International Journal of Mathematical Archive-14(1), 2023, 10-14
Available online through www.ijma.info ISSN 2229 – 5046

More on p*gb-closed Sets in Topological Spaces

ARUNA GLORY SUDHA. I*
Department of Mathematics,
Research Scholar, Registration Number: 21212152092002
Nazareth Margoschis College at Pillaiyanmanai, Nazareth
(Affiliated to ManonmaniamSundaranar University, Abishekapatti, Tirunelveli-627012)

DR. S. ZION CHELLA RUTH
Assistant Professor, Department of Mathematics,
Nazareth Margoschis College at Pillaiyanmanai, Nazareth
(Affiliated to ManonmaniamSundaranar University, Abishekapatti, Tirunelveli-627012)

(Received On: 18-12-22; Revised & Accepted On: 10-01-23)

ABSTRACT

Using the concept of pre-generalized b-closed and pre*-generalized b-open sets, we introduce and study the toplogical properties of pre*-generalized b-neighbourhood and pre*-generalized b-interior, pre*-generalized b-closure operators.*

Mathematics Subject Classification 2010: *54A05.*

Keywords: *p*gb-open, p*gb-closed,p*gb-nbhd, p*gb-interior, p*gb-closure.*

1. INTRODUCTION

In 2012, T. Selvi and A. PunithaDharani [3] introduced pre*-closed sets and investigated some of their properties. The characterizations of pre*-generalized b-closed sets and pre*-generalized b-open sets are given in [4]. In this paper, we introduce the notions of p*gb-neighbourhood of a subset of topological space, p*gb-interior and p*gb-closure of a set in a topological space and study their properties..

2. PRELIMINARIES

Throughout this paper (X, τ) represent a topological space on which no separation axiom is assumed unless otherwise mentioned. For a subset A of a topological space X, cl(A) and int(A) denote the closure of A and the interior of A respectively. (X, τ) will be replaced by X if there is no changes of confusion. We recall the following definitions and results.

Definition 2.1: [1] Let (X, τ) be a topological space. A subset A of the space X is said to be b-open [2] if $A \subseteq int(cl(A)) \cup cl(int(A))$ and b-closed if $int(cl(A)) \cap cl(int(A)) \subseteq A$.

Definition 2.2: [1] Let (X, τ) be a topological space and $A \subseteq X$. The b-closure of A, denoted by bcl(A) and is defined by the intersection of all b-closed sets containing A.

Definition 2.3: [2] Let (X, τ) be a topological space. A subset A of X is said to be generalized closed (briefly g-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ). The complements of the above mentioned closed sets are their respective open sets.

Definition 2.4: Let A be a subset of a topological space (X, τ). Then the union of all g-open sets contained in A is called the g-interior of A and it is denoted by int*(A). That is, $int^*(A) = \cup\{V : V \subseteq A \text{ and } V \in g\text{-}O(X)\}$.

Corresponding Author: Aruna Glory Sudha. I*
Department of Mathematics, Research Scholar, Registration Number: 21212152092002
Nazareth Margoschis College at Pillaiyanmanai, Nazareth
(Affiliated to ManonmaniamSundaranar University, Abishekapatti, Tirunelveli-627012)

Definition 2.5: Let A be a subset of a topological space (X, τ). Then the intersection of all g-closed sets in X containing A is called the g-closure of A and it is denoted by cl*(A). That is, cl*(A)=∩{F: A⊆F and F∈g-C(X)}.

Definition 2.6: [3] Let (X, τ) be a topological space. A subset A of the space X is said to be pre*-open if A ⊆ int*(cl(A)) and pre*-closed if cl*(int(A)) ⊆ A.

Definition 2.7: [4] A subset A of a topological space (X, τ) is called a pre* generalized b-closed set (briefly, p*gb-closed) if bcl(A) ⊆ U whenever A ⊆ U and U is pre*-open in (X, τ).

Lemma 2.8: [4] For a topological space (X, τ), Every open set is p*gb-open.

Lemma 2.9: [4]
- (a) Arbitrary intersection of p*gb-closed sets is p*gb-closed.
- (b) Arbitrary union of p*gb-open sets is p*gb-open.

Remark 2.10.[4]
- (a) The union of union of p*gb-closed sets need not be a p*gb-closed set.
- (b) The intersection of p*gb-open sets is p*gb-open.

3. p*gb-neighbourhood

Definition 3.1: Let X be a topological space and let x∈X. A subset N of X is said to be a p*gb-neighbourhood (shortly, p*gb-nbhd) of x if there exists a p*gb-open set U such that x∈U⊆N.

Definition 3.2: A subset N of a space X, is called a p*gb-nbhd of A⊆X if there exists an p*gb-open set U such that A⊆U⊆N.

Theorem 3.3: Every nbhd N of x∈X is a p*gb-nbhd of x.

Proof: Let N be anbhd of point x∈ X. Then there exists an open set U such that x∈U⊆N. Since every open set is p*gb-open, U is a p*gb-open set such that x∈U⊆N. This implies, N is a p*gb-nbhd of x.

Remark 3.4: The converse of the above theorem need not be true which is shown in the following example.

Example 3.5: Let X ={a, b, c} and τ = {ϕ, {a}, {a,b}, X}. In this space X, the p*gb-open sets are ϕ, {a}, {a,b}, {a,c}, X. The set {a, c} is the p*gb-nbhd of c, since {a,c} is p*gb-open set such that c∈{a,c}⊆{a,c}. But {a,c} is not a nbhd of the point c.

Remark 3.6: Every p*gb-open set is a p*gb-nbhd of each of its points.

Theorem 3.7: If F is a p*gb-closed subset of X and x∈X\F, then there exists a p*gb-nbhd N of x such that N∩F=ϕ.

Proof: Let F be p*gb-closed subset of X and x∈ X\F. Then X\F is p*gb-open set of X. By Theorem 3.6, X\F contains a p*gb-nbhd of each of its points. Hence there exists a p*gb-nbhd N of x such that N⊆ X\F. Hence N∩F=ϕ.

Definition 3.8: The collection of all p*gb-neighborhoods of x∈X is called the p*gb-neighborhood system of x and is denoted by p*gb-N(x).

Theorem 3.9: Let (X, t) be a topological space and x ∈ X. Then

(i) p*gb-N(x)≠ ϕ and x∈each member of p*gb-N(x)
(ii) If N ∈ p*gb-N(x) and N ⊆ M, then M ∈ p*gb-N(x).
(iii) Each member N ∈p*gb-N(x) is a superset of a member G∈p*gb-N(x) where G is a p*gb-open set.

Proof:
- (i) Since X is p*gb-open set containing x, it is a p*gb-nbhd of every x∈X. Thus for each x∈X, there exists atleast one p*gb-nbhd, namely X. Therefore, p*gb-N(x)≠ ϕ. Let N∈p*gb-N(x). Then N is a p*gb-nbhd of x. Hence there exists a p*gb-open set G such that x∈G ⊆N, so x ∈ N. Therefore x∈every member N of p*gb-N(x).
- (ii) If N p*gb-N(x), then there is a p*gb-open set G such that x∈G⊆N. Since N⊆M, M is p*gb-nbhd of x. Hence M∈p*gb-N(x).
- (iii) Let N∈p*gb-N(x). Then there is a p*gb-open set G, such that x ∈ G⊆N. Since G is p*gb-open and x∈G, G is p*gb-nbhd of x. Therefore G∈p*gb-N(x) and also G⊆N.

4. Pre* generalized b-interior operator

Definition 4.1: Let A be a subset of a topological space (X, τ). Then the union of all p*gb-open sets contained in A is called the p*gb-interior of A and it is denoted by p*gbint(A). That is, p*gbint(A)=∪{V:V⊆A and V∈p*gb-O(X)}.

The union of p*gb-open subsets of X is p*gb-open in X, then p*gbint(A) is p*gb-open in X.

Definition 4.2: Let A be a subset of a topological space X. A point x∈X is called a p*gb-interior point of A if there exists a p*gb-open set G such that x∈G⊆A.

Theorem 4.3: Let A be a subset of a topological space (X, τ). Then
(a) p*gbint(A) is the largest p*gb-open set contained in A.
(b) A is p*gb-open if and only if p*gbint(A)=A.

Proof:
(a) Being the union of all p*gb-open sets, p*gbint(A) is p*gb-open and contains every p*gb-open subset of A. Hence p*gbint(A) is the largest p*gb-open set contained in A.
(b) Necessity: Suppose A is p*gb-open. Then by Definition 4.1, A⊆p*gbint(A). But p*gbint(A)⊆A and therefore p*gbint(A)=A. Sufficiency: Suppose p*gbint(A)=A. Then, p*gbint(A) is p*gb-open set. Hence A is p*gb-open.

Theorem 4.4: Let A be a subset of a topological space (X, τ). Then
(a) p*gbint(A) is the set of all p*gb-interior points of A.
(b) A is p*gb-open if and only if every point of A is a p*gb-interior point of A.

Proof:
(a) Let x∈p*gbint(A)⇔x∈∪{V:V⊆A and V∈p*gb-O(X)}
⇔there exists a p*gb-open set G such that x∈G⊆A.
⇔x is a p*gb-interior point of A.
Hence p*gbint(A) is the set of all p*gb-interior points of A.
(b) Suppose A is p*gb-open. Then by Theorem 4.3(b) and by above part, we have every point of A is the p*gb-interior point of A.

Theorem 4.5: Let A and B be subsets of (X, τ). Then the following results hold.
(a) p*gbint(ϕ)= ϕ and p*gbint(X)=X.
(b) If B is any p*gb-open set contained in A, then B⊆p*gbint(A) .
(c) If A⊆B, then p*gbint(A)⊆p*gbint(B).
(d) int(A)⊆p*gbint(A)⊆A.
(e) p*gbint(p*gbint(A))=p*gbint(A).

Proof:
(a) Since ϕ is the only p*gb-open set contained in ϕ, then p*gbcl(ϕ)= ϕ. Since X is p*gb-open and p*gbint(X) is the union of all p*gb-open sets contained in X, p*gbint(X)=X.
(b) Suppose B is p*gb-open set contained in A. Since p*gbint(A) is the union of all p*gb-open set contained in A, then we have B⊆p*gbint(A).
(c) suppose A⊆ B. Let x∈p*gbint(A). Then x is a p*gb-interior point of A and hence there exists a p*gb-open set G such that x∈G⊆A. Since A⊆B, then x∈G⊆B. Therefore x is a p*gb-interior point of B. Hence x∈p*gbint(B).
(d) Since open set is p*gb-open, int(A)⊆p*gbint(A). Therefore int(A)p*gbint(A)⊆A.
(e) Since p*gbint(A) is p*gb-open and by Theorem 4.3(b), p*gbint(p*gbint(A))=p*gbint(A).

Theorem 4.6: Let A and B are the subsets of a topological space X. Then,
(a) p*gbint(A)∪p*gbint(B)⊆p*gbint(A∪B).
(b) p*gbint(A∩B)⊆p*gbint(A)∩p*gbint(B).

Proof:
(a) Let A and B be subsets of X. We have A⊆A∪B and B⊆A∪B. By Theorem 4.5(c), p*gbint(A)⊆p*gbint(A∪B) and p*gbint(B)⊆p*gbint(A∪B) which implies that, p*gbint(A)∪p*gbint(B)⊆p*gbint(A∪B).
(b) We have A∩B⊆A and A∩B⊆B. Then by Theorem 4.5(c), p*gbint(A∩B)⊆p*gbint(A) and p*gbint(A∩B)⊆p*gbint(B) which implies p*gbint(A∩B)⊆p*gbint(A)∩p*gbint(B).

Theorem 4.7: For any subset A of X,
(a) int(p*gbint(A))=int(A)
(b) p*gbint(int(A))=int(A).

Proof:
(a) Since p*gbint(A)⊆A , then int(p*gbint(A))⊆int(A). By Theorem 4.5(d), int(A)⊆(p*gbint(A)), we have int(A)=int(int(A))⊆int(p*gbcl(A)). Hence int(p*gbint(A))=int(A).
(b) Since int(A) is open and hence p*gb-open, by Theorem 4.3(b), p*gbint(int(A))=int(A).

5. p*gb-closure operator

Definition 5.1: Let A be a subset of a topological space (X, τ). Then the intersection of all p*gb-closed sets in X containing A is called the p*gb-closure of A and it is denoted by p*gbcl(A). That is, p*gbcl(A)=∩{F: A⊆F and F∈p*gb-C(X)}. The intersection of p*gb-closed set is p*gb-closed, then p*gbcl(A) is p*gb-closed.

Theorem 5.2: Let A be a subset of a topological space (X, τ). Then
(a) p*gbcl(A) is the smallest p*gb-closed set containing A.
(b) A is p*gb-closed if and only if p*gbcl(A)=A.

Proof:
(a) Being the intersection of all p*gb-closed sets, p*gbcl(A) is p*gb-closed and contained in every p*gb-closed set containing A. Hence p*gbcl(A) is the smallest p*gb-closed set containing A.
(b) Necessity: Suppose A is p*gb-closed. Then, p*gbcl(A)⊆A. But A⊆p*gbcl(A) and therefore p*gbcl(A)=A. Sufficiency: Suppose p*gbcl(A)=A. Since p*gbcl(A) isa p*gb-closed set, hence A is p*gb-closed.

Theorem 5.3: Let A and B be a two subsets of a topological space (X, τ). Then
(a) p*gbcl(ϕ)= $\phi$and p*gbcl(X)=X.
(b) If B is any p*gb-closed set containing A, then p*gbcl(A)⊆ B.
(c) If A⊆B, then p*gbcl(A)⊆p*gbcl(B).
(d) A⊆p*gbcl(A)⊆cl(A).
(e) p*gbcl(p*gbcl(A))=p*gbcl(A).

Proof:
(a) Since ϕ is p*gb-closed and p*gbcl(ϕ) is the intersection of all p*gb-closed sets containing ϕ, p*gbcl(ϕ)= ϕ. since X is the only p*gb-closed set containing X, then p*gbcl(X)=X.
(b) Suppose B is p*gb-closed set containing A. Since p*gbcl(A) is the intersection of all p*gb-closed set containing A, then p*gbcl(A)⊆ B.
(c) Suppose A⊆ B. Let F be any p*gb-closed set containing B. Since A⊆ B, then A⊆F and hence by (b), p*gbcl(A) ⊆ F. Therefore p*gbcl(A)⊆∩{F/ B⊆ F and F is p*gb-closed}=p*gbcl(B).
(d) Every closed set is p*gb-closed, p*gbcl(A)⊆cl(A). Therefore A⊆p*gbcl(A)⊆cl(A).
(e) p*gbcl(A) is p*gb-closed, by Theorem 5.2(b), p*gbcl(p*gbcl(A))=p*gbcl(A).

Theorem 5.4: Let A and B be subsets of a topological space (X, τ). Then,
(a) p*gbcl(A)∪p*gbcl(B)⊆p*gbcl(A∪B).
(b) p*gbcl(A∩B)⊆p*gbcl(A)∩p*gbcl(B).

Proof:
(a) Let A and B be subsets of X. We have A⊆A∪B and B⊆A∪B. By Theorem 5.3 (c), p*gbcl(A)⊆p*gbcl(A∪B) and p*gbcl(B)⊆p*gbcl(A∪B) which implies that, p*gbcl(A)∪p*gbcl(B)⊆p*gbcl(A∪B).
(b) We have A∩B⊆A and A∩B⊆B. Then by Theorem 5.3(c), p*gbcl(A∩B)⊆p*gbcl(A) andp*gbcl(A∩B)⊆p*gbcl(B) which implies p*gbcl(A∩B)⊆p*gbcl(A)∩p*gbcl(B).

Theorem 5.5: For a subset A of X and x∈X, x∈p*gbcl(A) if and only if V∩A≠ ϕ for every p*gb-open set V containing x.

Proof: Necessity: Let x∈p*gbcl(A). Suppose there is ap*gb-open set V containing x such that V∩A= ϕ. Then A⊆X\V and X\V is p*gb-closed and hence p*gbcl(A)⊆X\V. Since x∈p*gbcl(A), then x∈X\V which contradicts to x∈V.
Sufficiency: Assume that V∩A≠ ϕ for every p*gb-open set V containing x. Suppose x∉p*gbcl(A). Then there exists a p*gb-closed set F such that A⊆F and x∉F. Therefore x∈X\F, A∩(X\F)= ϕ and X\F is p*gb-open. This is a contradiction to our assumption. Hence x∈p*gbcl(A).

Theorem 5.6: For any subset A of X,

(a) cl(p*gbcl(A))=cl(A)
(b) p*gbcl(cl(A))=cl(A).

Proof:

(a) Since A$\subseteq$p*gbcl(A), then cl(A) $\subseteq$cl(p*gbcl(A)). By Theorem 5.3(d), p*gbcl(A)$\subseteq$cl(A), we have cl(p*gbcl(A)) $\subseteq$cl(cl(A)) = cl(A). Hence cl(p*gbcl(A)) = cl(A).
(a) Since cl(A) is closed and hence p*gb-closed, by Theorem 5.2(b), p*gbcl(cl(A)) = cl(A).

REFERENCES

1. Andrijevic D, On b-open sets, Mat.Vesnik 48(1996), 59-64.
2. Levine N, Generalized closed sets in topology, Rand. Circ. Mat. Palermo, 19(2)(1970), 89- 96.
3. Selvi, T, PunithaDharani, A, Some New Class of Nearly Closed and Open Sets, Asian Journal of Current Engineering and Maths, vol. 1(5), pp.305-307 (2012).
4. Aruna Glory Sudha. I, Zion Chella Ruth S, Pre* Generalized b-Closed Sets in Topological Spaces, Journal of Fundamental & Comparative Research, Vol. VIII, Issue-II, No.5(2022), 91-98.

Source of support: Nil, Conflict of interest: None Declared.