval-valued fuzzy graph on $G^{*}:(V, E)$. Then $G$ is said to be support neighbourly irregular interval-valued fuzzy graph if any two adjacent vertices of $G$ have distinct supports.

Definition 5.2: Let $G:(A, B)$ be an interval-valued fuzzy graph on $G^{*}:(V, E)$. Then $G$ is said to be support neighbourly totally irregular interval-valued fuzzy graph if any two adjacent vertices of $G$ have distinct total supports.

Remark 5.3: A support neighbourly irregular interval-valued fuzzy graph need not be support neighbourly totally irregular interval-valued fuzzy graph.

Example 5.4: Consider a fuzzy graph on graph on $G^{*}(V, E)$.

fig. 3
Here, $s_{G}(a)=(1.7,1.7), s_{G}(b)=(1.6,1.6), s_{G}(c)=(1.7,1.7), s_{G}(d)=(1.6,1.6), s_{G}(e)=(1.8,1.8)$.
But, $t s_{G}(a)=(2.2,2.2), t s_{G}(b)=(2.2,2.2), t s_{G}(c)=(2.1,2.1), t s_{G}(d)=(2.1,2.1), t s_{G}(e)=(2.3,2.3)$. Hence $G$ is support neighbourly irregular interval-valued fuzzy graph but not totally support neighbourly interval-valued fuzzy graph.

Remark 5.5: A totally support neighbourly irregular interval-valued fuzzy graph need not be support neighbourly irregular interval-valued fuzzy graph.

Example 5.6: Consider an interval-valued fuzzy graph $G:(A, B)$ on graph $G^{*}(V, E)$.


Here, $\quad s_{G}(u)=(1.1,1.1), s_{G}(v)=(2.1,2.1), s_{G}(w)=(2.1,2.1), s_{G}(x)=(2.2,2.2), s_{G}(y)=(1.1,1.1) \quad$. And $t s_{G}(u)=(1.6,1.6), t s_{G}(v)=(2.7,2.7), t s_{G}(w)=(2.9,2.9), t s_{G}(x)=(3.1,3.1), t s_{G}(y)=(1.6,1.6)$. Hence $G$ is totally support neighbourly irregular interval-valued fuzzy graph but not support neighbourly irregular interval-valued fuzzy graph.

Theorem 5.7: Let $G:(A, B)$ be an interval-valued fuzzy graph on $G^{*}(V, E)$. Then $A(u)=\left(\mu_{A}^{-}(u), \mu_{A}^{+}(u)\right)$, for all $u \in V$ is a constant function then the following are equivalent.

- G is a support neighbourly irregular interval-valued fuzzy graph.
- $G$ is a totally support neighbourly irregular interval-valued fuzzy graph.

Proof: Assume that $A(u)=\left(\mu_{A}^{-}(u), \mu_{A}^{+}(u)\right)=\left(c_{1}, c_{2}\right)$, for all $u \in V$. Let $\sigma(u)=c$, for all $u \in V$, where $c_{1}$ and $c_{2}$ are constant. Suppose $G$ is a support neighbourly irregular interval-valued fuzzy graph. Then, every pair of adjacent vertices in $G$ have distinct support. Let $v_{1}$ and $v_{2}$ be any two adjacent vertices of $G$ with distinct supports ( $l_{1}, l_{2}$ ) and ( $m_{1}, m_{2}$ ) respectively. Then $\left(l_{1}, l_{2}\right) \neq\left(m_{1}, m_{2}\right)$. Suppose $G$ is not a totally support neighbourly irregular interval-valued fuzzy graph. Then, there exist at least one pair of adjacent vertices $v_{1}$ and $v_{2}$ in $G$ with same total support $\Rightarrow t s_{G}\left(v_{1}\right)=t s_{G}\left(v_{2}\right) \Rightarrow d_{G}\left(v_{1}\right)+A\left(v_{1}\right)=d_{G}\left(v_{2}\right)+A\left(v_{2}\right) \Rightarrow\left(l_{1}, l_{2}\right)+$ $\left(c_{1}, c_{2}\right)=\left(m_{1}, m_{2}\right)+\left(c_{1}, c_{2}\right) \Rightarrow\left(l_{1}, l_{2}\right)=\left(m_{1}, m_{2}\right)$, which is a contradiction to $\left(l_{1}, l_{2}\right) \neq\left(m_{1}, m_{2}\right)$. Hence $G$ is totally support neighbourly irregular interval-valued fuzzy graph. Thus (ii) $\Rightarrow$ ( $i$ ) is proved. Hence (i) and (ii) are equivalent.

Now, suppose $G$ is a support neighbourly irregular interval-valued fuzzy graph. Then, every pair of adjacent vertices in $G$ have distinct total support. Let $v_{1}$ and $v_{2}$ be any pair of adjacent vertices in $G$ with distinct total support. Now $t_{G}\left(v_{1}\right) \neq t_{G}\left(v_{2}\right) \Rightarrow d_{G}\left(v_{1}\right)+A\left(v_{1}\right) \neq d_{G}\left(v_{2}\right)+A\left(v_{2}\right) \Rightarrow d_{G}\left(v_{1}\right)+\left(c_{1}, c_{2}\right) \neq d_{G}\left(v_{2}\right)+$ $\left(c_{1}, c_{2}\right) \Rightarrow d_{G}\left(v_{1}\right) \neq d_{G}\left(v_{2}\right)$. Hence $G$ is support neighbourly irregular interval-valued fuzzy graph. Thus (ii) $\Rightarrow(i)$ is proved. Hence (i) and (ii) are equivalent.

Remark 5.8: Converse of above theorem need not be true.
Example 5.9: Consider an interval-valued fuzzy graph $G:(A, B)$ on graph $G^{*}(V, E)$.


Here, $s_{G}(a)=(0.8,1.3), s_{G}(b)=(1.3,2), s_{G}(c)=(0.8,1.3), s_{G}(d)=(1.3,2), s_{G}(e)=(0.7,1), s_{G}(f)=(0.7,1)$. And $t s_{G}(a)=(1.4,1.9), t s_{G}(b)=(2.1,2.9), t s_{G}(c)=(1.1,1.8), t s_{G}(d)=(2,2.8), t s_{G}(e)=(1.1,1.5), t s_{G}(f)=(1.4,1.9)$. Hence the graph $G$ is both support neighbourly irregular and totally support neighbourly irregular interval-valued fuzzy graph but $A$ is not constant.

Theorem 5.10: Consider an interval-valued fuzzy graph $G:(A, B)$ on graph $G^{*}(V, E)$. If the support of all the vertices of $G$ are distinct, then $G$ is support neighbourly irregular interval-valued fuzzy graph.

Proof: Assume that the support of all the vertices of G are distinct. Then every pair of adjacent vertices of G have distinct supports and hence G is support neighbourly irregular intuitionistic fuzzy graph.

Theorem 5.11: Consider an interval-valued fuzzy graph $G:(A, B)$, a cycle of length $n$ and $B$ is a constant function, then $G$ is not a support neighbourly irregular interval-valued fuzzy graph.

Proof: Assume that $B$ is a constant function, say $B(v, w)=\left(k_{1}, k_{2}\right)$, for all $v, w \in V$. Since G is a cycle of length $n$, we have $s_{G}(v)=\left(4 k_{1}, 4 k_{2}\right)$, for all $v \in V$. Thus $s_{G}(v)$ is constant for all $v \in V$. Hence G is not a support neighbourly irregular interval-valued fuzzy graph.

Theorem 5.12: Consider an interval-valued fuzzy graph $G:(A, B)$, a cycle of length $n$ and $B$ is a constant function, then $G$ is a totally support neighbourly irregular interval-valued fuzzy graph.

Proof: Assume that $B$ is constant, say $B\left(v_{i}, v_{j}\right)=\left(k_{1}, k_{2}\right)$. Also, given $A\left(v_{i}\right)=\left(m_{i}, n_{i}\right)$ for all $v_{i} \in V$. Since $G$ is a cycle of length $n$, we have $s\left(v_{i}\right)=\left(4 k_{1}, 4 k_{2}\right)$, for all $v_{i} \in V$. Also given $\mu_{A}^{-}\left(v_{i}\right)=m_{i}$ and $\mu_{A}^{+}\left(v_{i}\right)=n_{i}$, for all $v_{i} \in V$. Thus $m_{1} \neq m_{2} \neq \cdots \neq m_{n}$ and $n_{1} \neq n_{2} \neq \cdots \neq n_{n}$.

Now $t s_{G}(v i)=s_{G}\left(v_{i}\right)+\left(\mu_{A}^{-}\left(v_{i}\right), \mu^{+}{ }_{A}\left(v_{i}\right)\right)=\left(4 k_{1}, 4 k_{2}\right)+\left(m_{j} n_{i}\right)$, for $i=1,2, \ldots n$
Hence $G$ is totally support neighbourly irregular interval - valued fuzzy graph.

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