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SOME COSMOLOGICAL ASPECTS OF MODIFIED NEWTONIAN DYNAMICS

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ABSTRACT

In the present work, we have derived a formula for gravitational force acting between highly relativistic massive particles, such as high-energy cosmic rays at long distances from each other in MOND region, moving at a speed approximately to a light c.In the low-acceleration limit we also deduced the MOND analogue of Newtonian equation of motion with MOND mass and thereafter calculated the MOND acceleration by taking into consideration the expressions derived by Sandars (1998) for critical length scale for three phases of the known universe that is for the radiation dominated , matter-dominated and the vacuum-energy-dominated Universe.

Key Words: *MOND, Newtonian Regime, MOND Regime, Critical radius, Radiation dominated Universe, Matter-dominated, Vacuum-energy-dominated Universe.*

I. INTRODUCTION

In the limit of small accelerations the MOND rules posits a deviation from standard Newtonian dynamics, and from General Relativity. There is a new theory of dynamics, a modified set of dynamical laws(MOND) or called Modified Newtonian Dynamics [4,5,6] was put forward by Mordehi Milgrom in the early 1980s. To deduce the gravitational masses MOND does not require the presence of large quantities of 'dark' components (**such as 'dark matter' and 'dark energy')** and it is used to describe the motions in galactic systems and, in particular, to obtain the masses of such systems. Though the original version of MOND is not relativistic, it is one of few extensions of gravitational theories with many observational predictions (The Modified Newtonian Dynamics as an Alternative to Hidden Matter Published online by Cambridge University Press: 04 August 2017, M. MILGROM AND J. BEKENSTEIN) [7].

In the basis of the modification the major predictions of MOND follow from the following assumptions [1, 2] (i) In the limit of small acceleration Newtonian dynamics breakdown. (ii) The acceleration of a test particle at a distance r from a mass M satisfies approximately

$$\frac{\boldsymbol{a}^2}{\boldsymbol{a}_0} \approx \frac{\boldsymbol{MG}}{\boldsymbol{r}^2} \tag{1.1}$$

in the limit $\frac{\boldsymbol{MG}}{\boldsymbol{r}^2} \ll \boldsymbol{a_0}$ or $\boldsymbol{a} \ll \boldsymbol{a_0}$, on the contrary of the standard expression $a = \frac{MG}{r^2}$, which holds for the region when $\boldsymbol{a} \gg \boldsymbol{a_0}$.A little more generally, if a_N is the Newtonian gravitational acceleration calculated from the mass distribution in the usual way then for $a_N \ll a_0$

$$a = \sqrt{a_0 a_N} = \frac{\sqrt{a_0\, MG}}{r} \tag{1.2}$$

The 1/r behaviour then provides a good fit to the rotation curves of galaxies at large r. Hence the proposed Universal law of gravitation in MOND region is

$$F = \frac{m\sqrt{GM_m\, a_0}}{r} \tag{1.3}$$

where M_m denotes MOND mass and r is the MOND radius and m is for the mass of the test particle.

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Here $\boldsymbol{a_0}$ is a fundamental acceleration parameter, central to MOND dynamics which appears in the role of a transition acceleration from the Newtonian for accelerations much larger thana_0 and the scale-invariant, deepMOND dynamics in the limit of small acceleration and the role of a proportionality constant in the modified equation of motion. Milgrom in his paper (1983, b) [5] determined the value of $\boldsymbol{a_0}$in few independent ways to be about $\boldsymbol{a_0 \approx 2 \times 10^{-8} cm/s^2}$. Interestingly, this value turns out to be of the same order as the natural cosmological acceleration parameter $\boldsymbol{CH_0 = 5 \times 10^{-8}(H_0/50kms^{-1}Mpc^{-1})cms^{-2}}$. In view of the fact that$a_0 \approx CH_0$, the universe is the only massive system which is both relativistic and involves accelerations not much larger than a_0 (Milgrom 1983, a) [4]. It is suggestive of a cosmological connection, but the structure of that cosmology is not at all evident (Sanders2005) [8].

The near equality of the MOND acceleration constant, a_0 as deduced from local, galactic phenomena – and cosmological parameters. Namely, $a_0 \sim cH_0 \sim c^2\Lambda^{1/2} \sim c^2/l_v$, where H_0 is the present value of the Hubble-Lemaˆıtre constant, $\Lambda$is the 'cosmological constant', and l_v is a cosmological characteristic length that is the de Sitter radius associated with Λ or the Hubble distance (Milgrom 2020)[10]. If think about a_0 its roles are same as $\hbar$ in quantum physics, or like c in relativity. All three play the roles of transition constants .It also shows ever where in MOND laws in the deep-MOND regime, where scale invariance directs that the constants of the theory G and a_0 appears only in the product$A_0 \equiv Ga_0$ that remain fixed and in a universe governed strictly by a DML (deep-MOND limit) theory neither a_0 nor G only A_0 appears (Milgrom, 2014, 2020) [9,10].

First study to gain some insight into pure MOND cosmology has been started by Felten (1984) [2]. He pointed out that uniform expansion of a spherical region is not possible with MOND. Friedman cosmology, based on uniform expansion, applies on the scale of the horizon, while MOND, in which the expansion is highly non-uniform, applies on sub horizon scales. At any era, while the Universe is homogeneous over all, density in homogeneities should be present on the scale of critical radius we say the MOND radius r_M at that time. At late cosmic time the Universe on large scales has only become 'MONDIAN'.

In the discussion of Cosmology and Relativity we first had the theory of relativity and then we use relativity to describe Cosmology, so cosmology is treated as one system to be described within relativity. This is not the correct path. Now here in MOND because of the connection of a_0 with cosmology, in the long turn we will understand MOND as a part and parcel of cosmology.

This would suggest that an isotropic and homogeneous universe, as described by the Robertson–Walker (RW) metric, is impossible in the case of Modified Newtonian Dynamics. Modified dynamics can only be valid inside a critical radius r_M. Some preliminary conclusions about a MOND universe can be draw by considering a finite expanding spherical region. It might be expected that some insight into a pure MOND cosmology might be gained by such a procedure using Milgrom's formula instead of Newton's.

Our work is assembled as follows: In section **II** we deduced MONDian Gravitational force acting on a particle moving at a speed closely to a light c. In section **III** derived and discuss the gravitational acceleration of an object with MOND mass and also deduced the MOND acceleration in the three phases of the known universe. We summarize our conclusions in section **IV**.

II. MONDian Gravitational force acting on a particle moving at a speed approximately to a light c:

In the phenomena involving highly relativistic massive particles, such as high-energy cosmic rays at long distances from each other in MOND region, moving at a speed approximately to a light c. (D Mahto, Md S Nadeem, M Ram,3 and K Vineeta2013)[1].

The mass-energy equivalence relation can be applied with the mass of light particle and we have

$$E = mc^2 \tag{2.1}$$

Or

$$m = \frac{E}{C^2} \tag{2.2}$$

Where involving parameters have their usual meaning.

According to quantum theory of radiation,

We have

$$E = h\nu \tag{2.3}$$

Where h is Planck constant, ν is the frequency of radiation having value equal to c/λ, and λ is the wavelength of radiation, that is, electromagnetic wave, specially visible wave.

Inserting value of E from equation (2.3) in (2.2),

We obtain $$m = \frac{h\nu}{c^2} \tag{2.4}$$
Or $$m = \frac{h}{c\lambda} \tag{2.5}$$

In view of equation (1.3) as discussed above the gravitational force of attraction between the MOND mass M_m and m the mass of test particle moving with a speed close to the speed of light at a distance r which is the MOND radius, is presented by
$$F = \frac{m\sqrt{GM_m a_0}}{r} \tag{2.6}$$

Inserting the value of the mass $m$of test particle from equation (2.5) equation (2.6) implies
$$F = \frac{\sqrt{GM_m a_0}}{r}\frac{h}{c\lambda} \tag{2.7}$$
Consequently by using the MOND radius $r_M$of mass M_m, $r_M = \left(\frac{M_m G}{a_0}\right)^{1/2}$ and MOND mass $M_m = \frac{C^4}{Ga_0}$ in equation (2.7), our resultant force becomes simply as
$$F = \frac{a_0 h}{\lambda C} \tag{2.8}$$

Equation (2.8) represents MONDian Gravitational force acting on a particle moving at a speed close to a speed of light c which includes three fundamental constants of nature—the speed of light c, Planck's constant h, and MOND acceleration constant a_0. The Planck's constant (h) governs the law of quantum world. The speed of light c is the cornerstone of the special theory of relativity and in the framework of MOND a_0 appears as the borderline acceleration between the MOND and the Newtonian regime. We can also introduce the cosmological constant $\wedge$ in the equation (2.8) of force by using the relation (James E Felten 1986) [3],
$$\wedge = c^2\lambda \tag{2.9}$$
Substituting the above value of cosmological constant $\wedge$ equation of force (5.8) turns into
$$F = \frac{a_0 hc}{\wedge} \tag{2.10}$$

Which includes one more constant in addition to three fundamental constants of nature.

III. The Cosmological Principle with MOND:

In case of standard cosmology, in the matter-dominated regime the dynamics of the universe can be inferred from Newtonian gravity. Sanders (1998) [11] used this path to realize the dynamics of the universe in the MONDian. So let us consider an isolated uniform spherical region of radius r, from the background in which $a \gg a_0$ then the Newtonian equation of motion is given by,
$$\ddot{r} = -\frac{GM}{r^2} \tag{3.1}$$

Here M the active relativistic mass, in the weak field static limit of the Einstein field equations, is presented by
$$M = \frac{4\pi G r^3}{3}(\rho + 3p) \tag{3.2}$$
Where ρ and p are the density and pressure of the cosmic fluid. Merging of equations (3.1) and (3.2) allows us to write down the Newtonian equation of motion as
$$\ddot{r} = -\frac{4\pi G r}{3}(\rho + 3p) \tag{3.3}$$

Now in MOND region equation (1.1) that is MOND acceleration with MOND mass $M_m = \frac{C^4}{\mathcal{A}_0}$ (Milgrom 2014, 2020) [9, 10], immediately leads to
$$\ddot{r} = \frac{C^2}{r} \tag{3.4}$$

Comparison of equations (3.3) and (3.4) gives,
$$(\rho + 3p) = \frac{3C^2}{4\pi G r^2} \tag{3.5}$$

On insertion of this value in equation (3.2) obtain the active gravitational mass
$$M = \frac{rM_m}{C^2}a_0 \tag{3.6}$$
in terms of Mondian Mass M_m.

In the high-acceleration limit the gravitational force is the usual Newtonian force, but in the low-acceleration limit $g = \sqrt{g_n}a_0$. As we are interested here only in a broad view of the overall dynamics of a MOND Universe, in the low-acceleration limit the MOND equivalence of equation (3.1) becomes,

$$\ddot{r} = \left[\frac{\mathcal{A}_0 M_m a_0}{c^2}\right][r]^{-1/2} \tag{3.7}$$

From equation (3.7), we can conclude that in the limit of small acceleration $a \ll a_0$ the acceleration of a particle at distance r from an active gravitational mass M in terms of mondian mass satisfies approximately$\ddot{r} = \left[\frac{\mathcal{A}_0 M_m a_0}{c^2}\right][r]^{-1/2}$. It is cleared that it does not follow Newtonian dynamics. This implies that some critical radius should be there. This critical radius is the MOND radius of a mass M_m is given by,$r_M \equiv \left(\frac{M_m G}{a_0}\right)^{1/2}$plays a parallel role to that of Schwarzschild radius in relativity. This may say MOND radiusr_M, beyond which the acceleration exceeds the MOND acceleration a_0 and the dynamics will be Newtonian. In any case, at any epoch in an evolving Universe it is likely that below the approximate scale$r_M$there exist MOND-induced inhomogeneities (R H Sandars 1998) [11].

If we substitute this value as the value of radius in equation (3.7) it is observed that the acceleration will be constant or we can say at a large radius around an active gravitational mass M in terms of Mondian mass M_m, the orbital speed on a circular orbit becomes independent of radius.

Cosmology is taking care of, or describing the universe at large as one system and there you may know that the universe is expanding, that is a fact and the way it expands depends on its material content. Whether it is a radiation field, or massive particles or non-relativistic particles. According to that the Radiation Dominated, Matter-dominated and the Vacuum-energy-dominated Universe are the three phases of the known universe. Now by taking into consideration the expressions derived by Sandars (1998) [11] for critical length scale for these three phases we will see what will happen with the MOND equivalence relation of the Newtonian equation of motion derived by us (3.7) after the use of these three different expressions. Expressions are

$$r_c = \frac{2a_0}{\Omega_0 \mathrm{H}_0^2} x^3, \tag{3.8}$$

$$r_c = \frac{a_0}{\Omega_r \mathrm{H}_0^2} x^4 \tag{3.9}$$

and

$$r_c = \frac{a_0}{\lambda \mathrm{H}_0^2} \tag{3.10}$$

When the Universe is Matter-dominated, Radiation-dominated and the Vacuum-Energy-dominated respectively. Here x is a time-dependent scalefactor and λ is the dimensionless cosmological constant.

In Matter dominated universe equation (3.7) becomes,

$$\ddot{r} = \left[\frac{\mathcal{A}_0 M_m a_0}{C^2}\right]^{1/2} \left[\frac{2a_0}{\Omega_0 \mathrm{H}_0^2} x^3\right]^{-1/2} \tag{3.11}$$

In radiation-dominated universe it will be,

$$\ddot{r} = \left[\frac{\mathcal{A}_0 M_m a_0}{C^2}\right]^{1/2} \left[\frac{a_0}{\Omega_r \mathrm{H}_0^2} x^4\right]^{-1/2} \tag{3.12}$$

and in vacuum-energy-dominated case it is

$$\ddot{r} = \left[\frac{\mathcal{A}_0 M_m a_0}{C^2}\right]^{1/2} \left[\frac{a_0}{\lambda \mathrm{H}_0^2}\right]^{-1/2} \tag{3.13}$$

From derived equations (3.11), (3.12) and (3.13) we can conclude that MOND acceleration in the Radiation dominated and Matter-dominated universe vary with the time-dependent scale factor x and in the vacuum-energy-dominated Universe it appears constant.

CONCLUSION

In this paper firstly we have derived two expressions for highly relativistic massive particles, such as high-energy cosmic rays at long distances from each other in MOND region, moving at a speed approximately to a light c, first includes three fundamental constants of nature and second involves one more constant which is cosmological constant. In the Cosmological Principle with MOND derived an equation of motion which does not follow Newtonian dynamics and at a large radius around an active gravitational mass M in terms of Mondianmass M_m, the orbital speed on a circular orbit becomes independent of radius. Also deduced the MOND acceleration in the three phases of the known universe and it is noticed that MOND acceleration in the Radiation dominated and Matter-dominated universe vary with the time-dependent scale factor x and in the vacuum-energy-dominated Universe it appears constant.

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