International Journal of Mathematical Archive-2(10), 2011, Page: 2051-2054
Available online through www.ijma.info ISSN 2229 – 5046

A FIXED POINT THEOREM SATISFYING GENERAL CONTRACTIVE CONDITION OF INTEGRAL TYPE USING TWO PAIR OF CONVERSE COMMUTING MAPPINGS IN METRIC SPACES

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(Received on: 03-09-11; Accepted on: 16-09-11)

ABSTRACT

In this paper, we use the concept of converse commuting maps of Lu[4], to prove a common fixed point theorem satisfies Meir-Killer Type contractive condition[2]. Our result is generalization of the results of various authors like Valeriu P. [5], Pathak H.K. and Verma R.K. [6].

Key words*: Commuting point, Coincidence point, Converse commuting maps.*

Subject classification*: 2000 AMS: 47H10, 54H25*

In 2002, Lu[4] introduced the concept of converse commuting maps as a reverse process of weakly compatible maps. Here we use the concept of converse commuting maps of Lu[4], to prove a common fixed point theorem satisfies Meir-Killer type contractive condition[2]. Our result is generalization of the results of various authors like Valeriu P.[5] , Pathak H. K. and Verma R. K.[6].

Recently, Pathak H.K. and Verma R.K. [6] proved the following result:

Theorem 1[6]: Let A, B, S and T be self maps defined on a metric space (X, d) satisfying the following conditions:

(a) the pairs (A, S) and (B, T) are conversely commuting, and
(b) the generalized contractive condition :

$$G\left(\int_0^{d(Ax,By)}\varphi(t)dt,\int_0^{d(Sx,Ty)}\varphi(t)dt,\int_0^{d(Ax,Sx)}\varphi(t)dt,\int_0^{d(By,Ty)}\varphi(t)dt,\int_0^{d(By,Sx)}\varphi(t)dt,\int_0^{d(Ax,Ty)}\varphi(t)dt,\right)\le 0$$

holds, for all $x, y\in X$ and t > 0 where ϕ : $R^+ \to R$ is a Lebesgue-integrable mapping which is summable, non-negative and such that $\int_0^{\in}\phi(t)dt > 0$ for each $\in$ > 0, and $G: R_+^6 \to R$ be a map satisfying

G(s, s, 0, 0, s, s) > 0, for all s > 0.

If A and S have a commuting point and B and T have a commuting point, then A, S, B and T have a unique common coincidence point.

Now, we use the concept of converse commuting maps of Lu [4], to prove a common fixed point theorem satisfies Meir-Killer Type contractive condition[2], which generalizes Theorem-1 as follows:

Theorem 2: Let A, B, S and T be self maps defined on a metric space (X, d) satisfying the following conditions:

(a) The pairs (A, S) and (B, T) are conversely commuting, and
(b) Given $\in > 0$, there exists $\delta > 0$ such that for all $x, y\in X$

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$\in < \int_0^{M(x,y)} \phi(t)\,dt < \in + \delta$ implies $\int_0^{d(Ax,By)} \phi(t)\,dt \le \in$

and for all $x, y \in X$, $k \in \left[0, \frac{1}{3}\right]$ such that

$$\int_0^{d(Ax,By)} \phi(t)\,dt < k \int_0^{[d(Sx,Ty)+d(Ax,Sx)+d(By,Ty)+d(Sx,By)+d(Ax,Ty)]} \phi(t)\,dt$$

where ϕ : $R^+ \to R$ is a Lebesgue-integrable mapping which is summable, non-negative and such that $\int_0^{\in} \phi(t)\,dt > 0$ for each $\in > 0$,

and $M(x, y) = Max\left\{ d(Sx,Ty), d(Ax,Sx), d(By,Ty), \frac{[d(Sx,By) + d(Ax,Ty)]}{2} \right\}$

If (A, S) and (B, T) have a commuting point, then A, S, B and T have a unique common coincidence point.

Proof: Let u be commuting point of (A, S) and v be commuting point of (B, T). As A and S are converse commuting we have ASu = SAu $\Rightarrow$ Au = Su. Hence d(Au, Su) = 0. It follows that ASu = SAu = AAu = SSu. Similarly, as B and T are converse commuting we have BTv = TBv $\Rightarrow$ Bv = Tv, hence d(Bv, Tv) = 0. It follows that BTv = TBv = TTv = BBv.

We claim that AAu = BBv. If not, take x = Au, y = Bv in condition (b), we have

$$\int_0^{d(AAu,BBv)} \phi(t)\,dt < k \int_0^{[d(SAu,TBv)+d(AAu,SAu)+d(BBv,TBv)+d(SAu,BBv)+d(AAu,TBv)]} \phi(t)\,dt$$

$$\int_0^{d(AAu,BBv)} \phi(t)\,dt < k \int_0^{[d(AAu,BBv)+d(AAu,AAu)+d(BBv,BBv)+d(AAu,BBv)+d(AAu,BBv)]} \phi(t)\,dt$$

$$= k \int_0^{3d(AAu,BBv)} \phi(t)\,dt$$

$$< 3k \int_0^{d(AAu,BBv)} \phi(t)\,dt \qquad (1)$$

which is a contradiction, since $k \in \left[0, \frac{1}{3}\right]$.

Hence from (1), we have AAu = BBv. Therefore AAu = SAu = ASu = SSu = BTv = TBv = BBv = TTv.

Now, we claim that Au = Bv. If not, then put x = u, y = v in condition (b), we have

$$\int_0^{d(Au,Bv)} \phi(t)\,dt < k \int_0^{[d(Su,Tv)+d(Au,Su)+d(Bv,Tv)+d(Su,Bv)+d(Au,Tv)]} \phi(t)\,dt$$

$$\int_0^{d(Au,Bv)} \phi(t)\,dt < k \int_0^{[d(Au,Bv)+d(Au,Au)+d(Bv,Bv)+d(Au,Bv)+d(Au,Bv)]} \phi(t)\,dt$$

$$= k\int_0^{3d(Au,Bv)}\phi(t)\,dt$$

$$< 3k\int_0^{d(Au,Bv)}\phi(t)\,dt \tag{2}$$

which is a contradiction, since $k\in\left[0,\frac{1}{3}\right]$.

Hence from (2), Au = Bv. So that Au = Su = Bv = Tv.

Now, we claim that Au = AAu. If not, then put x = Au, y = v, in condition (b), we have

$$\int_0^{d(AAu,Bv)}\phi(t)dt < k\int_0^{[d(SAu,Tv)+d(AAu,SAu)+d(Bv,Tv)+d(SAu,Bv)+d(AAu,Tv)]}\phi(t)dt$$

$$\int_0^{d(AAu,Bv)}\phi(t)dt < k\int_0^{[d(AAu,Bv)+d(AAu,AAu)+d(Bv,Bv)+d(AAu,Bv)+d(AAu,Bv)]}\phi(t)dt$$

$$= k\int_0^{3d(AAu,Bv)}\phi(t)dt$$

$$< 3k\int_0^{d(AAu,Bv)}\phi(t)dt \tag{3}$$

which is a contradiction, since $k\in\left[0,\frac{1}{3}\right]$.

Hence from (3), we have Au = AAu.

Therefore, Au = AAu = SAu = ASu = SAu = Bv = BBv = TBv = BTv = TTv.

Hence Au is a common fixed point of A, B, S, and T.

Finally now, we show that the common fixed point is unique. If possible, let x_0 and y_0 be two common fixed points of A, B, S, and T. Then by condition (b), take $x = x_0$ and $y = y_0$, we have

$$\int_0^{d(Ax_0,By_0)}\phi(t)dt < k\int_0^{[d(Sx_0,Ty_0)+d(Ax_0,Sx_0)+d(By_0,Ty_0)+d(Sx_0,By_0)+d(Ax_0,Ty_0)]}\phi(t)dt$$

$$\int_0^{d(x_0,y_0)}\phi(t)dt < k\int_0^{[d(x_0,y_0)+d(x_0,x_0)+d(y_0,y_0)+d(x_0,y_0)+d(x_0,y_0)]}\phi(t)dt$$

$$\int_0^{d(x_0,y_0)}\phi(t)dt < k\int_0^{[d(x_0,y_0)+d(x_0,y_0)+d(x_0,y_0)]}\phi(t)dt$$

$$= k\int_0^{3d(x_0,y_0)}\phi(t)dt$$

$$< 3k\int_0^{d(x_0,y_0)}\phi(t)dt \tag{4}$$

which is a contradiction, since $k \in \left[0, \frac{1}{3}\right]$.

Hence from (4), $x_0 = y_0$.

Therefore, the mappings A, B, S, and T have a unique common fixed point.

Example 2.1: Let X = $\left\{0, 1, \frac{1}{2}, \frac{1}{3}, \ldots\right\}$ and d is a usual metric d(x, y) = $|x - y|$. Define mappings A, S, B, T: X →X by

$Ax = \frac{1}{n+3}, x = \frac{1}{n}$ (n is odd), $Ax = \frac{1}{n+4}, x = \frac{1}{n}$ (n is even), A(0) = 0,

$Sx = \frac{1}{n+2}, x = \frac{1}{n}$ (n is odd), $Sx = \frac{1}{n+1}, x = \frac{1}{n}$ (n is even) , S(0) = 0,

$Bx = \frac{1}{n+4}, x = \frac{1}{n}$ (n is odd), $Bx = \frac{1}{n+3}, x = \frac{1}{n}$ (n is even), B(0) = 0,

$Tx = \frac{1}{n+1}, x = \frac{1}{n}$ (n is odd), $Tx = \frac{1}{n+2}, x = \frac{1}{n}$ (n is even) , T(0) = 0.

Next, define $\varphi(t)$ = max {0, $t^{(\frac{1}{t})-2}$ (1-logt)} for t > 0, $\varphi(0) = 0$.

Clearly all the conditions of above theorem and condition (b) satisfied for k = $\frac{1}{3}$. Also x = 0 is unique common fixed point of A, S, B and T.

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