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EFFECT OF THICKNESS OF THE POROUS MATERIAL ON THE PERISTALTIC PUMPING OF A JEFFRY FLUIDWITH NON-ERODIBLE POROUS LINING WALL

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ABSTRACT

This paper is devoted to study the effect of thickness of porous material on the peristaltic pumping of a Jeffry fluid when the tube wall is provided with non- erodible porous lining. Long wavelength and low Reynolds number approximation is used to linearize the governing equations. The expression for axial velocity, pressure gradient and frictional force are obtained by using Beavers-Joseph Boundary conditions. The effect of various parameters on pumping characteristics is discussed with the help of graphs.

Keywords: *Peristaltic pumping, Jeffry fluid, volume flow rate, pressure rise, Darcy's law.*

1. INTRODUCTION:

Peristalsis is known as a form of fluid transport that is used by many systems in the living body to propel or to mix the contents of a tube such as, in transport of urine from the kidney through the uerter to the bladder, food through the digestive tract, bile from the gall bladder in to the duodenum, moment of ovum in fallopian tube. This mechanism of fluid transport has received considerable attention in recent years in physiological science as well as Engineering.

The idea of peristaltic transport in mathematical point of view was first coined by Latham [8]. The initial mathematical model of peristalsis obtained by train of sinusoidal waves in an infinitely long symmetric channel or tube has been investigated by Shapiro et.al [13] and Fung and Yahi [2]. After these investigations, many authors have studied the peristaltic pumping for Newtonian and non- Newtonian fluids in different situations.

Peristaltic transport through uniform and non uniform annulus has been studied by Rathod and Asha [10]. A study of ureteral peristalsis in cylindrical tube through porous medium is made by Rathod and Channakote [11]. Rao and Rajagopal [9] analyzed some simple flow of Johnson- Segalman fluid. Vajarvelu et al. [17] studied the peristaltic transport of a Hershel- Bulkeley fluid in an inclined tube.

The importance of the study of peristaltic flow of non - Newtonian fluids is widely observed in industry and physiology. Hayat et.al [4-5] investigated the peristaltic transport of viscoelastic fluid with Jeffry model and they have also discussed the relaxation and retardation time on the peristaltic transport. Peristaltic transport of a Jeffry fluid under the effect of slip in an inclined asymmetric channel has been discussed by Srinivas and Mutturaja [15]. Krishna kumari et.al [6] studied the peristaltic pumping of a Jeffry fluid under the effect of magnetic field in an inclined channel. Recently, the effect of heat transfer on peristaltic flow of Jeffry fluid through porous medium in vertical annulus has been studied by Vasudev et. al [16]. The influence of heat transfer on peristaltic transport of Jeffry fluid in a vertical porous stratum has been studied by Vajarvelu et. al [18]. Krishna kumari [7] studied Peristaltic pumping of a Jeffry fluid in a porous tube. The influence of wall properties on MHD peristaltic transport of dusty fluid is studied by Rathod et.al. [20].

An experimental study of fluid flow at the interface between a porous medium and fluid layer with slip boundary condition was first investigated by Beavers and Joseph [1]. Vijayaraj et. al [19] studied the Peristaltic pumping of a fluid of variable viscosity in a non-uniform tube with permeable wall. Ravi Kumar et.al [12] studied the unsteady peristaltic pumping in a finite length tube with permeable wall. Sobh et.al [14] studied heat transfer in peristaltic Flow of viscoelastic Fluid in an Asymmetric Channel. Effect of thickness of the porous material on the peristaltic pumping when the tube wall is provided with non-erodible porous lining has been studied by Hemadri et.al [3].

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In this paper, our concern is to investigate the peristaltic transport of a simplest linear non-Newtonian model namely the Jeffry fluid model in a tube of radius 'a' and the wall of the tube is lined with non-erodible porous material of thickness h^1. The Navier stokes equations are governed by the free flow past the porous material and the flow in the permeable wall is described by Darcy's law. The axial velocity distribution, the stream function, the volume flow rate, the pressure rise and the frictional force are calculated. The effect of thickness of porous lining on the pumping characteristics is discussed.

2. MATHEMATICAL FORMULATION:

Consider the peristaltic flow of an incompressible Jeffry fluid in tube of radius 'a'. The wall of the tube is lined with porous material of permeability 'k'. The thickness of the porous lining is h^1. The axisymmetric flow in the porous lining is governed by Navier-Stokes equation. The flow in the porous layer is according to Darcy's law. In a cylindrical coordinate system ($\bar{r}$, $\bar{z}$) the dimensional equation for the tube radius for an infinite wave train is represented by

$$R = H(Z,t) = a + bSin\frac{2\pi}{\lambda}(Z - ct) \tag{2.1}$$

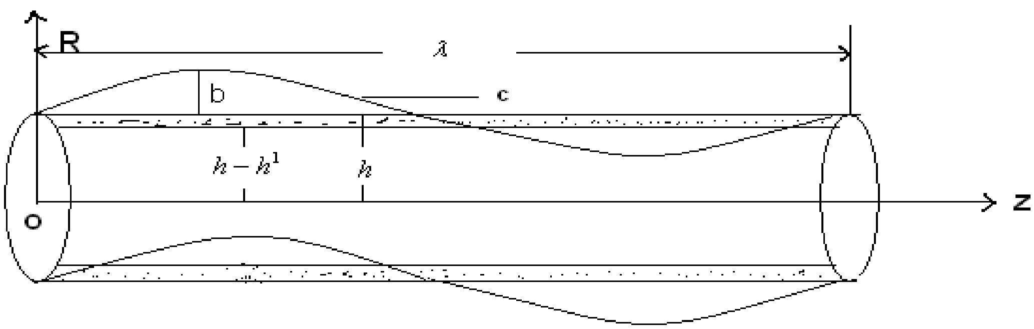

Fig1. Physical Model

The transformation from fixed frame to wave frame is given by

$$r = R; \qquad z = Z - ct; \qquad p(z) = P(Z,t); \qquad \psi = \Psi - \frac{R^2}{2} \tag{2.2}$$

where ψ and Ψ are stream functions in the wave and laboratory frames respectively. We assume that the flow is inertia- free and the wavelength is infinite.

The Constitutive equation for an incompressible Jeffry fluid is

$$S = \frac{\mu}{1+\lambda_1}(\dot{\gamma} + \lambda_2\,\ddot{\gamma}) \tag{2.3}$$

where μ is the dynamic viscosity, λ_1 is the ratio of relaxation to retardation times, λ_2 is the retardation time, $\dot{\gamma}$ is the shear rate and dots over the quantities denote differentiation.

In the wave frame, the equations governing the flow are

$$\rho\left(u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r}(rS_{rr}) + \frac{\partial}{\partial z}(S_{rz}) \tag{2.4}$$

$$\rho\left(u\frac{\partial w}{\partial r} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(rS_{rz}) + \frac{\partial}{\partial z}(S_{zz}) \tag{2.5}$$

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{2.6}$$

where p is the pressure and ρ is the density of the fluid.

Now introducing the following non- dimensional variables

$$\bar{r}=\frac{r}{a},\quad \bar{z}=\frac{z}{\lambda},\ \varepsilon=\frac{h^{1}}{\lambda},\ \ \bar{u}=\frac{u}{c\delta},\quad \bar{w}=\frac{w}{c},\quad \bar{p}=\frac{a^{2}}{\lambda\mu_1 c},\quad \bar{w}_B=\frac{w_B}{c},$$

$$Da=\frac{k}{a^{2}},\quad \bar{u}=-\frac{1}{r}\frac{\partial\bar{\psi}}{\partial\bar{z}},\quad \bar{w}=-\frac{1}{r}\frac{\partial\bar{\psi}}{\partial\bar{r}},\quad \bar{S}=\frac{aS}{\mu_1 c}$$

where $\bar{u}$ and $\bar{w}$ are the radial and axial velocities in the wave frame. Now the governing equations of motion (2.4), (2.5) and (2.6) becomes (dropping the bars),

$$\operatorname{Re}\delta^{3}\left(u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial r}+\frac{\delta}{r}\frac{\partial}{\partial r}(r\,S_{rr})+\delta^{2}\frac{\partial}{\partial z}(S_{zz}) \tag{2.7}$$

$$\operatorname{Re}\delta\left(u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}(r\,S_{rr})+\delta\frac{\partial}{\partial z}(S_{zz}) \tag{2.8}$$

$$\frac{\partial u}{\partial r}+\frac{\mu}{r}+\frac{\partial w}{\partial z}=0$$

where,

$$S_{rr}=\frac{2\delta}{1+\lambda_1}\left[1+\frac{\lambda_2\delta}{a}\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)\right]\frac{\partial u}{\partial r},$$

$$S_{rz}=\frac{1}{1+\lambda_1}\left[1+\frac{\lambda_2\delta}{a}\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)\right]\left(\frac{\partial w}{\partial r}+\delta^{2}\frac{\partial u}{\partial z}\right),$$

and $S_{zz}=\dfrac{2\delta}{1+\lambda_1}\left[1+\dfrac{\lambda_2\delta}{a}\left(u\dfrac{\partial}{\partial r}+w\dfrac{\partial}{\partial z}\right)\right]\dfrac{\partial w}{\partial z}$

Using the long wavelength approximation $(\delta<<1)$ and low Reynolds number (Re $\rightarrow$0), the equation (2.7) and (2.8) becomes

$$\frac{\partial p}{\partial r}=0 \tag{2.9}$$

$$\frac{\partial p}{\partial z}=\frac{1}{(1+\lambda_1)}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) \tag{2.10}$$

The dimensionless boundary conditions are

$$\frac{\partial w}{\partial r}=0,\quad at\quad r=0 \tag{2.11}$$

$$w=-1+w_B\quad at\quad r=h-\varepsilon \tag{2.12}$$

$$\frac{\partial w}{\partial r}=\frac{\alpha}{\sqrt{Da}}(w_B-Q)\quad at\quad r=h-\varepsilon \tag{2.13}$$

where $Q=\dfrac{Da}{\mu}\dfrac{\partial p}{\partial Z}$

3. SOLUTION:

Solving the equation (2.9) and (2.10) subject to the boundary conditions (2.11) - (2.13), we get the velocity as

$$w=\frac{p\left(1+\lambda_1\right)}{4}\left[r^2-(h-\varepsilon)^2+2(h-\varepsilon)\frac{\sqrt{Da}}{\alpha}-4\frac{Da}{\mu}\right]-1 \tag{3.1}$$

where, $p=\dfrac{dp}{dz}$,

Integrating the equation (3.1) and subjected to the boundary conditions $\psi=0\ \ at\ \ r=0$, we get the stream function as,

$$\psi=\frac{ph^2\left(1+\lambda_1\right)}{4}\left[\frac{r^2}{4h^2}-\frac{r^2}{2}\left\{\left(1-\frac{\varepsilon}{h}\right)^2-\frac{2}{h}\left(1-\frac{\varepsilon}{h}\right)\frac{\sqrt{Da}}{\alpha}-\frac{4Da}{\mu}\right\}\right]-\frac{r^2}{2} \tag{3.2}$$

The volume flux q through each cross section in the wave frame is given by

$$q=2\int_0^{h-\varepsilon} w\,r\,dr \tag{3.3}$$

$$q=-\frac{p\,(1+\lambda_1)(h-\varepsilon)^4}{8}\left[1-4\frac{\sqrt{Da}}{(h-\varepsilon)\alpha}+\frac{8Da}{\mu(h-\varepsilon)^2}\right]+\frac{(h-\varepsilon)^2}{2} \tag{3.4}$$

The instantaneous volume flow rate Q (z, t) in the laboratory frame between the centre line and the wall is

$$Q(Z,t)=2\int_0^{h-\varepsilon}(w+1)\,r\,dr \tag{3.5}$$

$$=-\frac{p\,(1+\lambda_1)(h-\varepsilon)^4}{8}\left[1-\frac{4\sqrt{Da}}{(h-\varepsilon)\alpha}+\frac{8Da}{\mu(h-\varepsilon)^2}\right] \tag{3.6}$$

From equation (3.6), we have

$$\frac{dp}{dz}=\frac{-8\left[q+(h-\varepsilon)^2\right]}{(h-\varepsilon)^4(1+\lambda_1)\left(1-4\dfrac{\sqrt{Da}}{(h-\varepsilon)\alpha}+\dfrac{8Da}{\mu(h-\varepsilon)^2}\right)} \tag{3.7}$$

Averaging equation (3.5) over one period yields the time mean flow (time averaged volume flow rate) $\bar{Q}$ as

$$\bar{Q}=\frac{2}{T}\int_0^T\int_0^{h-\varepsilon}(w+1)\,r\,dr\,dt=q+\frac{1}{T}\int_0^T(h-\varepsilon)^2\,dt \tag{3.7}$$

$$=q+(1-\varepsilon)^2+\frac{\phi^2}{2}$$

4. THE PUMPING CHARACTERISTIC:

Integrating the equation (3.7) with respect to z over one wavelength, we get the pressure rise (drop) over one cycle of the wave as

$$\Delta p=\int_0^1 \frac{-8\left[\left(\bar{Q}-(1-\varepsilon)^2-\frac{\phi^2}{2}\right)+(h-\varepsilon)^2\right]}{(h-\varepsilon)^4(1+\lambda_1)\left(1-\frac{4\sqrt{Da}}{(h-\varepsilon)\alpha}+\frac{8Da}{\mu(h-\varepsilon)^2}\right)}dz \tag{4.1}$$

The pressure rise required to produce zero average flow rate is denoted by Δp_0 and is given by

$$\Delta p_0=\int_0^1 \frac{dp}{dz}dz \tag{4.2}$$

It is observed that as ($\lambda \rightarrow 0$), equation (3.1), (3.2), (3.7) and (4.1) are reduces to the corresponding results of Hemadri et. al [3].

The dimensionless frictional force F at the wall across one wavelength in the tube is given by

$$F=\int_0^1 (h-\varepsilon)^2\left(-\frac{dp}{dz}\right)dz \tag{4.3}$$

5. RESULTS AND DISCUSSIONS:

In order to see the effect of various pertinent parameters such as the thickness of porous lining (ε), amplitude ratio (ϕ), ratios of viscosities in the free flow region and porous region (μ), Darcy number Da, slip parameter α , and Jeffry fluid λ on pumping characteristics we have plotted Figs. 2-13

Fig.2. shows the variation of pressure rise Δp with average flow rate Q for different values of ε with ϕ = 0.7, Da = 0.02, μ = 0.2, α = 0.6, λ = 0.8. It is found that, larger the thickness of porous lining, greater the pressure rise against which the pump works.

The variation of pressure rise Δp with average flow rate Q for different values of Da with ϕ = 0.7, ε = 0.01, μ = 0.2, α = 0.6, λ = 0.8 is presented in Fig. 3. Form this figure it is observed that, in pumping region the time averaged flow rate Q decreases with increasing Darcy number. Further, it is noted that larger the Darcy number, smaller the pressure rise.

The effect of amplitude ratio ϕ on the pumping performance is shown in Fig.4, it is noted that, larger the amplitude ratio, greater the pressure rise against which the pump works. For given Δp the flux Q for Jeffry fluid in tube depends on ϕ and it increases with increase in ϕ.

Fig.5. depicts the variation of μ on pumping characteristics with ϕ = 0.7, Da = 0.02, ε = 0.01, α = 0.6, λ = 0.8. It is seen that, an increase in μ increases the pressure rise Δp against which the pumping works. In addition, it is noted that the flux Q increases with increase of μ .

The variation of pressure rise Δp with average flow rate Q for different values of slip parameter α with ϕ = 0.7, ε = 0.01, μ = 0.2, Da = 0.02, λ = 0.8 is presented in Fig. 6. It is observed that, an increase in the value of slip parameter α , decreases the pressure rise Δp.

Fig.7. shows the relation between the pressure rise Δp and averaged flux Q for different value of λ with ϕ = 0.7, ε = 0.01, μ = 0.2, Da = 0.02, α = 0.6. It is found that the pumping rate decreases with increase of λ for pumping $(\Delta p > 0)$ and free pumping region $(\Delta p = 0)$. Also, it is noticed that the pumping is more for Newtonian fluid $(\lambda \rightarrow 0)$ than that of Jeffry fluid.

The variation of friction force F with averaged flow rate Q under the influence of all emerging parameters such as λ, Da, ε, μ, α, ϕ, it is observed that the effect of all the parameters on friction force are opposite to the effects on pressure with the averaged flow rate is observed in figs. 8-13

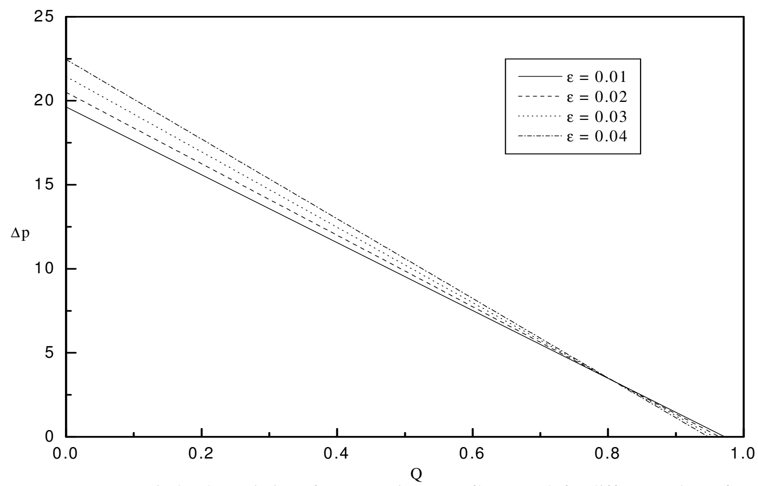

Fig.2. The variation of pressure rise Δp vs flow rate Q for different values of ε at $\phi = 0.7$, Da = 0.02, $\mu = 0.2$, $\alpha = 0.6$, $\lambda = 0.8$

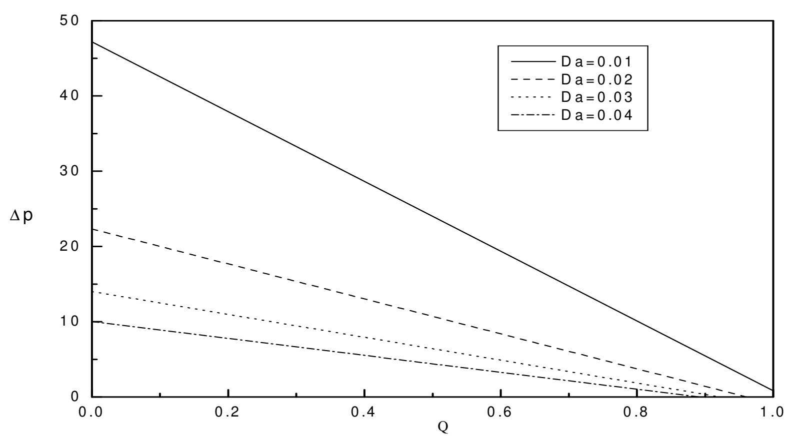

Fig.3. The variation of pressure rise Δp vs flow rate Q for different values of Da at $\phi = 0.7$, $\mu = 0.2$, $\alpha = 0.5$, $\varepsilon = 0.01$, $\lambda = 0.8$

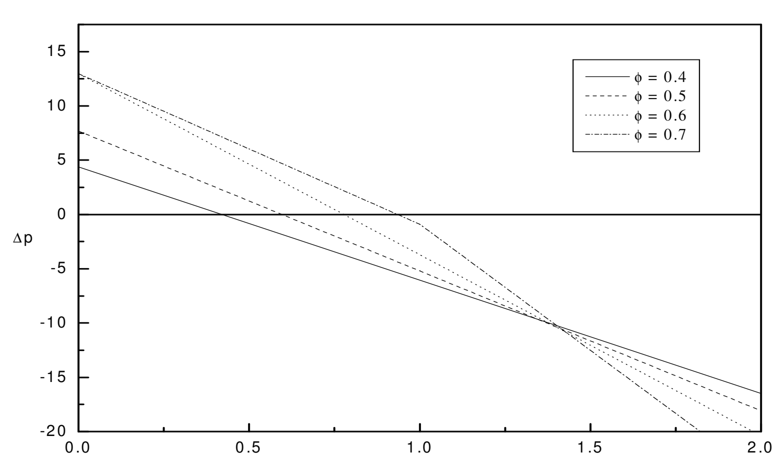

Fig. 4.The variation of pressure rise Δp vs. flow rate Q for different values of ϕ at $\mu = 0.2$, Da = 0.02, $\varepsilon = 0.01$, $\lambda = 0.8$, $\alpha = 0.6$

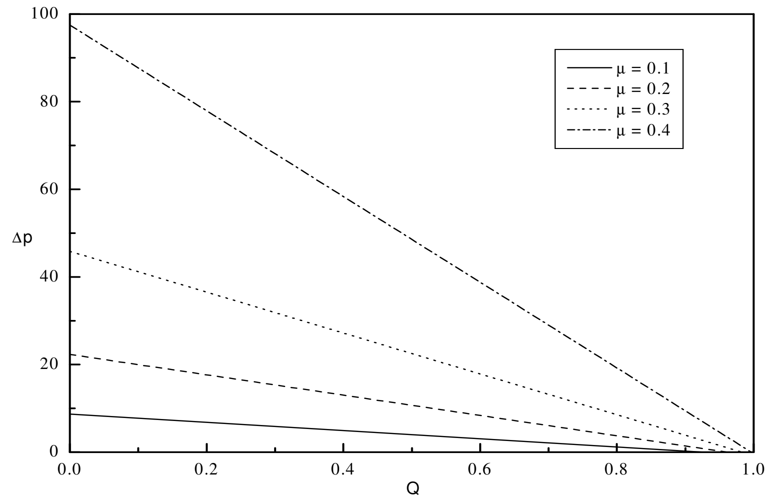

Fig.5.Thevariation of pressure rise Δp vs flow rate Q for different values of μ at $\phi = 0.7$, $\varepsilon = 0.01$, $Da = 0.02$, $\alpha = 0.6$, $\lambda = 0.8$

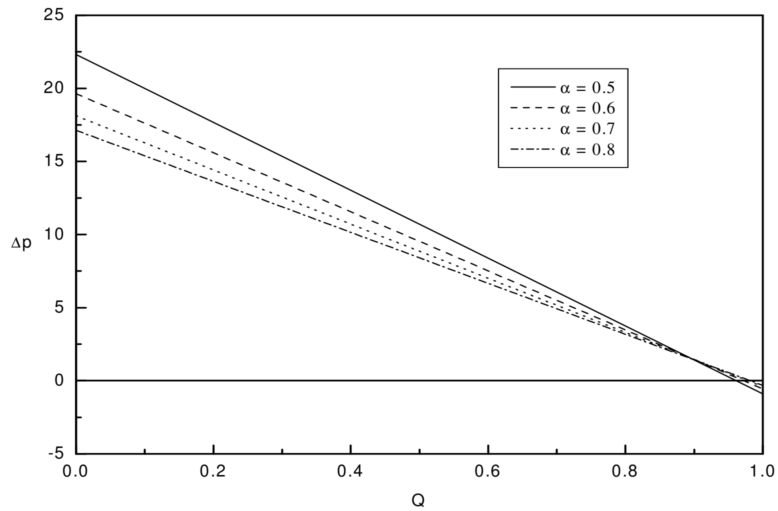

Fig. 6. The variation of pressure rise Δp vs flow rate Q for different values of α at $\phi = 0.7$, $\varepsilon = 0.01$, $\mu = 0.2$, $Da = 0.02$, $\lambda = 0.8$

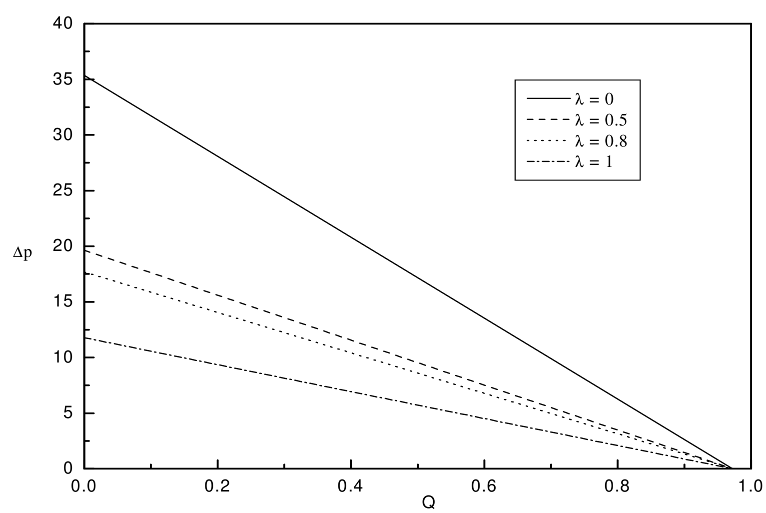

Fig.7. Thevariation of pressure rise vs flow rate Q for different values of λ at $\phi = 0.7$, $Da = 0.02$, $\varepsilon = 0.01$, $\alpha = 0.6$, $\mu = 0.2$

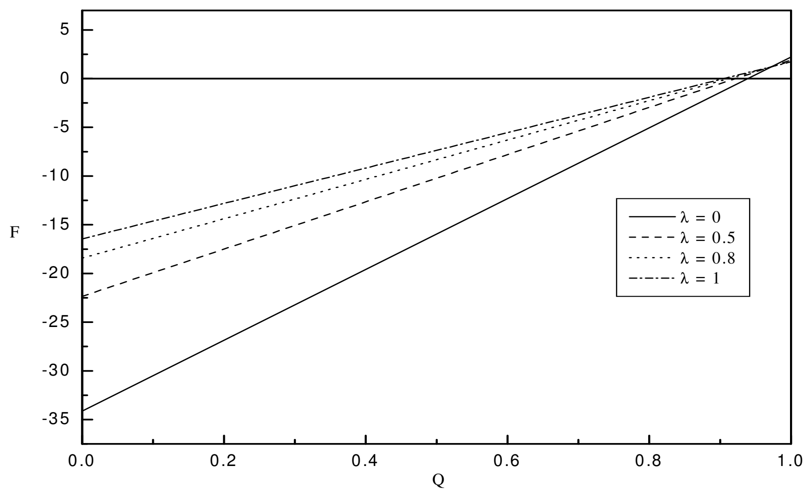

Fig.8. The variation of friction force F vs flow rate Q for different values of λ at $\phi = 0.7$, $D = 0.02$, $\mu = 0.2$, $\alpha = 0.6$, $\varepsilon = 0.01$.

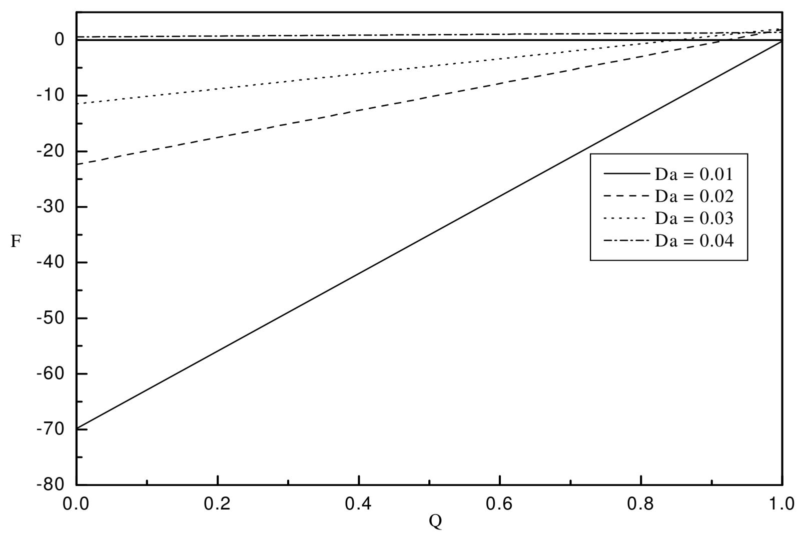

Fig.9. The variation of friction force F vs flo rate Q for different values of Da at $\phi = 0.7$, $\alpha = 0.6$, $\mu = 0.2$, $\varepsilon = 0.01$, $\lambda = 0.8$

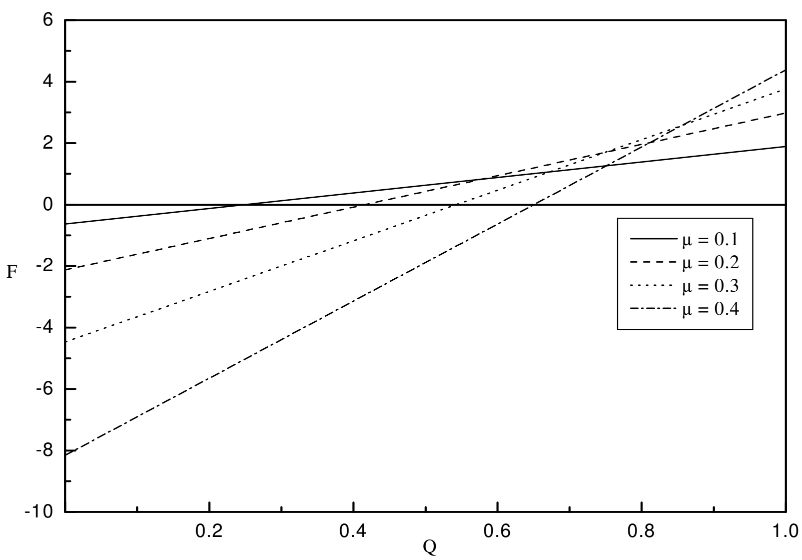

Fig.10. The variation of friction force F vs flow rate Q for different values of μ at $\phi = 0.7$, $\lambda = 0.8$, $\varepsilon = 0.01$, $Da = 0.02$, $\alpha = 0.6$.

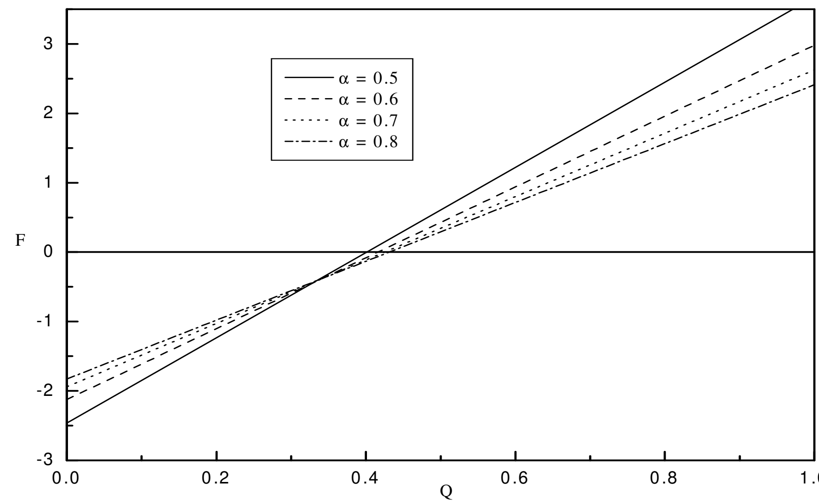

Fig.11. The variation of friction force F vs flow rate Q for different values of α at ϕ = 0.7, λ = 0.8, ε = 0.01, μ = 0.2

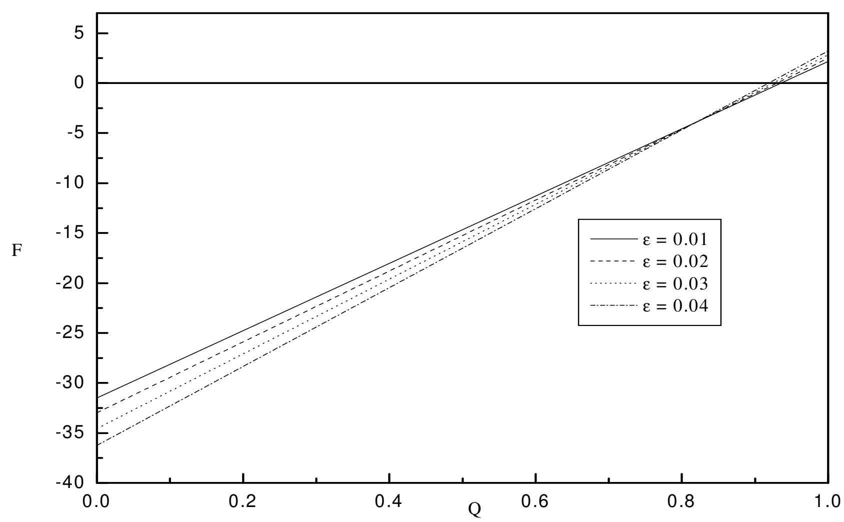

Fig.12. The variation of friction force F vs flow rate Q for different values ε at ϕ = 0.7, λ = 0.8, μ = 0.2, α = 0.6. Da = 0.02

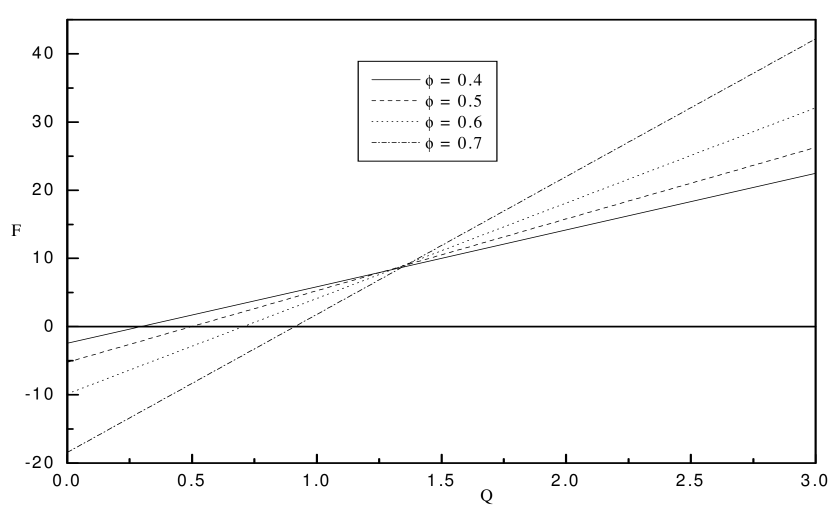

Fig.13. The variation of friction force F vsflow rate Q for different value of λ at ϕ = 0.7, ε =0.01, μ = 0.2, Da = 0.02, α = 0.6

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