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VAGUE $\hat{g}$ FEEBLY FUNCTIONS IN VAGUE TOPOLOGICAL SPACES

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ABSTRACT

This paper aims to study the concept of vague $\hat{g}$ feebly continuous mappings and contra- vague $\hat{g}$ feebly continuous mappings. We investigate the traditional connectedness and compactness for the new class as vague $\hat{g}$ feebly connectedness and vague $\hat{g}$ feebly compactness. Also we provide some characterizations of the above mappings.

Keywords: *Vague $\hat{g}$ feebly continuous mapping, contra- vague $\hat{g}$ feebly continuous mapping, vague $\hat{g}$ feebly connectedness and vague $\hat{g}$ feebly compactness.*

1. INTRODUCTION

In this paper we introduce the notion of vague $\hat{g}$ feebly continuous mappings and contra- vague $\hat{g}$ feebly continuous mappings and studied some of their properties. We also provide some characterizations of vague $\hat{g}$ feebly connectedness and vague $\hat{g}$ feebly compactness.

2. PRELIMINARIES

Definition 2.1: [2] A vague set $\mathcal{A}$ in the universe of discourse X is characterized by two membership functions given by:

- A true membership function $T_{\mathcal{A}} : X \to [0,1]$ and
- A false membership function $F_{\mathcal{A}} : X \to [0,1]$,

where $T_{\mathcal{A}}(x)$ is lower bound on the grade of membership of x derived from the "evidence for x", $F_{\mathcal{A}}(x)$ is a lower bound on the negation of x derived from the "evidence against x" and $T_{\mathcal{A}}(x) + F_{\mathcal{A}}(x) \leq 1$.

Thus the grade of membership of x in the vague set $\mathcal{A}$ is bounded by a subinterval $[T_{\mathcal{A}}(x), 1 - F_{\mathcal{A}}(x)]$ of $[0, 1]$.

Definition 2.2: [2] Let $\mathcal{A}$ and $\mathcal{B}$ be vague sets of the form $\mathcal{A} = \{\langle x, [T_{\mathcal{A}}(x), 1 - F_{\mathcal{A}}(x)]\rangle / x \in X\}$ and $\mathcal{B} = \{\langle x, [T_{\mathcal{B}}(x), 1 - F_{\mathcal{B}}(x)]\rangle / x \in X\}$ Then

a) $\mathcal{A} \subseteq \mathcal{B}$ if and only if $T_{\mathcal{A}}(x) \leq T_{\mathcal{B}}(x)$ and $1 - F_{\mathcal{A}}(x) \leq 1 - F_{\mathcal{B}}(x)$ forall $x \in X$
b) $\mathcal{A}^C = \{\langle x, F_{\mathcal{A}}(x), 1 - T_{\mathcal{A}}(x) / x \in X\rangle\}$
c) $\mathcal{A} \cap \mathcal{B} = \{\langle x, \min(T_{\mathcal{A}}(x), T_{\mathcal{B}}(x)), \min(1 - F_{\mathcal{A}}(x), 1 - F_{\mathcal{B}}(x))\rangle / x \in X\}$
d) $\mathcal{A} \cup \mathcal{B} = \{\langle x, \max(T_{\mathcal{A}}(x), T_{\mathcal{B}}(x)), \max(1 - F_{\mathcal{A}}(x), 1 - F_{\mathcal{B}}(x))\rangle / x \in X\}$

Definition 2.3: [4] A Vague set $\mathcal{A}$ of (X, τ) is said to be
VSCS if $\mathrm{Vint}(\mathrm{Vcl}(\mathcal{A})) \subseteq \mathcal{A}$, VSOS in short if $\mathcal{A} \subseteq \mathrm{Vcl}(\mathrm{Vint}(\mathcal{A}))$, VPCS if $\mathrm{Vcl}(\mathrm{Vint}(\mathcal{A})) \subseteq \mathcal{A}$, VPOS if $\mathcal{A} \subseteq \mathrm{Vint}(\mathrm{Vcl}(\mathcal{A}))$, VαCS if $\mathrm{Vcl}(\mathrm{Vint}(\mathrm{Vcl}(\mathcal{A}))) \subseteq \mathcal{A}$, VαOS if $\mathcal{A} \subseteq \mathrm{Vint}(\mathrm{Vcl}(\mathrm{Vint}(\mathcal{A})))$, VROS if $\mathcal{A} = \mathrm{Vint}(\mathrm{Vcl}(\mathcal{A}))$, VRCS if $\mathcal{A} = \mathrm{Vcl}(\mathrm{Vint}(\mathcal{A}))$.

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Definition 2.4: [7]A vague set $\mathcal{A}$ of (X, τ) is said to be a **vague $\hat{g}$-closed sets (V$\hat{G}$CS in short)** if $Vcl(\mathcal{A}) \subseteq U$ whenever $\mathcal{A} \subseteq U$ and U is a vague semi open set in X.

Definition 2.5: [9]A vague set $\mathcal{A}$ in a topological space X is called **Vague feebly open** in X if there exists an open set O such that $O \subseteq \mathcal{A} \subseteq Vscl(O)$. The complement of Vague feebly open set is **a Vague feebly closed set.**

Definition 2.6: [9]Vague feebly open set if $A \subseteq Vscl(Vint(A))$ and Vague feebly closed set if $Vsint(Vcl(A)) \subseteq A$.

Definition 2.7: [9]A vague set $\mathcal{A}$ of (X, τ) is said to be a **vague feebly generalised closed sets (V$\mathcal{F}$GCS in short)** if $V\not{f}cl(\mathcal{A}) \subseteq U$ whenever $\mathcal{A} \subseteq U$ and U is a vague feebly open set in X.

Definition 2.8: [9]A vague set $\mathcal{A}$ of (X, τ) is said to be a **vague generalised feebly closed sets (VG$\mathcal{F}$CS in short)** if $V\not{f}cl(\mathcal{A}) \subseteq U$ whenever $\mathcal{A} \subseteq U$ and U is a vague open set in X.

Definition 2.9: [9]A vague set $\mathcal{A}$ in a vague topological space (X, τ) is said to be a **vague $\hat{g}$ feebly closed sets (V$\hat{G}\mathcal{F}$CS in short)** if $V\not{f}cl(\mathcal{A}) \subseteq U$ whenever $\mathcal{A} \subseteq U$ and U is a vague semi open set in X.

Definition 2.10: Let f be a mapping from a VTS (X, τ) into a VTS (Y, σ). Then f is said to be a

(i) [7]**Vague continuous mapping** (V continuous mapping for short) if $f^{-1}(\mathcal{B}) \in VC(X)$ for each $VCS\, \mathcal{B} \in Y$.

(ii) [7]**Vague generalized continuous mapping** (VG continuous mapping for short) if $f^{-1}(\mathcal{B}) \in VGC(X)$ for each VCS $\mathcal{B} \in Y$.

(iii) **Vague α-continuous mapping** (V α −continuous mapping for short) if $f^{-1}(\mathcal{B}) \in V\alpha C(X)$ for each $VCS\, \mathcal{B} \in Y$.

(iv) [4]**Vague semi-continuous mapping** (V semi - continuous mapping for short) if $f^{-1}(\mathcal{B}) \in VSC(X)$ for each $VCS\, \mathcal{B} \in Y$.

(v) [4]**Vague semi pre-continuous mapping** (V semi pre−continuous mapping for short) if $f^{-1}(\mathcal{B}) \in VSPC(X)$ for each $VCS\, \mathcal{B} \in Y$.

(vi) [8]**Vague $\hat{g}$ continuous mapping** (V$\hat{g}$ continuous for short) mapping if $f^{-1}(\mathcal{B}) is\ a\ V\hat{g}C(X)$ for every $VCS\, \mathcal{B} \in Y$.

(vii) **Vague feebly continuous mapping** (V $\not{f}$continuous mapping for short) if $f^{-1}(\mathcal{B}) \in V\not{f}C(X)$ for each $VCS\, \mathcal{B} \in Y$.

(viii) **Vague generalised feebly continuous mapping** (V $g\not{f}$continuous mapping for short) if $f^{-1}(\mathcal{B}) \in Vg\not{f}C(X)$ for each $VCS\, \mathcal{B} \in Y$.

(ix) **Vague feebly generalised continuous mapping** (V $\not{f}g$ continuous mapping for short) if $f^{-1}(\mathcal{B}) \in V\not{f}gC(X)$ for each $VCS\ \mathcal{B} \in Y$.

(x) **Contra vague continuous mapping** (Contra V continuous mapping for short) if $f^{-1}(\mathcal{B}) \in VC(X)$ for each $VOS\ \mathcal{B} \in Y$.

(xi) **Contra vague generalized continuous mapping** (Contra VG continuous mapping for short) if $f^{-1}(\mathcal{B}) \in VGC(X)$ for each $VOS\ \mathcal{B} \in Y$.

3. VAGUE $\hat{g}$ FEEBLY CONTINUOUS MAPPINGS

In this section we introduce vague $\hat{g}\ \not{f}$ continuous mapping and investigate some of its properties.

Definition 3.1: A mapping $f: (X, \tau) \to (Y, \sigma)$ is called vague $\hat{g}\not{f}$ continuous (V$\hat{g}\not{f}$ continuous for short) mapping if $f^{-1}(V) is\ a\ V\,\hat{g}\not{f}$ CS in (X, τ) for every VCS in (Y, σ).

Example 3.2: $X = \{a, b\}$, $Y = \{u, v\}$ and $\tau = \{0, G_1, 1\}$and $\sigma = \{0, G_2, 1\}$ are VT_s on X and Y respectively.
$G_1 = \{< x, [0.5,0.9], [0.2,0.5] >\}$, $G_2 = \{< y, [0.5,0.9], [0.4,0.6] >\}$. Define a mapping $f: (X, \tau) \to (Y, \sigma)\ by\ f(a) = u\ and\ f(b) = v$. Since the inverse image of a vague closed set $A = \{< y, [0.1,0.5], [0.4,0.6] >\}$ in (Y, σ) is a V $\hat{g}\not{f}$ CS in (X, τ). Hence f is a vague $\hat{g}$ feebly continuous mapping.

Proposition 3.3:

1. Every vague continuous map is vague $\hat{g}$ feebly continuous.
2. Every vague semi continuous map is vague $\hat{g}$ feebly continuous.
3. Every vague semi pre - continuous map is vague $\hat{g}$ feebly continuous
4. Every vague α −continuous map is vague $\hat{g}$ feebly continuous.
5. Every vague $\hat{g}$ − continuous map is vague g - continuous.
6. Every vague g - continuous map is vague $\hat{g}$ feebly continuous.
7. Every vague $\hat{g}$ −continuous map is vague $\hat{g}$ feebly continuous.
8. Every vague $\hat{g}\ \not{f}$ −continuous map is vague generalised feebly continuous.
9. Every vague $\hat{g}\ \not{f}$ − continuous map is vague feebly generalised continuous.
10. Every vague feebly continuous map is vague $\hat{g}$ feebly continuous.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be a vague continuous map. Let V be a vague closed set in (Y,σ). Since f is vague continuous map, $f^{-1}(V)$ is a vague closed set in (X,τ). Every vague closed set is vague ĝ feebly closed. Hence $f^{-1}(V)$ is a vague ĝ feebly closed set in (X,τ). Hence f is a vague ĝ feebly continuous.

Similarly we can prove the other propositions. The converses are not true as can be seen from the following examples.

Example 3.4: $\mathrm{X}=\{\mathrm{a,b}\}, \mathrm{Y}=\{\mathrm{u,v}\}$ and $\tau=\{0,\mathrm{G}_1,1\}$and $\sigma=\{0,\mathrm{G}_2,1\}$ are $\mathrm{VT_s}$ on X and Y respectively. $\mathrm{G}_1=\{<x,[0.5,0.9],[0.2,0.5]>\}, \mathrm{G}_2=\{<y,[0.5,0.9],[0.4,0.6]>\}$. Define a mapping $f:(X,\tau)\to(Y,\sigma)\ by\ f(a)=u\ and\ f(b)=v$. Since the inverse image of a vague closed set $\mathrm{A}=\{<y,[0.1,0.5],[0.4,0.6]>\}$ in (Y,σ) is a V ĝ∮ CS in (X,τ), but A is not vague closed in (X,τ) Hence f is a vague ĝ feebly continuous mapping but not vague continuous.

Example 3.5: $\mathrm{X}=\{\mathrm{a,b}\}, \mathrm{Y}=\{\mathrm{u,v}\}$ and $\tau=\{0,\mathrm{G}_1,1\}$and $\sigma=\{0,\mathrm{G}_2,1\}$ are $\mathrm{VT_s}$ on X and Y respectively. $\mathrm{G}_1=\{<x,[0.4,0.5],[0.5,0.6]>\}, \mathrm{G}_2=\{<y,[0.6,0.7],[0.2,0.4]>\}$. Define a mapping $f:(X,\tau)\to(Y,\sigma)\ by\ f(a)=u\ and\ f(b)=v$. Since the inverse image of a vague closed set $\mathrm{A}=\{<y,[0.3,0.4],[0.6,0.8]>\}$ in (Y,σ) is a V ĝ∮ CS in (X,τ), but A is not vague semi closed in (X,τ) Hence f is a vague ĝ feebly continuous mapping but not vague semi continuous.

Example 3.6: $\mathrm{X}=\{\mathrm{a,b}\}, \mathrm{Y}=\{\mathrm{u,v}\}$ and $\tau=\{0,\mathrm{G}_1,\mathrm{G}_2,1\}$and $\sigma=\{0,\mathrm{G}_3,1\}$ are $\mathrm{VT_s}$ on X and Y respectively. $\mathrm{G}_1=\{<x,[0.7,0.8],[0.8,0.9]>\}, \mathrm{G}_2=\{<x,[0.1,0.2],[0.2,0.3]>\}$ and $\mathrm{G}_3=\{<y,[0.1,0.4],[0.6,0.9]>\}$ Define a mapping $f:(X,\tau)\to(Y,\sigma)\ by\ f(a)=u\ and\ f(b)=v$. Since the inverse image of a vague closed set $\mathrm{A}=\{<y,[0.6,0.9],[0.1,0.4]>\}$ in (Y,σ) is a V ĝ∮ CS in (X,τ), but A is not vague semi pre-closed in (X,τ), since $Vint\,(B)\nsubseteq A\subseteq B$. Hence f is a vague ĝ feebly continuous mapping but not vague semi pre - continuous.

Example 3.7: $\mathrm{X}=\{\mathrm{a,b}\}, \mathrm{Y}=\{\mathrm{u,v}\}$ and $\tau=\{0,\mathrm{G}_1,1\}$and $\sigma=\{0,\mathrm{G}_2,1\}$ are $\mathrm{VT_s}$ on X and Y respectively. $\mathrm{G}_1=\{<x,[0.5,0.9],[0.5,0.5]>\}, \mathrm{G}_2=\{<y,[0.5,0.9],[0.4,0.6]>\}$. Define a mapping $f:(X,\tau)\to(Y,\sigma)\ by\ f(a)=u\ and\ f(b)=v$. Since the inverse image of a vague closed set $\mathrm{A}=\{<y,[0.1,0.5],[0.4,0.6]>\}$ in (Y,σ) is a V ĝ∮ CS in (X,τ), but A is not vague $\alpha-$ closed in (X,τ) Hence f is a vague ĝ feebly continuous mapping but not vague $\alpha-$ continuous.

Example 3.8: $\mathrm{X}=\{\mathrm{a,b}\}, \mathrm{Y}=\{\mathrm{u,v}\}$ and $\tau=\{0,\mathrm{G}_1,\mathrm{G}_2,\mathrm{G}_3,\mathrm{G}_4,1\}$and $\sigma=\{0,\mathrm{G}_5,1\}$ are $\mathrm{VT_s}$ on X and Y respectively. $\mathrm{G}_1=\{<x,[0.2,0.5],[0.4,0.5]>\}, \mathrm{G}_2=\{<x,[0.5,0.6],[0.3,0.4]>\}, \mathrm{G}_3=\{<x,[0.5,0.9],[0.5,0.6]>\}, \mathrm{G}_4=\{<x,[0.2,0.5],[0.3,0.4]>\}$and $\mathrm{G}_5=\{<y,[0.4,0.5],[0.5,0.6]>\}$ Define a mapping $f:(X,\tau)\to(Y,\sigma)\ by\ f(a)=u\ and\ f(b)=v$. Since the inverse image of a vague closed set $\mathrm{A}=\{<y,[0.5,0.6],[0.4,0.5]>\}$ in (Y,σ) is a V g CS in (X,τ), but A is not vague ĝ closed in (X,τ), when $B=\{<x,[0.5,0.8],[0.5,0.6]>\}$ is a vague semi closed set containing A. Hence f is a vague g continuous mapping but not vague ĝ continuous.

Example 3.9: $\mathrm{X}=\{\mathrm{a,b}\}, \mathrm{Y}=\{\mathrm{u,v}\}$ and $\tau=\{0,\mathrm{G}_1,1\}$and $\sigma=\{0,\mathrm{G}_2,1\}$ are $\mathrm{VT_s}$ on X and Y respectively. $\mathrm{G}_1=\{<x,[0.4,0.7],[0.2,0.2]>\}, \mathrm{G}_2=\{<y,[0.2,0.4],[0.8,0.9]>\}$. Define a mapping $f:(X,\tau)\to(Y,\sigma)\ by\ f(a)=u\ and\ f(b)=v$. Since the inverse image of a vague closed set $\mathrm{A}=\{<y,[0.6,0.8],[0.1,0.2]>\}$ in (Y,σ) is a V ĝ∮ CS in (X,τ), but A is not vague g - closed in (X,τ) Hence f is a vague ĝ feebly continuous mapping but not vague g - continuous.

Example 3.10: $\mathrm{X}=\{\mathrm{a,b}\}, \mathrm{Y}=\{\mathrm{u,v}\}$ and $\tau=\{0,\mathrm{G}_1,\mathrm{G}_2,\mathrm{G}_3,\mathrm{G}_4,1\}$and $\sigma=\{0,\mathrm{G}_5,1\}$ are $\mathrm{VT_s}$ on X and Y respectively. $\mathrm{G}_1=\{<x,[0.2,0.5],[0.4,0.5]>\}, \mathrm{G}_2=\{<x,[0.5,0.6],[0.3,0.4]>\}, \mathrm{G}_3=\{<x,[0.5,0.9],[0.5,0.6]>\}, \mathrm{G}_4=\{<x,[0.2,0.5],[0.3,0.4]>\}$and $\mathrm{G}_5=\{<y,[0.4,0.5],[0.5,0.6]>\}$ Define a mapping $f:(X,\tau)\to(Y,\sigma)\ by\ f(a)=u\ and\ f(b)=v$. Since the inverse image of a vague closed set $\mathrm{A}=\{<y,[0.5,0.6],[0.4,0.5]>\}$ in (Y,σ) is a V ĝ∮CS in (X,τ), but A is not vague ĝ closed in (X,τ), when $B=\{<x,[0.5,0.8],[0.5,0.6]>\}$ is a vague semi closed set containing A. Hence f is a vague ĝ∮ continuous mapping but not vague ĝ continuous.

Example 3.11: $\mathrm{X}=\{\mathrm{a,b}\}, \mathrm{Y}=\{\mathrm{u,v}\}$ and $\tau=\{0,\mathrm{G}_1,1\}$and $\sigma=\{0,\mathrm{G}_2,1\}$ are $\mathrm{VT_s}$ on X and Y respectively. $\mathrm{G}_1=\{<x,[0.4,0.7],[0.2,0.4]>\}, \mathrm{G}_2=\{<y,[0.6,0.8],[0.4,0.7]>\}$. Define a mapping $f:(X,\tau)\to(Y,\sigma)\ by\ f(a)=u\ and\ f(b)=v$. Since the inverse image of a vague closed set $\mathrm{A}=\{<y,[0.2,0.4],[0.3,0.6]>\}$ in (Y,σ) is a Vg∮CS in (X,τ), but A is not vague ĝ∮ closed in (X,τ), when $B=\{<x,[0.4,0.7],[0.3,0.6]>\}$ is a vague semi open set in X. Hence f is a vague g feebly continuous mapping but not vague ĝ feebly continuous.

Example 3.12: $X = \{a, b\}, Y = \{u, v\}$ and $\tau = \{0, G_1, 1\}$and $\sigma = \{0, G_2, 1\}$ are VT_s on X and Y respectively. $G_1 = \{< x, [0.1,0.6], [0.2,0.4] >\}, G_2 = \{< y, [0.4,0.8], [0.7,0.8] >\}$. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ $by\ f(a) = u\ and\ f(b) = v$. Since the inverse image of a vague closed set $A = \{< y, [0.2,0.6], [0.2,0.3] >\}$ in (Y, σ) is a Vg$\not{f}$CS in (X, τ), but A is not vague $\hat{g}\not{f}$ closed in (X, τ), when $B = \{< y, [0.2,0.6], [0.2,0.4] >\}$ is a vague semi open set in X. Hence f is a vague feebly g continuous mapping but not vague $\hat{g}$ feebly continuous.

Example 3.13: $X = \{a, b\}, Y = \{u, v\}$ and $\tau = \{0, G_1, 1\}$and $\sigma = \{0, G_2, 1\}$ are VT_s on X and Y respectively. $G_1 = \{< x, [0.5,0.9], [0.2,0.5] >\}, G_2 = \{< y, [0.5,0.9], [0.4,0.6] >\}$. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ $by\ f(a) = u\ and\ f(b) = v$. Since the inverse image of a vague closed set $A = \{< y, [0.1,0.5], [0.4,0.6] >\}$ in (Y, σ) is a V $\hat{g}\not{f}$ CS in (X, τ), but A is not vague feebly closed in (X, τ) Hence f is a vague $\hat{g}$ feebly continuous mapping but not vague feebly continuous.

Theorem 3.14: The following statements are equivalent for a function $f: (X, \tau) \rightarrow (Y, \sigma)$

(i) f is vague $\hat{g}\not{f}$ continuous.

(ii) For every vague open set V of Y, $f^{-1}(V)$ is vague $\hat{g}\not{f}$ open set in X.

Proof: $(\boldsymbol{i}) \Longrightarrow (\boldsymbol{ii})$ Let V be vague open subset of Y and let $x \in f^{-1}(V)$ be any arbitrary point. Since $f(x) \in V$ by (i), there exist vague $\hat{g}\not{f}$ open set U_x in X, containing x such that arbitrary union of vague $\hat{g}\not{f}$ open sets is vague $\hat{g}\not{f}\ open$, $f^{-1}(V)$ is vague $\hat{g}\not{f}$open in X.
$(ii) \Longrightarrow (i)$ it is obvious.

Theorem 3.15: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is vague $\hat{g}\not{f}$ continuous then $f(V\hat{g}\not{f}cl(A)) \subset V\hat{g}\not{f}cl(f(A))$ for every vague subset A of X.

Proof: Let $A \subseteq X$. Then $V\hat{g}\not{f}cl(f(A))$ is a vague closed in Y, since f is vague $\hat{g}\not{f}$ continuous, $f^{-1}(V\hat{g}\not{f}cl(f(A)))$ is vague $\hat{g}\not{f}$ closed in X. And $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(V\hat{g}\not{f}cl\big(f(A)\big)$, Therefore $V\hat{g}\not{f}cl(A) \subseteq V\hat{g}\not{f}cl(f^{-1}\left(V\hat{g}\not{f}cl\big(f(A)\big)\right) = f^{-1}(V\hat{g}\not{f}cl(f(A)))$. Hence $f\big(V\hat{g}\not{f}cl(A)\big) \subseteq V\hat{g}\not{f}cl(f(A))$ for every vague subset A of X.

Theorem 3.16: Let (X, τ) and (Y, σ) be any two VTS. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a vague $\hat{g}\not{f}$ continuous mapping. Then for every vague set A in Y, $V\hat{g}\not{f}cl\big(f^{-1}(A)\big) \subseteq f^{-1}\big(V\hat{g}\not{f}cl(A)\big)$.

Proof: Let A be a vague set in (Y, σ). Let $B = f^{-1}(A)$. Then $f(B) = f(f^{-1}(A)) \subseteq A$. Then by the theorem 3.15 $f(V\hat{g}\not{f} - cl\big(f^{-1}(A)\big)) \subseteq V\hat{g}\not{f}cl(f\big(f^{-1}(A)\big))$. Thus $V\hat{g}\not{f}cl\big(f^{-1}(A)\big) \subseteq f^{-1}\big(V\hat{g}\not{f}cl(A)\big)$.

Theorem 3.17: The composition of two $V\hat{g}\not{f}$ −continuous mapping may not be $V\hat{g}\not{f}$ −continuous.

Example 3.18: $X = \{a, b\}, Y = \{u, v\}, Z = \{c, d\}$ and $\tau = \{0, G_1, 1\}, \sigma = \{0, G_2, 1\}, \lambda = \{0, G_3, 1\}$are VT_s on X , Y and Z respectively. $G_1 = \{< x, [0.5,0.5], [0.4,0.6] >\}, G_2 = \{< y, [0.5,0.5], [0.3,0.7] >\}$ and $G_3 = \{<$ y,0.4,0.6,0.3,0.5>. Define a mapping $f{:}X,\ \tau \rightarrow Y,\ \sigma\ by\ fa{=}u\ and\ fb{=}v$ and $g{:}Y{,}\sigma{\rightarrow}Z{,}\lambda$. Then the mappings f and g are vague $\hat{g}$ feebly continuous mapping but the mapping $gof: (X, \tau) \rightarrow (Z, \lambda)$ is not vague $\hat{g}$ feebly continuous.

Theorem 3.19: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is vague $\hat{g}$ feebly continuous and $g: (Y, \sigma) \rightarrow (Z, \lambda)$ is vague continuous. Then $gof: (X, \tau) \rightarrow (Z, \lambda)$ is vague $\hat{g}$ feebly continuous.

Proof: Let A is a vague closed set in (Z, λ), then $g^{-1}(A)$ is vague closed in (Y, σ), since g is vague continuous. Therefore $(gof)^{-1}(A) = f^{-1}(g^{-1}(A)\,)$ is vague $\hat{g}$ feebly closed in (X, τ). Hence $gof: (X, \tau) \rightarrow (Z, \lambda)$ is vague $\hat{g}$ feebly continous.

4. CONTRA - VAGUE $\hat{g}$ FEEBLY CONTINUOUS MAPPINGS

Definition 4.1: Let (X, τ) and (Y, σ) be two VTSs and let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function. Then f is said to be **contra – vague feebly continuous** if $f^{-1}(V)$ is vague feebly closed set of (X, τ), for every vague open set V in (Y, σ).

Definition 4.2: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be a **contra – vague $\hat{g}$ continuous**, if $f^{-1}(V)$ is vague $\hat{g}$ closed set of (X, τ), for every vague open set V in (Y, σ).

Definition 4.3: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be a **contra – vague $\hat{g}$ feebly continuous**, if $f^{-1}(V)$ is vague $\hat{g}$ feebly closed set of (X, τ), for every vague open set V in (Y, σ).

Definition 4.4: A function $f: (X, \tau) \to (Y, \sigma)$ is said to be a **contra – vague feebly generalised continuous**, if $f^{-1}(V)$ is vague $\not{f}$g closed set of (X, τ), for every vague open set V in (Y, σ).

Definition 4.5: A function $f: (X, \tau) \to (Y, \sigma)$ is said to be a **contra – vague generalised feebly continuous**, if $f^{-1}(V)$ is vague g$\not{f}$ closed set of (X, τ), for every vague open set V in (Y, σ).

Definition 4.6: A vague subset $'A'$ of a VTS (X, τ) is called **vague – clopen** if it is both vague open and vague closed.

Proposition 4.7:

1. Every contra vague continuous map is contra vague $\hat{g}$ feebly continuous.
2. Every contra vague g - continuous map is contra vague $\hat{g}$ feebly continuous.
3. Every contra vague $\hat{g}$ −continuous map is contra vague $\hat{g}$ feebly continuous.
4. Every contra vague $\hat{g}\,\not{f}$ −continuous map is contra vague generalised feebly continuous.
5. Every contra vague feebly continuous map is contra vague $\hat{g}$ feebly continuous.

Proof: Let $f: (X, \tau) \to (Y, \sigma)$ be a contra vague continuous map. Let V be a vague open set in (Y, σ). Since f is contra vague continuous map, $f^{-1}(V) \in VC(X)$ for each $VOS\ V \in Y$.

Similarly we can prove the other propositions. The converses are not true as we can see from the following examples.

Example 4.8: $\mathrm{X} = \{\mathrm{a}, \mathrm{b}\}, \mathrm{Y} = \{\mathrm{u}, \mathrm{v}\}$ and $\tau = \{0, \mathrm{G}_1, 1\}$and $\sigma = \{0, \mathrm{G}_2, 1\}$ are $\mathrm{VT_s}$ on X and Y respectively. $\mathrm{G}_1 = \{< x, [0.5,0.9], [0.2,0.5] >\}, \mathrm{G}_2 = \{< y, [0.1,0.5], [0.4,0.6] >\}$. Define a mapping $f: (X, \tau) \to (Y, \sigma)\ by\ f(a) = u\ and\ f(b) = v$. Since the inverse image of a vague open set $\mathrm{A} = \{< y, [0.1,0.5], [0.4,0.6] >\}$ in (Y, σ) is a V $\hat{g}\not{f}$ CS in (X, τ), but A is not vague closed in (X, τ) Hence f is a contra vague $\hat{g}$ feebly continuous mapping but not contra vague continuous.

Example 4.9: $\mathrm{X} = \{\mathrm{a}, \mathrm{b}\}, \mathrm{Y} = \{\mathrm{u}, \mathrm{v}\}$ and $\tau = \{0, \mathrm{G}_1, 1\}$and $\sigma = \{0, \mathrm{G}_2, 1\}$ are $\mathrm{VT_s}$ on X and Y respectively. $\mathrm{G}_1 = \{< x, [0.4,0.7], [0.2,0.4] >\}, \mathrm{G}_2 = \{< y, [0.2,0.4], [0.3,0.6] >\}$. Define a mapping $f: (X, \tau) \to (Y, \sigma)\ by\ f(a) = u\ and\ f(b) = v$. Since the inverse image of a vague open set $\mathrm{A} = \{< y, [0.2,0.4], [0.3,0.6] >\}$ in (Y, σ) is a V$\hat{g}\not{f}$CS in (X, τ), but A is not vague g closed in (X, τ). Hence f is a contra vague $\hat{g}$ feebly continuous mapping but not contra vague g continuous.

Example 4.10: $\mathrm{X} = \{\mathrm{a}, \mathrm{b}\}, \mathrm{Y} = \{\mathrm{u}, \mathrm{v}\}$ and $\tau = \{0, \mathrm{G}_1, \mathrm{G}_2, \mathrm{G}_3, \mathrm{G}_4, 1\}$and $\sigma = \{0, \mathrm{G}_5, 1\}$ are $\mathrm{VT_s}$ on X and Y respectively. $\mathrm{G}_1 = \{< x, [0.2,0.5], [0.4,0.5] >\}, \mathrm{G}_2 = \{< x, [0.5,0.6], [0.3,0.4] >\}, \mathrm{G}_3 = \{< x, [0.5,0.9], [0.5,0.6] >\}, \mathrm{G}_4 = \{< x, [0.2,0.5], [0.3,0.4] >\}$and $\mathrm{G}_5 = \{< y, [0.5,0.6], [0.4,0.5] >\}$ Define a mapping $f: (X, \tau) \to (Y, \sigma)\ by\ f(a) = u\ and\ f(b) = v$. Since the inverse image of a vague open set $\mathrm{A} = \{< y, [0.5,0.6], [0.4,0.5] >\}$ in (Y, σ) is a V $\hat{g}\not{f}$CS in (X, τ), but A is not vague $\hat{g}$ closed in (X, τ), when $B = \{< x, [0.5,0.8], [0.5,0.6] >\}$ is a vague semi closed set containing A. Hence f is a contra vague $\hat{g}\not{f}$ continuous mapping but not contra vague $\hat{g}$ continuous.

Example 4.11: $\mathrm{X} = \{\mathrm{a}, \mathrm{b}\}, \mathrm{Y} = \{\mathrm{u}, \mathrm{v}\}$ and $\tau = \{0, \mathrm{G}_1, 1\}$and $\sigma = \{0, \mathrm{G}_2, 1\}$ are $\mathrm{VT_s}$ on X and Y respectively. $\mathrm{G}_1 = \{< x, [0.4,0.7], [0.2,0.4] >\}, \mathrm{G}_2 = \{< y, [0.2,0.4], [0.3,0.6] >\}$. Define a mapping $f: (X, \tau) \to (Y, \sigma)\ by\ f(a) = u\ and\ f(b) = v$. Since the inverse image of a vague open set $\mathrm{A} = \{< y, [0.2,0.4], [0.3,0.6] >\}$ in (Y, σ) is a Vg$\not{f}$CS in (X, τ), but A is not vague $\hat{g}\not{f}$ closed in (X, τ), when $B = \{< x, [0.4,0.7], [0.3,0.6] >\}$ is a vague semi open set in X. Hence f is a contra vague g feebly continuous mapping but not contra vague $\hat{g}$ feebly continuous.

Example 4.12: $\mathrm{X} = \{\mathrm{a}, \mathrm{b}\}, \mathrm{Y} = \{\mathrm{u}, \mathrm{v}\}$ and $\tau = \{0, \mathrm{G}_1, 1\}$and $\sigma = \{0, \mathrm{G}_2, 1\}$ are $\mathrm{VT_s}$ on X and Y respectively. $\mathrm{G}_1 = \{< x, [0.5,0.9], [0.2,0.5] >\}, \mathrm{G}_2 = \{< y, [0.1,0.5], [0.4,0.6] >\}$. Define a mapping $f: (X, \tau) \to (Y, \sigma)\ by\ f(a) = u\ and\ f(b) = v$. Since the inverse image of a vague open set $\mathrm{A} = \{< y, [0.1,0.5], [0.4,0.6] >\}$ in (Y, σ) is a V $\hat{g}\not{f}$ CS in (X, τ), but A is not vague feebly closed in (X, τ) Hence f is a contra vague $\hat{g}$ feebly continuous mapping but not contra vague feebly continuous.

Remark 4.13: The composition of two Contra $V\hat{g}\not{f}$ −continuous mapping may not be Contra $V\hat{g}\not{f}$ −continuous.

Theorem 4.14: Let $f: (X, \tau) \to (Y, \sigma)$ be a mapping.Then the following statements are equivalent.

a) f is a contra vague $\hat{g}$ feebly continuous mapping,
b) $f^{-1}(V)$ is a V$\hat{g}\not{f}CS(X)$ for every VOS $'V'$ in Y.

Proof: (i) $\Rightarrow (ii)$ Let $'V'$ be a VCS in Y. Then $'V^{C}{}'$ is a VOS in Y. By hypothesis, $f^{-1}(V^C)$ is a V$\hat{g}\not{f}CS$ in X. (i.e)., $f^{-1}(V)$ is a V$\hat{g}\not{f}OS$ in X.
(ii) ⇒(i) Let $'V'$ be a VOS in Y. Then $'V^{C}{}'$ is a VCS in Y. By hypothesis, $f^{-1}(V^C)$ is a V$\hat{g}\not{f}OS$ in X. (i.e)., $f^{-1}(V)$ is a V$\hat{g}\not{f}CS$ in X. Thus f is a contra vague $\hat{g}$ feebly continuous mapping.

Theorem 4.15: If $f: (X,\tau) \to (Y,\sigma)$ is contra vague ĝ feebly continuous and $g: (Y,\sigma) \to (Z,\lambda)$ is vague continuous. Then $gof: (X,\tau) \to (Z,\lambda)$ is a contra vague ĝ feebly continuous.

Proof: Let A is a vague open set in (Z,λ), then $g^{-1}(A)$ is vague open in (Y,σ), since g is vague continuous. Therefore $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is vague ĝ feebly closed in (X,τ). Hence $gof: (X,\tau) \to (Z,\lambda)$ is contra vague ĝ feebly continuous.

Theorem 4.16: If $f: (X,\tau) \to (Y,\sigma)$ is contra vague ĝ feebly continuous and $g: (Y,\sigma) \to (Z,\lambda)$ is contra vague continuous. Then $gof: (X,\tau) \to (Z,\lambda)$ is a vague ĝ feebly continuous.

Proof: Let A is a vague open set in (Z,λ), then $g^{-1}(A)$ is vague closed in (Y,σ), since g is contra vague continuous. Therefore $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is vague ĝ feebly open in (X,τ). Hence $gof: (X,\tau) \to (Z,\lambda)$ is vague ĝ feebly continuous.

Theorem 4.17: A vague continuous mapping $f: (X,\tau) \to (Y,\sigma)$ is a contra vague ĝ feebly continuous if Vĝ$\not{f}O(X) =$ Vĝ$\not{f}C(X)$

Proof: Let $A \subseteq Y$ be a vague open set in (Y,σ), then by hypothesis $f^{-1}(A)$ is vague open in (X,τ) and hence $f^{-1}(A)$ is a Vĝ$\not{f}OS$ in X. since Vĝ$\not{f}O(X) =$ Vĝ$\not{f}C(X)$, $f^{-1}(A)$ is a Vĝ$\not{f}CS$ in (X,τ). Therefore $f: (X,\tau) \to (Y,\sigma)$ is contra vague ĝ feebly continuous mapping.

5. VAGUE ĝ FEEBLY COMPACTNESS & VAGUE ĝ FEEBLY CONNECTEDNESS

Definition 5.1: A collection $\{U_\alpha\}_{\alpha\in\Delta}$ of vague ĝ feebly open sets in VTS (X,τ) is said to a **vague ĝ feebly open cover** of a vague subset 'A' of X if $A \subseteq \cup\{U_\alpha\}_{\alpha\in\Delta}$.

Definition 5.2: A VTS (X,τ) is said to be a **vague ĝ feebly compact** if every vague ĝ feebly open cover of X has a finite vague sub cover.

Definition 5.3: A vague set B of VTS (X,τ) is said to be a vague ĝ compact relative to X, if for every collection $\{U_\alpha\}_{\alpha\in\Delta}$ of vague ĝ open subset of X such that $B \subseteq \cup\{U_\alpha\}_{\alpha\in\Delta}$ there exist a finite subset Δ_0 of Δ such that $B \subseteq\cup \{U_\alpha\}_{\alpha\in\Delta_0}$.

Definition 5.4: If B is vague ĝ feebly compact as a subspace of X then a subset of a VTS X is said to be vague ĝ feebly compact.

Theorem 5.5: Every Vĝ feebly closed subset A of a Vĝ feebly compact space is Vĝ feebly compact relative to X.
Proof is similar to the case of V ĝ compactness so omitted.

Theorem 5.6: The V ĝ$\not{f}$- continuous image of a vague ĝ feebly compact is vague ĝ feebly compact.

Proof: Let $f: (X,\tau) \to (Y,\sigma)$ be a V ĝ$\not{f}$- continuous map from a vague ĝ feebly compact space (X,τ) onto a VTS. Let $\{U_\alpha\}_{\alpha\in\Delta}$ be an vague open cover of Y then $f^{-1}(\{U_\alpha\}_{\alpha\in\Delta})$ is a V ĝ$\not{f}$- open cover in X. Since (X,τ) is a vague ĝ feebly compact this V ĝ$\not{f}$ -open cover has a finite sub cover $f^{-1}(\{U_i\}_{i=1,2\ldots n})$. Since f is onto $(\{U_i\}_{i=1,2\ldots n})$ is a finite vague sub cover of Y, so Y is vague ĝ feebly compact.

Definition 5.7: A vague topological space X is said to be a **vague ĝ feebly connected** if X cannot be written as a disjoint union of two non empty vague ĝ feebly open sets.

Definition 5.8: If B is vague ĝ feebly connected as a subspace of X then a subset of a VTS X is said to be vague ĝ feebly connected.

Theorem 5.9: For a VTS (X,τ) , the following are equivalent:

i. (X,τ) is vague ĝ feebly connected.
ii. The only vague subset of (X,τ) which are both V ĝ$\not{f}$ $-$open and V ĝ$\not{f}$ $-$closed are 0_v and 1_v.

Proof: $(i) \Rightarrow (ii)$ Let U_v be a V ĝ$\not{f}$ $-$open and V ĝ$\not{f}$ $-$closed subset of (X,τ) then ${U_v}^c$ is both V ĝ$\not{f}$ $-$open and V ĝ$\not{f}$ $-$closed. Since X is disjoint union of V ĝ$\not{f}$ $-$open sets U_v and ${U_v}^c$, one of these must be empty (i.e)., $U_v = 0_v$ or $U_v = 1_v$.
$(ii) \Rightarrow (i)$ Let $X = U_v \cup V_v$, where U_v and V_v are disjoint non empty V ĝ$\not{f}$ $-$open subsets of X then U_v is both V ĝ$\not{f}$ $-$open and V ĝ$\not{f}$ $-$closed. By assumption $U_v = 0_v$ or $U_v = 1_v$. Hence (X,τ) is vague ĝ feebly connected.

Theorem 5.10: Let $f:(X,\tau)\rightarrow(Y,\sigma)$ is a V $\hat{g}\mathcal{f}$ −continuous, surjection and (X,τ) is vague ĝ feebly connected then (Y,σ) is also vague ĝ feebly connected.

Proof: Suppose that (Y,σ) is not vague ĝ feebly connected, then $Y = U_v \cup V_v$, where U_v and V_v are disjoint non empty sets in Y. Since f is V $\hat{g}\mathcal{f}$ −continuous and surjection, $X = f^{-1}(U_v) \cup f^{-1}(V_v)$, where $f^{-1}(U_v)$ and $f^{-1}(V_v)$ are disjoint non empty and V $\hat{g}\mathcal{f}$ −open in X. This contradicts the fact that X is vague ĝ feebly connected. Hence Y is vague ĝ feebly connected.

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